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VARDANYAN, Anna

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DEVELOPMENT AND APPLICATION OF FLAT SHELL ELEMENT IN HYBRID-TYPE PENALTY METHOD

Anna VARDANYAN
Chief Examiner: Doctor Professor Norio TAKEUCHI
Assistant Examiners: Yutaka TANAKA, Hideaki HAMADA

Graduate School of Engineering and Design, Hosei University

The purpose of this study is to develop an algorithm and a shell model that utilizes the advantages of HPM, for material nonlinear large displacement analysis using the shell model. First, the displacement field represented by the local coordinate system, which is different for each element, is converted into the local coordinate system of one of the adjacent elements. Subsequently, it is converted into the coordinate system of the element boundary edge. Finally, we propose a method to calculate the relative displacement from the displacement of the boundary edge of the adjacent elements obtained in this manner. In this study, we propose a numerical algorithm of material nonlinear large displacement analysis method using flat plate shell elements in HPM. First, we describe the flat plate elements of HPM using the local coordinate system. Even with a flat plate, adjacent elements have an angle when a large displacement occurs. Next, we propose the relative displacement in this case. The "step by step method" is used as the algorithm for large displacement analysis. Finally, the accuracy of the solution of the proposed method is verified by a simple numerical examples.

. ***Key Words** : Large Displacement, Hinge, Flat Shell Element, Hybrid-type Penalty Method*

1. Introduction

Simulation of a series of phenomena in which fracture progresses from an elastic state, forms a collapse mechanism, and then moves discretely is called multi-stage fracture simulation (MSFS) [1]. In brittle materials, large displacement states, mainly rigid body displacement, often occur under the load state that forms the collapse mechanism. Therefore, in MSFS, large displacement analysis is required as a numerical method under the load state that forms the collapse mechanism.

On the other hand, it is convenient to use a plate element or a flat shell element for thin plate fracture analysis such as glass plate fracture. However, a flat plate initially represented by a flat surface will have a curved surface when it is in a large displacement state. Therefore, in MSFS, the use of plate elements is not appropriate, and it is necessary to use flat shell elements.

In addition, the model order reduction (MOR) method [2] has attracted attention in the field of stress analysis, which improves the efficiency of processing calculations by reducing the number of dimensions of the model. For example, numerical results can be obtained in a short time by simplifying the computation process with low-dimensional elements, such as flat shell elements. Flat shell elements can also be conveniently used to analyze other problems in sheet steel conditions, such as stress analysis of sheet glass.

Even in relation to the MOR mentioned at the beginning, a method of using an interface element for the deformed connection has been proposed [3]. In recent years, models used in isogeometric analysis [4] have been developed [5][6], and methods such as the isogeomeytric inverse finite element method have also been studied [7].

Generally, in order to evaluate the safety of a structure, it is necessary to understand the destruction state and collapse load of the structure. To solve these problems, a hybrid-type penalty method (HPM) has been proposed [8]-[11]. In the HPM, an independent displacement field is assumed for each element. This displacement field is composed of rigid body displacement, strain, and its gradient. This is suitable for the analysis of large displacement problems in which rigid body displacement is predominant.

The continuity of displacement between elements is approximated using a penalty function. Furthermore, the surface force is obtained from the relative displacement along the boundary between adjacent elements. By applying the fracture condition to the penalty function on the element boundary using this surface force, it is possible to introduce the fracture phenomenon such as slip, crack, and hinge.

2. Brief Formulation of Flat Shell Element

The deformed state in the flat shell problem is shown in Fig. 1. Horizontal displacement in the flat shell problem is the sum of displacement u_1 of in-plane deformation and displacement $-z(\partial w_1/\partial x)$ of out-of-plane deformation as shown.

The deformation of the flat shell problem is expressed by the sum of the plane stress and plate bending deformation.

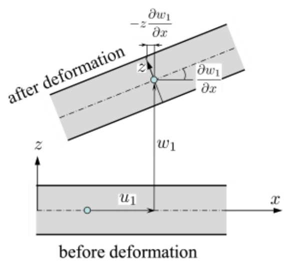


Fig.1 Flat shell deformation

This relation is expressed as follows:

$$\begin{aligned} u &= u_1 - z\frac{\partial w_1}{\partial x} \\ v &= v_1 - z\frac{\partial w_1}{\partial y} \\ w &= w_1 \end{aligned} \tag{1}$$

Hybrid-type virtual work equation with respect to the *M* subdomain and *N* intersection boundary is the following:

$$\begin{aligned} &\sum_{e=1}^{M} \delta W^{(e)} + \sum_{s=1}^{N} H_{<s>} = \\ &\sum_{e=1}^{M} \left(\int_{\Omega^{(e)}} \boldsymbol{\sigma} : \mathrm{grad}\delta\boldsymbol{u}\, dV - \int_{\Omega^{(e)}} \boldsymbol{f} \cdot \delta\boldsymbol{u} dV - \int_{\Gamma^{(e)}} \boldsymbol{t} \cdot \delta\boldsymbol{u}\, dS \right) \\ &- \sum_{s=1}^{N} \left(\delta \int_{\Gamma_{<s>}} \boldsymbol{\lambda} \cdot (\boldsymbol{u}^{(a)}_{<ab>} - \boldsymbol{u}^{(b)}_{<ab>})\, dS \right) = 0 \quad {}^{\forall}\delta\boldsymbol{u} \in \mathbb{V} \end{aligned} \tag{2}$$

In HPM, the discretization equation is derived based on the hybrid virtual work equation of equation (3).

$$\sum_{e=1}^{M} \delta W^{(e)} + \sum_{s=1}^{N} H_{<s>} = 0 \tag{3}$$

Here, *M* is the number of elements and *N* is the number of element boundary edges. Here, (e) represents the *e* element $\Omega^{(e)}$. Also, <ab> represents the common boundary $\Gamma_{<ab>} := \partial\Omega^{(a)} \cap \partial\Omega^{(b)}$ of adjacent elements.

The first term represents the virtual work formula, and the second term represents the formula for subsidiary conditions, as shown below.

$$\delta W^{(e)} = \int_{\Omega^{(e)}} \boldsymbol{\sigma} : \mathrm{grad}\delta\boldsymbol{u}\, dV - \int_{\Omega^{(e)}} \boldsymbol{f} \cdot \delta\boldsymbol{u} dV - \int_{\Gamma^{(e)}} \boldsymbol{t} \cdot \delta\boldsymbol{u}\, dS \tag{4}$$

$$H_{<s>} = -\delta \int_{\Gamma_{<s>}} \boldsymbol{\lambda} \cdot (\boldsymbol{u}^{(a)}_{<ab>} - \boldsymbol{u}^{(b)}_{<ab>})\, dS \tag{5}$$

Also, $\boldsymbol{\sigma}$ is stress, $\boldsymbol{f}$ is physical strength, $\boldsymbol{u}$ is displacement, and $\delta\boldsymbol{u}$ is virtual displacement. $\boldsymbol{\lambda}$ is Lagrange multiplier.

Figure 2 shows the degrees of freedom of the flat shell elements of HPM. The proposed displacement field is shown in equations (6)-(8). The red letters represent the rigid body displacement shown in Fig.2.

$$u = u_0 - y\theta_z + x\varepsilon_x + \frac{1}{2} y\gamma_{xy} \tag{6}$$

$$v = v_0 + x\theta_z + y\varepsilon_y + \frac{1}{2} x\gamma_{xy} \tag{7}$$

$$w = w_0 + y\theta_x - x\theta_y - \frac{1}{2}x^2\varepsilon_{x,z} - \frac{1}{2}y^2\varepsilon_{y,z} - \frac{1}{2}xy\gamma_{xy,z} \tag{8}$$

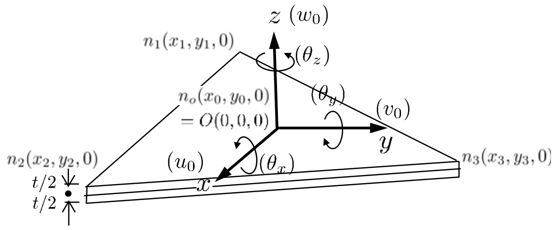


Fig.2 DOF of Flat Shell Element

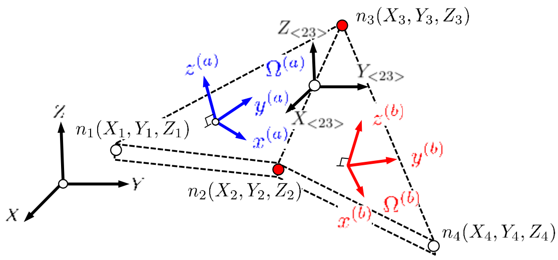


Fig.3 Local Coordinate System for Each Subdomain

As shown in Fig.3, $\boldsymbol{x}$ is a value related to the local coordinate system *x-y-z* for each element, and $\boldsymbol{X}$ is a value related to the global coordinate system *X-Y-Z*. Also, $\boldsymbol{R}_x$ is a coordinate transformation matrix between the global coordinate system and the local coordinate system. There is the following relationship between these.

$$\boldsymbol{x} = \boldsymbol{R}_x \boldsymbol{X} \tag{9}$$

Using these relationships, there are the following relationships between the local coordinate systems of adjacent elements.

$$\boldsymbol{x}^{(a)}_{<23>} = \boldsymbol{R}^{(a)}_x \left(\boldsymbol{R}^{(b)}_x\right)^{-1} \boldsymbol{x}^{(b)}_{<23>} \tag{10}$$

Here, (*a*) and (*b*) represent the adjacent elements $\Omega^{(a)}$ and $\Omega^{(b)}$, and <23> represents the boundary edge n_2-n_3 as shown in Fig.2.

The displacement field shown in equations (6)-(8) is simply written as follows:

$$\boldsymbol{u}^{(e)} = \boldsymbol{N}^{(e)}\boldsymbol{U}^{(e)} \tag{11}$$

Here, $\boldsymbol{u}^{(e)}$ is the displacement at an arbitrary point in the element *e*, $\boldsymbol{U}^{(e)}$ is the degree of freedom of the element *e*, and $\boldsymbol{N}^{(e)}$ is the coefficient matrix relating these. The displacement $\boldsymbol{u}^{(e)}_{<s>}$ at the boundary edge <s> of the element (e) is defined as follows:

$$\boldsymbol{u}^{(e)}_{<s>} \overset{\text{def.}}{=} \boldsymbol{u}^{(e)}\Big|_{\Gamma_{<s>}} \tag{12}$$

The displacement $\boldsymbol{u}^{(e)}_{<s>}$ is represented by a local coordinate system provided for each element, and the displacement $\boldsymbol{u}^{(e)}_{n<s>}$ converted into the coordinate system of the boundary edge of the element (e) is as follows:

$$\boldsymbol{u}^{(e)}_{n<s>} = \boldsymbol{R}^{(e)}_{<s>}\boldsymbol{u}^{(e)}_{<s>} \tag{13}$$

$\boldsymbol{R}^{(e)}_{<s>}$ is the matrix that transforms the local coordinate system of $\Omega^{(e)}$ into the coordinate system along the element boundary <s>.

At this time, the displacement on the boundary side of the adjacent elements based on the local coordinate system of $\Omega^{(a)}$ is expressed as follows:

$$\boldsymbol{u}^{(a|a)}_{n<ab>} = \boldsymbol{R}^{(a)}_{<ab>}\boldsymbol{u}^{(a)}_{<ab>} \tag{14}$$

$$\boldsymbol{u}^{(b|a)}_{n<ab>} = \boldsymbol{R}^{(a)}_{<ab>}\boldsymbol{R}^{(a)}_{x}\left(\boldsymbol{R}^{(b)}_{x}\right)^{-1}\boldsymbol{u}^{(b)}_{<ab>} \tag{15}$$

Here, (b|a) is the displacement field of $\Omega^{(b)}$ represented by the local coordinate system of $\Omega^{(a)}$. From the above, the relative displacement $\boldsymbol{\delta}^{(a)}_{<ab>}$ with respect to the $\Omega^{(a)}$ coordinate system can be obtained as follows.

$$\boldsymbol{\delta}^{(a)}_{<ab>} = \boldsymbol{u}^{(a|a)}_{n<ab>} - \boldsymbol{u}^{(b|a)}_{n<ab>} \tag{16}$$

By introducing the above displacement field relation into equation (3), the discretization equation in HPM based on the Kirchhoff theory can be obtained as follows:

$$\boldsymbol{K}_s\boldsymbol{U} = \boldsymbol{P}_s \tag{17}$$

$$\boldsymbol{K}_s = \sum_{e=1}^{M}\boldsymbol{K}^{(e)}_s + \sum_{s=1}^{N}\boldsymbol{K}_{s<s>},\ \ \boldsymbol{P}_s = \sum_{e=1}^{M}\boldsymbol{P}^{(e)}_s$$

Here, $\boldsymbol{K}^{(e)}_s$ represents a coefficient matrix obtained from the virtual work equation (4), and $\boldsymbol{K}_{s<s>}$ is a coefficient matrix obtained from the incidental condition equation (5). In addition, $\boldsymbol{P}^{(e)}_s$ means a load term for each element.

(1) *Accuracy of Elastic Solution*

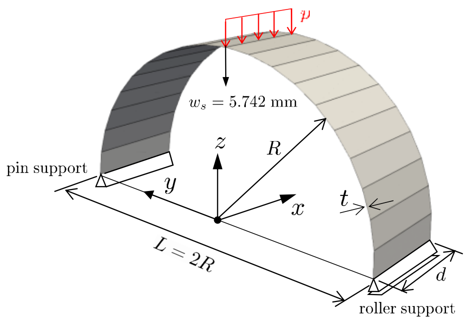
As a first example, we consider semi-circular curved beam with a pin support on one end and roller support on the other end, as shown in Fig. 4(a). The shape of semi-circular curved beam is *R* = 0.16m, *d* = 0.1m, *t* = 0.002m. Young's modulus is E = 190GN/m^2, Poisson's ratio is 0. Line load is applied at the top middle of the model *p* = 1kN/m.

Fig. 4(b) shows the beam model representation of the same problem.

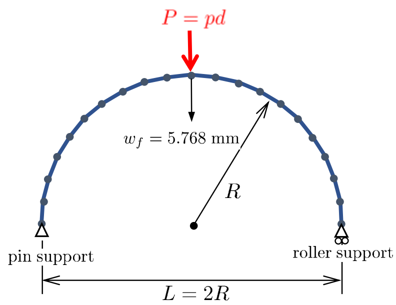
Theoretical result of vertical displacement obtained:

$$w_{\text{Theory}} = \frac{PR^3}{4EI}\left(\frac{3\pi}{2} - 4\right) = 5.759\ \text{mm}$$

Also vertical displacement of frame model showing Fig.4 (b) is computed as $w_f = 5.768\ \text{mm}$.



(a) flat shell model



(b) frame model

Fig. 4 Numerical Model for Accuracy of Elastic Solution

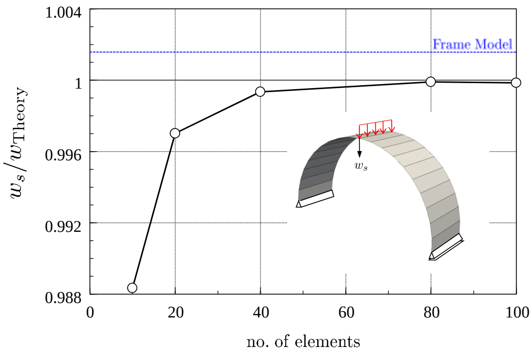


Fig.5 Convergence of Displacement Solution by Number of Elements

Figure 5 shows the convergence of the displacement according to the number of elements. The circles are the results of the flat shell model, and the blue line is the results of the frame model. The horizontal axis represents the number of elements, and the vertical axis represents the ratio of the analytical solution to the vertical displacement of the center. The solution by the proposed method has an error of approximately 0.4% even with a rough element division of nearly 20 elements.

Figure 6 shows the displacement mode when the number of element divisions is 40. As a scale, the displacement is multiplied by 5.

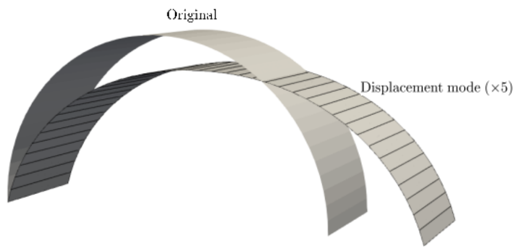


Fig.6 Distribution of vertical displacements

(2) *Pinched Cylinder Analysis*

As a second example, we consider the pinched cylinder with both ends free, as shown in Fig. 7.

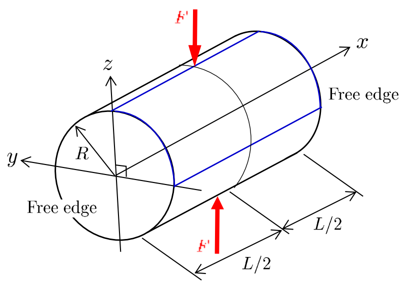


Fig.7 Pinched Cylinder with Free Edges

The shape of cylinder is L = 0.9m, R = 0.16m, t = 0.002m. Elasticity modulus E = 190GPa and Poisson's ratio n = 0.265. Point load F = 8.386kN is applied at the top middle of the model.

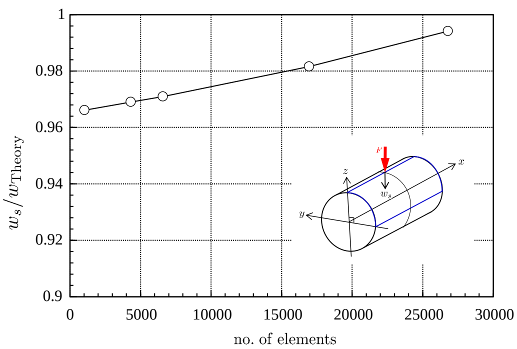


Fig.8 Vertical Displacement of Loading Point

Figure 8 shows the convergence state of the vertical displacement of the loading point. The vertical axis is the vertical displacement divided by the analytical solution, and the horizontal axis is the number of elements.

3. Material Non-Linear Analysis

(1) *Constitutive Equation for Surface Force*

Discrete analysis using HPM deals with two fractures: one within the element and the other between adjacent elements. We deal only with failure at the edge between adjacent elements. In shell problems, fractures such as bending fractures, slip failures, and tensile cracking occur at the edge. We analyzed the progress of a hinge with bending fracture.

Generally, the yield function $f(\boldsymbol{M},\boldsymbol{\sigma})$ and plastic potential $\Phi(\boldsymbol{M},\boldsymbol{\sigma})$ are

$$f(\boldsymbol{M},\boldsymbol{\sigma}) = 0 \tag{18}$$

$$\Phi(\boldsymbol{M},\boldsymbol{\sigma}) = 0 \tag{19}$$

Here, we assume $f = \Phi$ according to the associate flow rule. A plastic hinge is assumed as the fracture condition of the flat shell model in HPM. The fracture conditions in this case are as follows:

$$f(\boldsymbol{M}) = \frac{|M|}{M_y} - 1 \tag{20}$$

where M_y represents the full plastic moment.

We considered the increasing strain on the plastic hinge using the flow theory of plasticity. For this case, the incremental bending moment is obtained as follows:

$$\Delta M_n = k^{(p)} \Delta\delta \tag{21}$$

where $\Delta\delta$ represents the relative displacement, and the penalty function is given as

$$k_{ij}^{(p)} = k_i^{(e)}\delta_{ij} - \frac{1}{\sum k_i^{(e)} f_i^2} f_i f_j k_i^{(e)} k_j^{(e)} \tag{22}$$

(2) *Load Incremental Method*

For nonlinear analysis, “the R-min method” [14] in the load increment method shown in Fig.10 was used. The method first searches for the boundary of the element for the smallest rate of load increment. At the element boundary where the hinge occurs, the coefficient matrix of equation (17) is obtained using the incremental relationship of equation (21), and the minimum rate of load increment is searched again. This process was repeated until all the loads were applied

Assumably, the stress state shifts from point P to point R owing to the loading. Because the stress state cannot exceed the yield surface, it is necessary to return it to point *Q*. The load whose stress state does not exceed the yield surface is obtained by multiplying stress at point *R* by the increment rate, which is given as follows:

$$r = \frac{\overline{PQ}}{\overline{PR}} \tag{23}$$

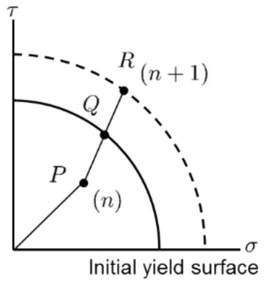


Fig. 9 Relationship between yield surface and stress state

Figure 9 shows a conceptual diagram for explaining the load increment rate; P is the position of the surface force up to the previous time, and R is the position where the incremental surface force obtained at the current time and the previous surface force is added. That is, the incremental surface force of the current time is PR. If R clearly exceeds point Q of the initial yield surface, it implies that an extra surface force QR is acting. Therefore, the surface force in R is returned to the position of Q, and only the elastic part of PQ is left. For this purpose, the following load increment rate is used.

Accordingly, the (i + 1) th acting load $\boldsymbol{P}^{(i+1)}$ can be obtained from the i-th load $\boldsymbol{P}^{(i)}$ as follows:

$$\boldsymbol{P}^{(i+1)} = (1 - r_i)\boldsymbol{P}^{(i)} \tag{24}$$

The term r_i denotes the rate of load increment, and it can be obtained from equation (20) as follows:

$$f(\boldsymbol{M} + r \cdot \Delta \boldsymbol{M}) \leq 0 \tag{25}$$

When the yield function is given by equation (20), the rate of load increment is computed as follows:

$$\left(\frac{M_n + r \cdot \Delta M_n}{M_y} \right) - 1 = 0 \tag{26}$$

We obtain r as follows:

$$r = \frac{M_y + M_n}{\Delta M_n} \tag{27}$$

Therefore, the bending moment $\boldsymbol{M}^{n+1}$ after the increment can be obtained by adding the incremental bending moment ΔM multiplied by r to the previous bending moment $\boldsymbol{M}^n$. One has

$$\boldsymbol{M}^{n+1} = \boldsymbol{M}^n + r \cdot \Delta \boldsymbol{M} \tag{28}$$

In the case of bending moment, residual load at the n^{th} step will be

$$\boldsymbol{P}^{(n)} = \prod_{i=0}^{n-1} \left[(1 - r_i) \right] \Delta \boldsymbol{P} \tag{29}$$

The cumulative rate of load increment is as follows:

$$r_{TOTAL} = \sum_{k=1}^{n} \left(\prod_{i=0}^{k-1} \left[(1 - r_i) \right] \right) r_k \tag{30}$$

When $r_{TOTAL} = 1$, an iteration is finished.

4. Large Displacement Analysis

(1) *Numerical Algorithm by Step-by-Step Method*

Large displacement analysis is difficult to handle as a linear analysis like small deformation analysis because the stiffness matrix changes with the deformation of the object. In this paper, as shown in Fig. 10, we use the step-by-step method to analyze the large displacement problem by repeating the micro-deformation analysis.

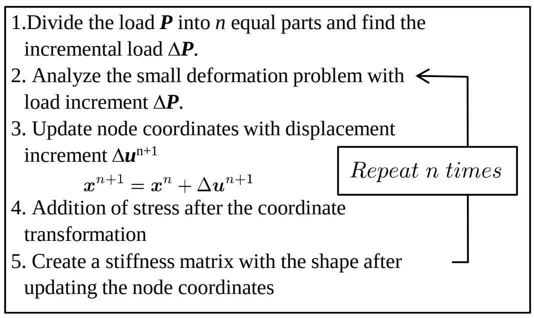


Fig.10 Large displacement analysis method by step-by-step method

As shown in the figure, the load acting on the object is divided into several incremental loads, and the small deformation problem is analyzed for each incremental load. The obtained displacement $\Delta \boldsymbol{u}^{n+1}$ is added to the coordinate value $\boldsymbol{x}^n$ before deformation, the coordinate value is updated, and the shape after deformation is created as $\boldsymbol{x}^{n+1}$.

The previous stress is added to the incremental stress to obtain the total stress, a rigidity matrix is created with deformation, the coordinate value is updated, and the shape after deformation is created as $\boldsymbol{x}^{n+1}$. The previous stress is added to the incremental stress to obtain the total stress, a rigidity matrix is created with the new node coordinate values, and the linear analysis is repeated.

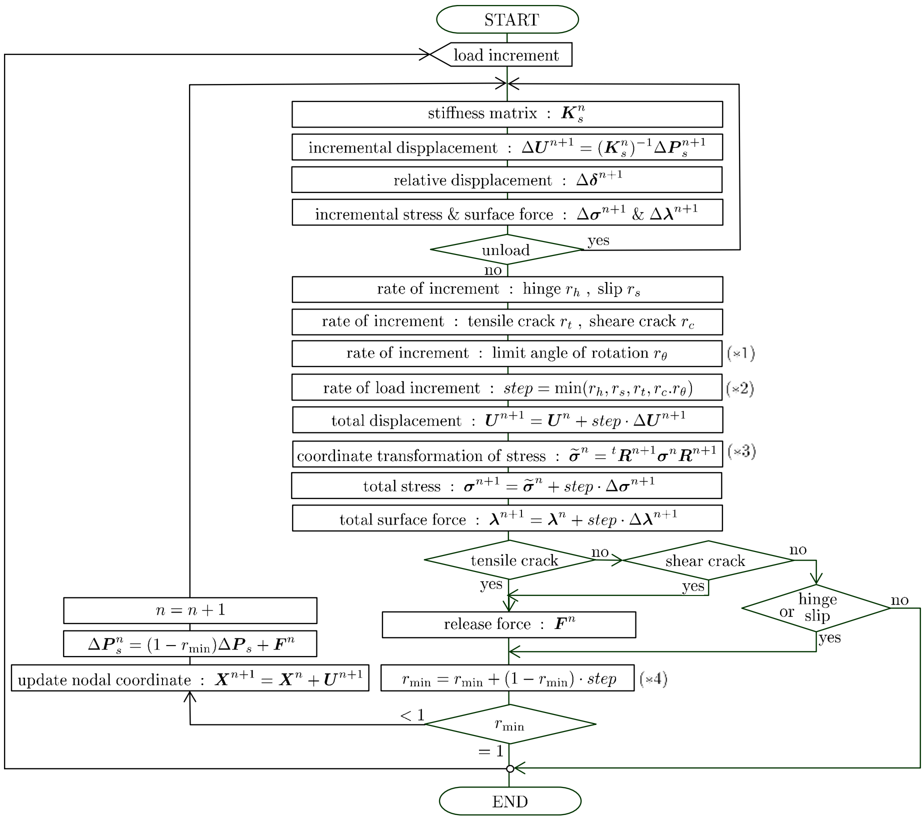


Fig. 11 Analysis Flow of Large Displacement Problem by Step-by-step Method with Hinge Condition

Figure 11 shows the analysis flowchart of the load increment method proposed in this study. In this method, the stiffness matrix is recreated for each load increment; therefore, the computation process takes time. However, a stable solution can be obtained even under a load state near the collapse load. In addition, the load increment can be calculated from the load increment rate obtained by analysis, and a more accurate collapse load can be obtained.

Four conditions are assumed as factors that determine the load increment rate: plastic hinges, slips, shear cracks, and tensile cracks. These load increment rates are found for all unfractured element boundaries, and the smallest of them is set to the current load increment rate.

For shear cracks and tensile cracks under the four conditions, a process to release the surface force has been added. In this study, the surface force is fully released, but it is also possible to gradually release it by changing the release rate.

The above operation is repeated until the load increment rate r_{min} is 1; that is, the assumed total load is applied. However, if the mechanism is formed before it becomes 1, the load value at that time becomes the collapse load.

(2) *Numerical algorithm of Material Nonlinear Large Displacement Analysis*

Figure 12 shows the analysis flow of the large displacement problem with material nonlinearity in this study. As shown in the figure, the solution is first obtained by linear analysis of the small deformation problem.

Next, the load increment rate for the material nonlinear problem is obtained, and the load increment rate due to the limit rotation angle for the large deformation problem is obtained in (*1). Then, the minimum load increment rate is calculated by (*2). For stress, the coordinate transformation is applied in (*3). After that, the total load increment rate used up to now is computed by (*4). If this value is less than 1, the coordinates are updated and the linear analysis is repeated again with the remaining load.

(3) *Numerical Example of Large Displacement Analysis*

In this section, we verify the accuracy of the solution of the large displacement analysis by the proposed step-by-step method for the elastic problem of the flat shell. Figure 12 shows the model and mesh division used in the analysis.

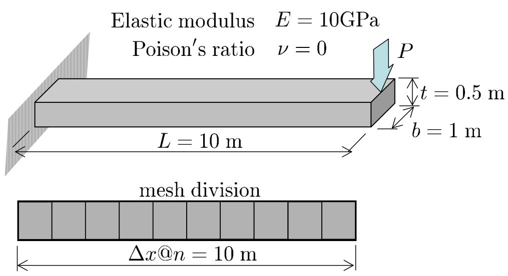


Fig.12 Numerical Model for Cantilever

As shown in the figure, the numerical model is a flat plate with one end fixed, and the dimensions are as shown in the figure. The material constants used in the analysis are also shown in the figure. The mesh is divided as shown in the lower part of the figure, but the number of divisions is analyzed assuming various cases.

In Fig. 13, the horizontal axis is the limit rotation angle, and the vertical axis is the value obtained by dividing the deflection at the free end by the solution of the beam theory. The red circle represents the deflection at the free end, and the blue triangle represents the horizontal displacement. A convergent solution is obtained when the limit rotation angle is set to 0.05 or less. Expressed in degrees, it is about 2.86o. Sin2.86o is about 0.0499, which is similar to the limit rotation angle.

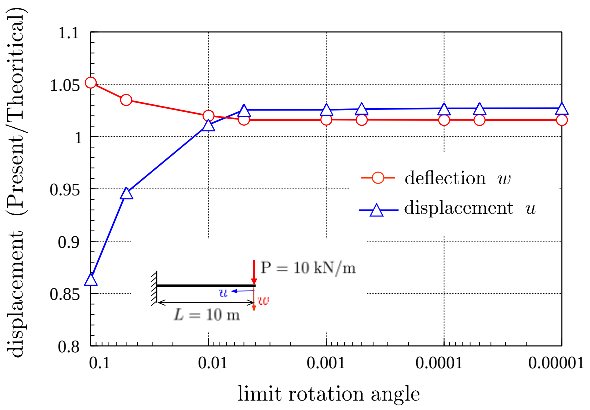


Fig.13 Accuracy of Displacement u and w for Limit Rotation Angle

Figure 14 shows the dimensionless deflection of the free end of the flat plate on the horizontal axis and the dimensionless load on the vertical axis. The blue circle is the solution by this method, and the results are almost the same.

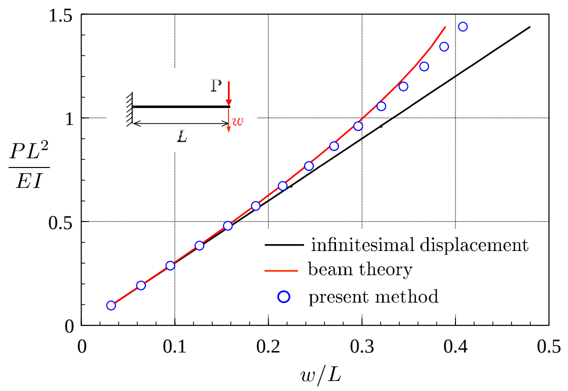


Fig.14 Relationship between Load and Large Deflection

On the other hand, in Fig.15, the horizontal axis shows the dimensionless horizontal displacement value, and the vertical axis shows the dimensionless load. The blue circle is the solution by this method, and the results are almost the same.

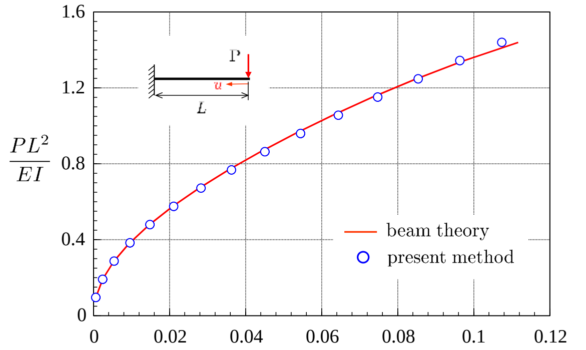


Fig.15 Relationship between Load and Lateral Displacement

Figure 16 shows the displacement mode. The figure shows an example of 20 divisions, and the deflection and horizontal displacement are shown on a real scale.

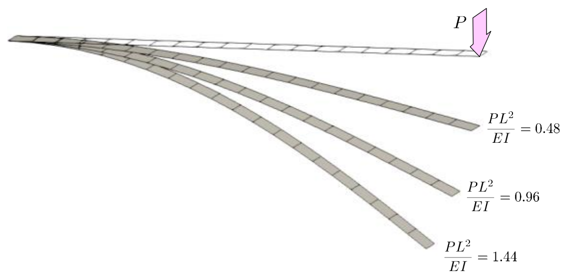
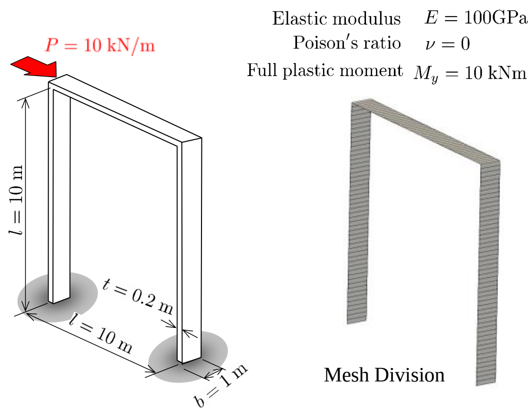


Fig.16 Displacement Mode for each Load

(4) *Numerical Example of Large Displacement Analysis with Hinge*

Figure 17 shows the model and mesh division used in the analysis. As shown in the figure, the numerical model is a portal frame with both end fixed, and the dimensions are as shown in the figure.



(a) Model (b) Mesh

Fig.17 Numerical Model for Gate Frame

In the nonlinear analysis, it was assumed that only the plastic hinge was generated, and only the total plastic moment was set. The material constants used in the analysis are shown in the figure. As shown on the right of the figure, the mesh was divided into 50 rectangular elements for columns and beam members, and 150 in total.

Figure 18 shows the dimensionless displacement of upper right corner of the frame on the horizontal axis and the dimensionless load on the vertical axis. The collapse load of the large displacement solution is slightly higher than that.

Figure 19 shows the displacement mode.

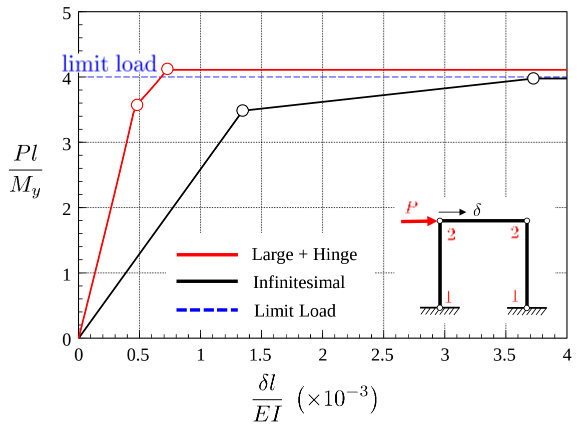


Fig.18 Load-Displacement Curve for Large Displacement Analysis with Hinge Condition

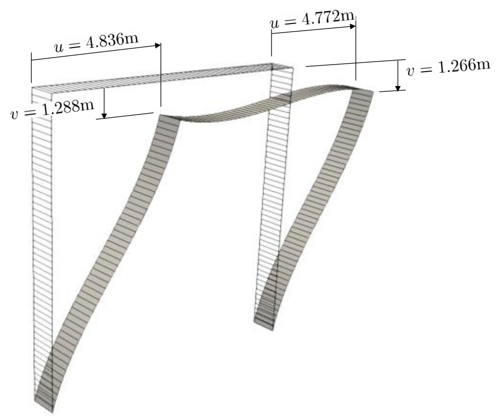


Fig.19 Displacement Mode at Collapse Load for Portal Frame Fixed at Both End

5. Conclusion

In this paper, we proposed a numerical method to the material nonlinear large displacement problem using flat shell elements in HPM. Since HPM uses an independent displacement field for each element, the displacement field can be expressed using the local coordinate system set for each element. The application of this displacement field is convenient for shell elements based on the Kirchhoff-Love theory, which can superimpose in-plane deformation and out-of-plane deformation.

However, in HPM, which introduces the continuity of displacement using a penalty function, it is difficult to handle relative displacement in the case of a shell structure in which adjacent elements are not flatly connected. In this paper, we proposed a method to transform the displacement into one of the local coordinate systems of adjacent elements in the calculation of this relative displacement.

Finally, we proposed an algorithm for material nonlinear large displacement analysis that combines the algorithm of large displacement analysis and material nonlinear analysis. We were able to show the characteristics of the solution obtained by proposed method from a simple numerical example.

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