Borda Count Method for Fiscal Policy : A Political Economic Analysis

Ishida, Ryo / Oguro, Kazumasa

(出版者 / Publisher)
The Institute of Comparative Economic Studies, Hosei University / 法政大学比較経済研究所

(雑誌名 / Journal or Publication Title)
Journal of International Economic Studies

(巻 / Volume)
36

(開始ページ / Start Page)
25

(終了ページ / End Page)
40

(発行年 / Year)
2022-03

(URL)
https://doi.org/10.15002/00025446

Journal of International Economic Studies (2022), No.36, 25–40

Borda Count Method for Fiscal Policy - A Political Economic Analysis -

Ryo Ishida[1]
and
Kazumasa Oguro[2]

Abstract

Survey data reveals that government budgets tend to go into the red. Public Choice economists as well as public finance economists have been interested in this phenomenon and have come up with several explanations, such as fiscal illusion. This paper presents a new explanation for this tendency from the political economic point of view; the current voting system might have a tendency to bring about a budget deficit. If policy choices only deal with the current tax rate and do not take into account the intertemporal tax rate, it may be difficult to select a budget-balanced choice. Even if voting choices take into account intertemporal aspects, as there exist so many choices to increase tax rates to reimburse government deficits, votes from people who support a balanced budget are split and therefore it may be difficult to select a budget-balanced choice under the relative majority rule even if a balanced budget is supported by a majority of voters. We further demonstrate that the Borda count method, known to mitigate vote-splitting problems, might overcome this issue.

Keywords**:** relative majority rule, Borda count method, deficit

JEL**: D72; H41; H62**

Remark

The authors thank the following people for helpful comments: Toru Nakazato (Sophia University), Shinji Yamashige (Hitotsubashi University), participants from a workshop at the Policy Research Institute, Ministry of Finance Japan, including Yoichi Nemoto, Kiyoshi Takata, Hiroyuki Matsuoka, Yumiko Ozeki and Daisuke Ishikawa, and participants from the 2018 Spring meeting of Japan Association for Applied Economics, especially the discussant Masaya Yasuoka. The views expressed herein are those of the authors and do not necessarily reflect the opinions of the organizations to which the authors belong. Any remaining errors are the sole responsibility of the authors.

---

[1] Corresponding author. Ph.D. in Economics, Visiting Scholar, Policy Research Institute, Ministry of Finance Japan (rrishida112358 at gmail.com)
Address: 3-1-1, Kasumigaseki, Chiyoda-Ku, Tokyo, Japan
Tel: +81-3-3581-4111
[2] Ph.D. in Economics, Professor, Faculty of Economics, Hosei University

1. Introduction

Macroeconomists almost always assume that the intertemporal government budget is balanced. In other words, the government budget is not necessarily balanced today but is to be balanced in the future. This assumption seems natural because, in our daily life, everyone is required to settle her deficit at some time. However, in reality, is not easy for the government to implement a balanced budget. Buchanan (1987, p.471) asserts that the government budget tends to be in deficit by stating, "*The most elementary prediction from public choice theory is that in the absence of moral or constitutional constraints democracies will finance some share of current public consumption from debt issue rather than from taxation and that, in consequence, spending rates will be higher than would accrue under budget balance.*" An IMF survey[3] also reveals that 139 to 159 countries among 189 countries were in deficit during 2010-2015. Moreover, 112 countries had been in deficit for all six consecutive years during this period.

There are several explanations why government budgets tend to be in deficit. Orthodox tax smoothing (Barro, 1979) cannot explain this tendency. As seen above, Buchanan explains this phenomenon by saying that debt finance is preferred to taxation finance under democracy, which is sometimes called a *fiscal illusion* (Buchanan and Wagner, 1977). Weingast et al. (1981) and Cogan (1993) explain this phenomenon in a microeconomic way, suggesting that a tragedy of the commons brings about government deficits. Alesina and Tabellini (1990) propose a theory that the current government has an incentive to constrain a future government's activity by accumulating public debt.[4]

The purpose of this paper is to propose another explanation. Our explanation, which is based on the study of political economy, is that current voting rules may have a tendency to bring about a non-budget-balanced choice.[5] Firstly, if the policy choices are only for today's tax rate, not only non-budget-balancing people but also budget-balancing people who prefer future tax increases to today's tax increase may opt for no tax increase today. Moreover, even if policy choices take into account an intertemporal tax rate, the relative majority rule might not reflect voter opinion. It is well-known that, although widely implemented, the relative majority rule has several pitfalls. For example, it is vulnerable to vote splitting. If there are several policy choices with similar ideologies, the vote may be split and thus it will turn out to be difficult for these choices to obtain a relative majority. As a result, even though ideology A is supported by the majority, where ideology B is supported by the minority, if there are many policy choices embodying ideology A, a candidate with ideology B may win under the relative majority voting rule. This effect, known as the *spoiler effect*, can be seen in many electoral campaigns, such as in the 2000 U.S. Presidential Elections (The New York Times, 2004). We show that there is a possibility that, even if budget-balancing people are in the majority, they may suffer from the spoiler effect.

There are several voting rules that can be used to overcome the caveats of the relative majority rule. Among these, the Borda count method (Borda, 1784) and Condorcet method (Condorcet, 1785) are well known. We will focus on the Borda count method hereafter.[6] Under the Borda count method,

---

[3] IMF Fiscal Monitor

[4] There are other explanations. For example, Oguro et al. (2013) explain it by incorporating the political powers of different generations and intergenerational altruism using an overlapping generations model.

[5] The caveat and possible remedies of the current voting system are analyzed by many studies, including Ishida and Oguro (2018) and Ishida (2015). Among these, Oguro et al. (2018) focus on the proof-of-work system introduced in cryptocurrencies. The mechanism of cryptocurrencies is empirically analyzed in Hattori and Ishida (2020a) and Hattori and Ishida (2020b).

[6] The Condorcet method requires the maximum likelihood method if there are four or more choices (Young, 1988), which is difficult to implement using our model.

if there are n policy choices, each person gives n points, $n-1$ points, ⋯, 1 point respectively to each choice. The points for each choice are then summed up. The choice that collects the highest points is elected. This count method is robust for split votes and thus overcomes the spoiler effect. Further, this count method is not an armchair theory, but is used in special legislative seats for ethnic minorities in Slovenia, and a similar method, called the Dowdall method, is implemented in Nauru (Golder, 2005; Fraenkel and Grofman, 2014). We will see that the Borda count method works for our issue as well.

This explanation has some similarities with Alesina and Drazen (1991), where two groups of people play a war of attrition game in order for a group to shift the burden of public debt onto another group. Since a cost must be paid for the postponement of redemption in each period, this war-of-attrition makes social welfare sub-optimal. In this model, people know that non-sustainable debt must be redeemed, but totally rational people act sub-optimally. In our model, the majority of people know that non-sustainable debt must be redeemed, but, due to the current voting system, the social consensus postpones redemption and acts sub-optimally. However, the mechanism is totally different from that in Alesina and Drazen (1991).

Our model is also similar to the agenda-setting model proposed by Romer and Rosenthal (1978) where agenda setters influence the final social consensus. Our model claims that the final social consensus depends on whether the voting agenda is only for today's tax rate or the intertemporal tax rate. However, our model goes further; the final social consensus does depend on what kind of voting system is used.

The remainder of this paper is organized as follows. A simple model is presented in the next section. This simple model is generalized and an individual utility function is specified in the following section. Finally, a conclusion is presented.

2. Simple model

Assume there is an initial government deficit that is normalized to unity. Our interest is people's voting behavior that chooses a policy from several possible plans to reimburse the government deficit in a two-period model ($t=1,2$). For the sake of simplicity, we assume that there are only three different plans for reimbursing the government deficit. Plan X reimburses the government deficit only at $t=1$ by tax increases, where plan Y reimburses the government deficit only at $t=2$ by tax increases. Both plan X and plan Y are budget-balanced plans. However, plan Z is non-budget-balanced; it does not reimburse the government deficit either at $t=1$ or at $t=2$. Denoting each plan by (τ_1, τ_2) where τ_i refers to a tax increase at $t=i$, plan X is denoted as $(1,0)$, whereas plan Y is $(0,1)$ and plan Z is $(0,0)$. We assume people's voting behavior depends only on tax profile. Note that, if a policy is chosen, it will definitely be implemented. In other words, the selected policy fully binds not only this period's tax rate but also next period's tax rate.

Let there be $n_a+n_b+n_c$ people. n_a people have preference A, n_b people have preference B, and n_c people have preference C. People with preference A or B prefer balanced-budget plans. Among balanced-budget plans, people with preference A prefer plan X to plan Y (early reimbursement is preferred), where people with preference B prefer plan Y to plan X (late reimbursement is preferred). People with preference C prefer a non-balanced-budget plan and consider that plan Y is at least better than plan X.[7] These preferences are described in Figure 1, where $\mathrm{S} \succ \mathrm{T}$, meaning that S is preferred to T. We assume that budget-balancing people are greater

[7] An identical discussion is possible when people with preference C consider that plan X is at least better than plan Y. Without loss of generality, we assume here that people with preference C consider that plan Y is at least better than plan X.

in number than non-budget-balancing people; i.e. $n_a + n_b > n_c$. We also assume $n_c > n_a$ and $n_c > n_b$. An example of what we are considering is $n_a = n_b = 3$ and $n_c = 4$. Finally, we assume truthful voting hereinafter.

A: X ≻ Y ≻ Z
B: Y ≻ X ≻ Z
C: Z ≻ Y ≻ X

Figure 1: Voting preferences for people A, B and C

As follows, we will see that a balanced-budget plan is not chosen in this framework if policy choices do not take into account the intertemporal aspect or if relative majority rule is implemented.

(1) Let there be two policy choices that do not take into account the intertemporal aspect; tax increase at $t = 1$ and no-tax-increase at $t = 1$. People with preference B as well as preference C will then select the latter choice whichever voting rule is implemented. Therefore, a no-tax-increase is chosen even though budget-balancing people are in the majority.

(2) Let there be three policy choices that take into account the intertemporal aspect; $(\tau_1, \tau_2) = (1,0), (0,1), (0,0)$. People with preference A vote for the first choice, people with preference B the second, and people with preference C the third. Therefore, under the relative majority rule, $(\tau_1, \tau_2) = (0,0)$ is chosen even though budget-balancing people are in the majority.

The reasons for this result are summarized in the following two points. First, if the choices only take into account today's tax rate ($t = 1$), not only non-budget-balancing people (preference C) but also budget-balancing people who prefer late reimbursement (preference B) choose no-tax-increase today. Since $n_b + n_c > n_a$ holds, the choice of no-tax-increase is selected. This result holds whichever voting rule is implemented. Second, even if the choices take into account the future tax rate ($t = 2$), votes from budget-balancing people will be split between plan X and plan Y, which will result in a win for plan Z under the relative majority voting rule because $n_c > n_a$ and $n_c > n_b$ hold. As a result, even if budget-balancing people are the majority, the non-balanced-budget choice is selected.

This issue is overcome by incorporating the intertemporal aspect in policy choices and implementing the Borda count method. Under the Borda count method, each person gives n points, $n - 1$ point, ⋯, 1 point respectively to n choices. The points for each choice are summed, the choice collecting the largest number of points being selected. Using the Borda count method for the three policy choices (plan X, Y and Z), people with preference A give 3 points to plan X, 2 points to plan Y, and 1 point to plan Z. People with preference B give 3 points to plan Y, 2 points to plan X, and 1 point to plan Z. People with preference C give 3 points to plan Z, 2 points to plan Y, and 1 point to plan X. This is described in Figure 2.

	X	Y	Z
A	3	2	1
B	2	3	1
C	1	2	3
Total points	$3n_a + 2n_b + n_c$	$2n_a + 3n_b + 2n_c$	$n_a + n_b + 3n_c$

Figure 2: Voting result under the Borda count method

In this case, budget-balanced plan Y is chosen because $2n_a + 3n_b + 2n_c > n_a + n_b + 3n_c$ and $2n_a + 3n_b + 2n_c > 3n_a + 2n_b + n_c$. We will generalize this idea in the following section.

3. General model

As we have seen in the previous section, we show that the Borda count method is efficient in overcoming the aforementioned issue. However, the model presented in the previous section does not specify an individual utility function. Also, it considers only three policy choices; $(\tau_1, \tau_2) = (1,0), (0,1), (0,0)$. In order to generalize the previous model, we specify the voter's utility function and consider many possible policy choices hereafter.

Assume that there are k budget-balancing people and l non-budget-balancing people. If $l < k$, budget-balancing people are in the majority where $l > k$ implies non-budget-balancing people are in the majority.[8] Each person is indexed by $i \in [1, k + l]$. Before period 0, the initial government debt that is to be reimbursed by period 3 is $D > 0$, and the number of budget-balancing and non-budget-balancing people and everyone's future endowments are already common knowledge. The initial government debt might be used to finance public goods provided before period 0. In period 0, the lump-sum tax schedule is determined by voting, the details of which are discussed later. In period 1 and period 2, each person is endowed with $w_i \in (0, n_i)$ and $n_i - w_i \in (0, n_i)$, respectively, and has to pay a lump-sum tax, $\tau_1 \in [0, \min_i w_i)$ and $\tau_2 \in [0, \min_i (n_i - w_i))$, respectively. Note that the size of the lump-sum tax is unaffected by the size of one's endowment. Each person enjoys private consumption, $c_i^1 = w_i - \tau_1$ at period 1 and $c_i^2 = n_i - w_i - \tau_2$ at period 2, respectively, which implies that saving is prohibited by any person.

We will consider a case where tax rates τ_1 and τ_2 are restricted to non-negative integers, and budget-balancing people prefer (τ_1, τ_2) to (τ'_1, τ'_2) and non-budget-balancing people prefer (τ'_1, τ'_2) to (τ_1, τ_2) if $\frac{D}{k+l} \geq \tau_1 + \tau_2 > \tau'_1 + \tau'_2 \geq 0$. If $\tau_1 + \tau_2 = \tau'_1 + \tau'_2$, the preference depends on each person's characteristics independent from whether they are for or against balanced budgets. In order only to demonstrate such preferences, we specify utility functions as follows.

The government reimburses its debt at period 3 from tax revenue during period 1 to period 2; $(k + l)(\tau_1 + \tau_2)$. Since the tax schedule is solely determined by voting, the government has no objective function.

Each person's utility function is

$$U_i = U(c_i^1, c_i^2, \xi, \theta) = \frac{(c_i^1)^{1-\sigma}}{1-\sigma} + \frac{(c_i^2)^{1-\sigma}}{1-\sigma} - \theta v(\xi) \qquad (1)$$

where $\sigma > 0$ measures the degree of relative risk aversion and function $v(\cdot)$ measures the disutility from final government debt per capita.[9] Remaining government debt per capita at period

---

[8] We are interested in a case where budget-balancing people are in the majority. Supporting evidence of this case is Mochida (2016). Using an Internet-based questionnaire of 1,000 answers randomly sampled from approximately 3.27 million people in Japan, he reports that three quarters of people are budget-balancing people (immediate redemption, 33.1%; gradual redemption, 42.4%) where a quarter of people are non-budget-balancing.

[9] People do not receive utility or disutility directly from government debt. However, it is natural to assume that people may expect future tax increases after period 3, which they themselves or their descendants have to bear. Maybe people expect a future default and take into account its consequence, in which they themselves or their descendants will suffer if there is substantial debt outstanding. Assuming that people take into account their descendants' utility, as appeared in Buchanan (1976), function $v(\cdot)$ is understood to include all these effects. It is worth noting that Alesina and Drazen (1991) also assume that postponing debt redemption is costly because of the distortion of taxation and lobbying costs. Such an effect may also be included in function $v(\cdot)$. The form of the function may be derived by analyzing the probability of default (e.g. Cuadra et

3 is $\xi \equiv \frac{D}{k+l} - (\tau_1 + \tau_2)$. $\theta = 1$ for budget-balancing people where $\theta = 0$ for non-budget-balancing people. We consider neither interest rate nor discount rate. An interpretation of function $v(\cdot)$ is that it measures possible future consequences caused by government debt. Therefore, it is assumed that $v(\xi)$ is a positive, differentiable and strictly increasing function when $\xi > 0$ and a weakly increasing function when $\xi \leq 0$. For technical reasons, we also assume that $v'(\xi) > 1$ when $\xi > 0$ hereafter.[10]

Assume that everyone is given sufficient endowments, i.e. $\frac{D}{k+l} + 1 \leq \min_i w_i$ and $\frac{D}{k+l} + 1 \leq \min_i (n_i - w_i)$ in order to avoid corner solutions.

We also assume that the degree of relative risk aversion σ is small. More specifically, we assume the following condition.

Assumption 1 (sufficiently small relative risk aversion σ): We assume that, for any person i, the following inequality always holds.

$$U(c_i^1, c_i^2, \xi, \theta = 1) > U\left(\widetilde{c_i^1}, \widetilde{c_i^2}, \tilde{\xi}, \theta = 1\right) \quad (2)$$

and

$$U(c_i^1, c_i^2, \xi, \theta = 0) < U\left(\widetilde{c_i^1}, \widetilde{c_i^2}, \tilde{\xi}, \theta = 0\right) \quad (3)$$

Where

$$c_i^1 = w_i - \tau_1 \geq 1 \quad (4)$$

$$c_i^2 = n_i - w_i - \tau_2 \geq 1 \quad (5)$$

$$\frac{D}{k+l} - (\tau_1 + \tau_2) = \xi \quad (6)$$

$$\widetilde{c_i^1} = w_i - \widetilde{\tau_1} \geq 1 \quad (7)$$

$$\widetilde{c_i^2} = n_i - w_i - \widetilde{\tau_2} \geq 1 \quad (8)$$

$$\frac{D}{k+l} - (\widetilde{\tau_1} + \widetilde{\tau_2}) = \tilde{\xi} \quad (9)$$

$$\tilde{\xi} - \xi \geq \frac{1}{k+l} \quad (10)$$

$$\xi, \tilde{\xi} \in \left[0, \frac{D}{k+l}\right]. \quad (11)$$

That is, for both budget-balancing -people and non-budget-balancing people, the total amount of tax (in other words, the remaining amount of government debt at period 3) is of primary importance and its allocation is of secondary importance. In other words, we assume that utility is close to the quasi-linear utility function $U_i = U(c_i^1, c_i^2, \xi, \theta) = c_i^1 + c_i^2 - \theta v(\xi)$ and that deviation from the quasi-linear utility function is only for technical purposes.[11] This condition is satisfied when the degree of relative risk aversion σ is sufficiently small.

---

al., 2010), which is not our research focus. Similar methods can be seen in Caselli (1997) and Müller et al. (2016), where a fraction of debt defaults or probability of defaults explicitly appears in the cost function or utility function.

[10] Our discussion can be extended where $v'(\xi) > 1$ holds only for $\xi > \xi_0 > 0$. In such a case, we will define $\bar{D} = D - (k + l)\xi_0$ and consider a situation where debt $\bar{D}$ (excessive debt) is to be reimbursed by period 3.

[11] We assume that people have a slight preference on tax allocation. In other words, some people prefer $(\tau_1, \tau_2) = (3,0)$ to $(\tau_1, \tau_2) = (2,1)$ where other people have the opposite preference. However, this preference is only of secondary importance.

In addition, for the sake of simplicity, we assume integer restriction as follows. As we will consider voting later, a discrete profile is easier to handle than a continuum profile.

Assumption 2 (step size): We assume that w_i and n_i are positive integers and τ_1, τ_2 and $\frac{D}{k+l}$ (initial government debt per capita) are integers. We also assume that $n_i - \frac{D}{k+l}$ is an even number for all i.

Note that the integer step size is arbitrary. Therefore, if a statement requires $\frac{D}{k+l}$ to be sufficiently large, such a statement also holds when the step size is sufficiently small, and vice versa.

Due to Assumption 2, consumptions at period 1 and period 2 are always positive integers. In order to reimburse a government debt whose size is D, possible tax schedules are restricted to

$$(\tau_1, \tau_2) = \left(0, \frac{D}{k+l}\right), \left(1, \frac{D}{k+l} - 1\right), \cdots, \left(\frac{D}{k+l}, 0\right). \qquad (12)$$

Following these settings, we propose the tax allocation most preferred by a particular person i.

Proposition 1: If person i is budget-balancing, her first preference would be the following tax schedule.

$$\begin{cases} (\tau_1, \tau_2) = \left(0, \frac{D}{k+l}\right) \; if \; 0 > w_i - \frac{n_i - \frac{D}{k+l}}{2} \\ (\tau_1, \tau_2) = \left(m, \frac{D}{k+l} - m\right) \; if \; m = w_i - \frac{n_i - \frac{D}{k+l}}{2} \in [0, \frac{D}{k+l}] \\ (\tau_1, \tau_2) = \left(\frac{D}{k+l}, 0\right) \; if \; \frac{D}{k+l} < w_i - \frac{n_i - \frac{D}{k+l}}{2} \end{cases}$$

If person i is non-budget-balancing, her best preference would be tax allocation $(\tau_1, \tau_2) = (0,0)$.

Proof of Proposition 1: A non-budget-balancing person is assumed not to take into account the remaining government debt at period 3 in her utility at period 0. Therefore, at period 0, a non-budget-balancing person always prefers a small lump-sum tax, i.e. $(\tau_1, \tau_2) = (0,0)$.

For budget-balancing people, due to Assumption 1, a balanced-budget tax plan, namely $\tau_1 + \tau_2 = \frac{D}{k+l}$, is preferred. Among balanced-budget tax plans, these people would prefer a smoothed consumption because the CRRA utility function is concave. Therefore,

(a) $(\tau_1, \tau_2) = \left(0, \frac{D}{k+l}\right)$ is the first preference if $0 > w_i - \frac{n_i - \frac{D}{k+l}}{2}$ holds, because $c_i^1 = w_i - \tau_1 = w_i$ is still less than $c_i^2 = n_i - w_i - \tau_2 = n_i - w_i - \frac{D}{k+l}$ and $(\tau_1, \tau_2) = \left(0, \frac{D}{k+l}\right)$ is the corner solution.

(b) $(\tau_1, \tau_2) = \left(m, \frac{D}{k+l} - m\right)$ is the first preferene if $m = w_i - \frac{n_i - \frac{D}{k+l}}{2} \in [0, \frac{D}{k+l}]$ holds, because $c_i^1 = w_i - \tau_1 = w_i - m = \frac{n_i - \frac{D}{k+l}}{2}$ is equal to $c_i^2 = n_i - w_i - \tau_2 = n_i - w_i - \frac{D}{k+l} + m = \frac{n_i - \frac{D}{k+l}}{2}$.

(c) $(\tau_1, \tau_2) = \left(\frac{D}{k+l}, 0\right)$ is the first preference if $\frac{D}{k+l} < w_i - \frac{n_i - \frac{D}{k+l}}{2}$ holds, because $c_i^1 = w_i - \tau_1 = w_i - \frac{D}{k+l}$ is still greater than $c_i^2 = n_i - w_i - \tau_2 = n_i - w_i$ and $(\tau_1, \tau_2) = \left(\frac{D}{k+l}, 0\right)$ is the corner solution.

(Q.E.D.)

Still, a budget-balancing person i prefers any tax schedule that satisfies $\tau_1 + \tau_2 = \frac{D}{k+l}$ to any tax schedule that does not satisfy this condition ($\because$ Condition of small σ).

The abovementioned utility function is given only for demonstrating people's preferences where budget-balancing people prefer (τ_1, τ_2) to (τ'_1, τ'_2) and non-budget-balancing people prefer (τ'_1, τ'_2) to (τ_1, τ_2) if $\frac{D}{k+l} \geq \tau_1 + \tau_2 > \tau'_1 + \tau'_2 \geq 0$. Proposition 1 is shown only to demonstrate that people's preferences between (τ_1, τ_2) and (τ'_1, τ'_2) depend on each person's characteristics (independently of whether they are for or against balanced budgets) if $\tau_1 + \tau_2 = \tau'_1 + \tau'_2$. The following discussion does not depend on the specification of the utility function.

Definition 1 (classification of budget-balancing people): Among budget-balancing people, let the number of people whose first preference is $(\tau_1, \tau_2) = \left(m, \frac{D}{k+l} - m\right)$ be k_m. Note that $\sum_{m=0}^{\frac{D}{k+l}} k_m = k$ is satisfied.

We hereafter consider what kind of tax plan will be considered when voting. We assume that the budget surplus tax schedule be eliminated, namely, people only consider tax schedules where total tax revenue is either equal to or smaller than the current government debt.

Definition 2 (government debt per capita): Define $X \equiv \frac{D}{k+l}$.

Assumption 3 (possible tax schedule): We assume $X < \min_i w_i$ and $X < \min_i (n_i - w_i)$ are satisfied. The government provides possible tax schedules (τ_1, τ_2) that satisfy $\tau_1 + \tau_2 \leq X$, $\tau_1 \in \mathbb{N} \cup \{0\}$ and $\tau_2 \in \mathbb{N} \cup \{0\}$ and people choose the tax schedule they prefer by voting, i.e. we exclude the possibility that a (strict) budget surplus tax schedule be adopted.

Finally, we assume people vote truthfully and do not consider strategic voting.

Let us consider several voting schemes to enable us to verify how the Borda count method works.

Proposition 2: Assume that the voting agenda is only about today's tax rate. Then, if $l + k_0 > \max_{m \geq 1} k_m$ is satisfied, the non-budget-balanced solution $\tau_1 = 0$ is chosen under the relative majority voting rule. If $l + k_0 < \max_{m \geq 1} k_m$ is satisfied, $\tau_1 = 0$ is not chosen.

Proof of Proposition 2: Under the relative majority rule, policy choice $\tau_1 = 0$ collects $l + k_0$ votes, where $\tau_1 = m > 0$ collects k_m votes. Therefore, if $l + k_0 > \max_{m \geq 1} k_m$ is satisfied, the non-budget-balanced solution $\tau_1 = 0$ is chosen even if a majority of people are in favor of a balanced budget, i.e. $l < k = \sum_{m=0}^{\frac{D}{k+l}} k_m$. It is easy to see that, if $l + k_0 < \max_{m \geq 1} k_m$, $\tau_1 = \bar{m}$ for $\bar{m} = \operatorname{argmax}_m k_m$ is adopted. (Q.E.D.)

It can be easily seen that it is difficult for a budget-balanced tax plan to be adopted if the voting agenda is only for today's tax rate. Therefore, we can consider taking the intertemporal aspect into account in our voting agenda. However, we can see that it is still difficult to adopt the budget-balanced tax plan under the relative majority voting rule.

Proposition 3: Assume that the voting agenda is the intertemporal tax rate. Then, if $l > \max_m k_m$ is satisfied, the non-budget-balanced solution $(\tau_1, \tau_2) = (0,0)$ is chosen under the relative majority voting rule. If $l < \max_m k_m$ is satisfied, $(\tau_1, \tau_2) = (0,0)$ is not chosen.

Proof of Proposition 3: Each person votes for $(\tau_1, \tau_2) \in \{(\tau_1, \tau_2) | \tau_1 + \tau_2 \leq \mathrm{X},\ \tau_1 \in \mathbb{N} \cup \{0\}$ and $\tau_2 \in \mathbb{N} \cup \{0\}\}$. Consequently, $(0,0)$ collects l votes and $(m, \mathrm{X} - m)$ collects k_m votes, respectively. If $l > \max_m k_m$ is satisfied, the non-budget-balanced solution $(0,0)$ is chosen even if budget-balancing people are in the majority ($l < k$), in which case the majority of people prefer $(m, \mathrm{X} - m)$, whatever value m takes, to the non-budget-balanced solution $(0,0)$. It is easy to see that, if $l < \max_m k_m$, $(\tau_1, \tau_2) = (\bar{m}, X - \bar{m})$ for $\bar{m} = \mathrm{argmax}_m\, k_m$ is adopted. (Q.E.D.)

The result of Proposition 3 results from the fact that the budget-balancing people's vote is split into multiple choices, $(\tau_1, \tau_2) = (m, \mathrm{X} - m)$ for $m \in [0, X]$. We can see that the Borda count method is effective in coping with this spoiler effect.

Proposition 4: Under the Borda count method,

(1) *An almost* budget-balanced tax plan is *asymptotically* chosen if budget-balancing people are in the majority ($k > l$) and the step size of government debt per capita X defined under Assumption 2 is sufficiently small. To be precise, $\forall \varepsilon_1 > 0,\ \forall \varepsilon_2 > 0, \exists X_0$ s.t. $\forall X > X_0$[12]; $\frac{k}{l} > 1 + \varepsilon_1 \Rightarrow$tax plan $(\overline{\tau_1}, \overline{\tau_2})$ is chosen where $\overline{\tau_1} + \overline{\tau_2} > (1 - \varepsilon_2)X$.

(2) *An almost* no-tax plan is *asymptotically* chosen if non-budget-balancing people are in the majority ($k < l$) and the step size of government debt per capita X defined under Assumption 2 is sufficiently small. To be precise, $\forall \varepsilon_1 > 0,\ \forall \varepsilon_2 > 0, \exists X_0$ s.t. $\forall X > X_0$; $\frac{l}{k} > 1 + \varepsilon_1 \Rightarrow$tax plan $(\overline{\tau_1}, \overline{\tau_2})$ is chosen where $\overline{\tau_1} + \overline{\tau_2} < \varepsilon_2 X$.

(3) If budget-balancing people are a $\frac{3}{4}$ majority or more ($k \geq 3l$), budget-balanced tax plan $(\overline{\tau_1}, \overline{\tau_2})$ with $\overline{\tau_1} + \overline{\tau_2} = X$ is always chosen whatever value X takes.

(4) If non-budget-balancing people are more than a $\frac{2}{3}$ majority ($2k < l$), the no-tax plan $(0,0)$ is always chosen whatever value X takes.

Proof of Proposition 4: See Appendix 1

This proposition reveals that the Borda count method assures that the majority's opinion is, at least asymptotically, reflected in tax policy. If budget-balancing people are in the majority and government debt per capita is sufficiently large, a tax plan close to a budget-balanced tax plan is chosen asymptotically. This proposition does not specifically state which tax plan is to be chosen, but says that a tax plan that is approximately budget balanced is to be chosen. If non-budget-balancing people are in the majority and government debt per capita is sufficiently large, a tax plan close to a no-tax plan is chosen asymptotically. Moreover, it is proven that, if budget-balancing people are in a supermajority (75% majority), a budget-balanced tax plan is always chosen and if non- budget-balancing people are in a supermajority (66.7% majority), a no-tax plan is always chosen.

One may complain that, not all possible policy choices are placed on the agendas of political

---

[12] As noted in Assumption 2, this proposition requires either the step size to be sufficiently small or government debt per capita X to be sufficiently large.

parties during actual political campaigns, and thus Proposition 4, in which all possible tax policies are on the agenda, is unrealistic. Realistically, we can consider a case where some policy choices are selected for the agenda. In such a case, we can show that the majority's opinion will *almost certainly* be reflected in tax policy. To be precise, if the number of policy choices on the agenda is sufficiently small, the majority's opinion will almost certainly be reflected in tax policy (Proposition 5). If the number of policy choices on the agenda is sufficiently large, similar to Proposition 4, the opinion of the supermajority of people (75% or 66.7%) will be reflected in tax policy (Proposition 6). These results reinforce our message provided in Proposition 4.

Proposition 5: Consider a case where, for a sufficiently small N, N tax policies are on the agenda. At least a budget-balanced tax policy and a no-tax policy are included on this agenda. Under the Borda count method,

(1) A budget-balanced tax plan is *almost certainly* chosen if budget-balancing people are in a majority ($k > l$), policy choices are randomly distributed, and the step size of government debt per capita X defined under Assumption 2 is sufficiently small.

To be precise, assume there are $N \geq 3$ policy choices where at least one policy is budget balanced ($\tau'_1 + \tau'_2 = X$), one policy is a no-tax plan ($(\tau_1, \tau_2) = (0,0)$) and other choices are uniformly distributed according to $\{(\tau_1, \tau_2) \in \mathbb{Z}^2 | \tau_1 \geq 0, \tau_2 \geq 0, \tau_1 + \tau_2 \leq \mathrm{X}, (\tau_1, \tau_2) \neq (\tau'_1, \tau'_2), (\tau_1, \tau_2) \neq (0,0)\}$. For a sufficiently small N, $\forall \varepsilon > 0$, $\exists X_0$ s.t. $\forall X > X_0$; the budget-balanced tax plan $(\overline{\tau_1}, \overline{\tau_2})$ with $\overline{\tau_1} + \overline{\tau_2} = X$ is chosen with a probability of $1 - \varepsilon$ or more.

(2) A no-tax plan is *almost certainly* chosen if non-budget-balancing people are in a majority ($k < l$), policy choices are randomly distributed, and the step size of government debt per capita X defined under Assumption 2 is sufficiently small.

To be precise, assume there are $N \geq 3$ policy choices where at least one policy is budget balanced ($\tau'_1 + \tau'_2 = X$), one policy is a no-tax plan ($(\tau_1, \tau_2) = (0,0)$) and other choices are uniformly distributed according to $\{(\tau_1, \tau_2) \in \mathbb{Z}^2 | \tau_1 \geq 0, \tau_2 \geq 0, \tau_1 + \tau_2 \leq \mathrm{X}, (\tau_1, \tau_2) \neq (\tau'_1, \tau'_2), (\tau_1, \tau_2) \neq (0,0)\}$. For a sufficiently small N, $\forall \varepsilon > 0$, $\exists X_0$ s.t. $\forall X > X_0$; the no-tax plan $(0,0)$ is chosen with a probability of $1 - \varepsilon$ or more.

Proof of Proposition 5: See Appendix 2

Proposition 5 holds for a sufficiently small number of policy choices on the agenda. If there is a sufficiently large number of policy choices on the agenda, the following proposition holds.

Proposition 6: Consider a case where, for a sufficiently large N, N tax policies are on the agenda. Assume that policy choices on the agenda include at least one budget-balanced policy ($\tau'_1 + \tau'_2 = X$) and one no-tax policy ($(\tau_1, \tau_2) = (0,0)$), and assume that other choices are uniformly distributed according to $\{(\tau_1, \tau_2) \in \mathbb{Z}^2 | \tau_1 \geq 0, \tau_2 \geq 0, \tau_1 + \tau_2 \leq \mathrm{X}, (\tau_1, \tau_2) \neq (\tau'_1, \tau'_2), (\tau_1, \tau_2) \neq (0,0)\}$. Under the Borda count method,

(1) *An almost* budget-balanced tax plan is *asymptotically* and *almost certainly* chosen if budget-balancing people are in the majority ($k > l$) and the step size of government debt per capita X defined under Assumption 2 is sufficiently small.

(2) *An almost* no-tax plan is *asymptotically* and *almost certainly* chosen if non-budget-balancing people are in the majority ($k < l$) and the step size of government debt per capita X defined under Assumption 2 is sufficiently small.

(3) If budget-balancing people are a $\frac{3}{4}$ majority or more ($k \geq 3l$), the budget-balanced tax plan $(\overline{\tau_1}, \overline{\tau_2})$ with $\overline{\tau_1} + \overline{\tau_2} = X$ is *almost certainly* chosen whatever value and stem size X takes.

(4) If non-budget-balancing people are more than a $\frac{2}{3}$ majority ($2k < l$), the no-tax plan $(0,0)$ is

almost certainly chosen whatever value and stem size X takes.

Proof of Proposition 6: See Appendix 3

Integrating these results, the majority's opinion will be *asymptotically* and *almost certainly* reflected in tax policy under the Borda count method, both when N is sufficiently large and when N is sufficiently small. These results represent the superiority of the Borda count method for representing people's preferences.

4. Conclusion

The current voting system is not one-size-fits-all. If policy choices do not incorporate intertemporal aspects, voters have little opportunity to express their real preferences. Furthermore, in reality, the relative majority rule is, although criticized in many ways, widely implemented. However, this voting rule is vulnerable to vote splitting. If there are many choices in which similar ideologies are upheld, it will be difficult for these choices to be chosen under the relative majority rule. Even if policy choices incorporate intertemporal aspects, the relative majority voting rule might not reflect voter preferences, a situation that could be remedied by the Borda count method.

Our paper does not intend to refute the existing explanations regarding why there is a tendency for budget deficits. Our paper intends to present another explanation for widely-implemented voting systems to have a tendency to bring about budget deficits.

Our paper only considers two types of voters; budget-balancing people and non-budget-balancing people. Further research may consider other types of voters, such as semi-budget-balancing people. A caveat of our paper is that one of our main results is valid if budget-balancing people are in the majority, which is not obvious at all. Another caveat of our paper is that it assumes truthful voting and excludes the possibility of strategic voting. Considering the possibility of strategic voting may polish this paper in a theoretical way.

Appendix 1 (Proof of Proposition 4)

Each person votes for $(\tau_1, \tau_2) \in \{(\tau_1, \tau_2) | \tau_1 + \tau_2 \leq \mathrm{X},\ \tau_1 \in \mathbb{N} \cup \{0\} \text{ and } \tau_2 \in \mathbb{N} \cup \{0\}\}$ by Borda voting.

(a) First, we calculate how many points the no-tax plan $(0,0)$ collects. Non-budget-balancing people give maximum points $\frac{(X+1)(X+2)}{2}$ to the no-tax plan $(0,0)$. Therefore, it collects $\frac{(X+1)(X+2)}{2}l$ points from non-budget-balancing people. Budget-balancing people give a minimum of 1 point to the no-tax plan $(0,0)$. Therefore, it collects k points from budget-balancing people. In sum, the no-tax plan $(0,0)$ collects $k + \frac{(X+1)(X+2)}{2}l$ points.

(b) Second, we calculate how many points budget-balanced tax plans (τ_1, τ_2) with $\tau_1 + \tau_2 = \mathrm{X}$ collect *on average*. Non-budget-balancing people give 1 point to $X+1$ points, respectively, on budget-balanced tax plans. Therefore, they collect $\frac{(X+1)(X+2)}{2}l$ points in total from non-budget-balancing people. Budget-balancing people give $\frac{(X+1)(X+2)}{2}$ points to $\frac{(X+1)(X+2)}{2} - X$ points, respectively, to budget-balanced tax plans. Therefore, they collect $(X+1)\frac{(X+1)(X+2)-X}{2}k$ points in total from budget-balancing people. In sum, budget-balanced tax plans collect $(X+1)\frac{(X+1)(X+2)-X}{2}k + \frac{(X+1)(X+2)}{2}l$ points in total and

$\frac{(X+1)(X+2)-X}{2}k+\frac{X+2}{2}l=\frac{X^2+2X+2}{2}k+\frac{X+2}{2}l$ points *on average*.

(c) Third, we calculate how many points a tax plan (τ_1,τ_2) with $\tau_1+\tau_2=p\in(0,X)$ can collect *at most*. Non-budget-balancing people strictly prefer any tax plan (τ_1,τ_2) with $\tau_1+\tau_2<p$ to a tax plan (τ_1,τ_2) with $\tau_1+\tau_2=p$. Therefore, a tax plan (τ_1,τ_2) with $\tau_1+\tau_2=p\in(0,X)$ collects *at most* $(X+1)+X+\cdots+(p+1)=\frac{(X+p+2)(X-p+1)}{2}$ points from a non-budget-balancing person. Budget-balancing people strictly prefer any tax plan (τ_1,τ_2) with $\tau_1+\tau_2>p$ to a tax plan (τ_1,τ_2) with $\tau_1+\tau_2=p$. Therefore, a tax plan (τ_1,τ_2) with $\tau_1+\tau_2=p\in(0,X)$ collects *at most* $1+2+\cdots+(p+1)=\frac{(p+1)(p+2)}{2}$ points from a budget-balancing person. In sum, a tax plan (τ_1,τ_2) with $\tau_1+\tau_2=p\in(0,X)$ can collect *at most* $\frac{(p+1)(p+2)}{2}k+\frac{(X+p+2)(X-p+1)}{2}l$ points.

(d) Consider a case where budget-balancing people are in the majority ($k>l$). As $\frac{X^2+2X+2}{2}k+\frac{X+2}{2}l>k+\frac{(X+1)(X+2)}{2}l$ holds, the no-tax plan $(0,0)$ is never chosen. If a balanced tax plan (τ_1,τ_2) with $\tau_1+\tau_2=\mathrm{X}$ is chosen, (1) and (3) are already proven. Therefore, we will focus on the case where tax plan (τ_1,τ_2) with $\tau_1+\tau_2=p\in(0,X)$ is chosen. In such a case, $\frac{(p+1)(p+2)}{2}k+\frac{(X+p+2)(X-p+1)}{2}l>\frac{X^2+2X+2}{2}k+\frac{X+2}{2}l$ is necessary. Equivalently, $\frac{X^2+2X-(p^2+p)}{2}l>\frac{X^2+2X-(p^2+3p)}{2}k\Leftrightarrow\frac{X^2+2X-(p^2+p)}{X^2+2X-(p^2+3p)}>\frac{k}{l}\Leftrightarrow 1+\frac{2p}{X^2+2X-(p^2+3p)}>\frac{k}{l}$ is necessary. Note that $\frac{2p}{X^2+2X-(p^2+3p)}=0$ when $p=0$ and $\frac{2p}{X^2+2X-(p^2+3p)}$ is a strictly increasing function with p. Let $\varepsilon_1\equiv\frac{k}{l}-1$ and $\varepsilon_2\equiv 1-\frac{p}{X}$. Then, a necessary condition that a tax plan (τ_1,τ_2) with $\tau_1+\tau_2=p\in(0,X)$ is chosen is that $\frac{2(1-\varepsilon_2)}{(2\varepsilon_2-\varepsilon_2^2)X+3\varepsilon_2-1}>\varepsilon_1$ holds. This inequality shows that, whatever value $\varepsilon_1>0$ and $\varepsilon_2>0$ take, a sufficiently large X prevents this inequality from holding. In other words, whatever value $\varepsilon_1>0$ and $\varepsilon_2>0$ take, if X is sufficiently large, $\tau_1+\tau_2$ must be greater than $(1-\varepsilon_2)X$ to be chosen. ε_1 is increasing on the right-hand side of $\frac{2(1-\varepsilon_2)}{(2\varepsilon_2-\varepsilon_2^2)X+3\varepsilon_2-1}>\varepsilon_1$ and ε_2 decreasing on the left-hand side. Thus, (1) is proven and *An almost* balanced-budget tax plan is always *asymptotically* chosen if government debt per capita X is sufficiently large.

(e) Let's continue to consider a case where budget-balancing people are in the majority ($k>l$). (3) is proven because $\max_{p\in(0,X)}\left[1+\frac{2p}{X^2+2X-(p^2+3p)}\right]=\left[1+\frac{2p}{X^2+2X-(p^2+3p)}\right]_{p=X-1}=1+\frac{2(X-1)}{X+2}=\frac{3X}{X+2}<3$ holds. If $\frac{k}{l}\geq 3$, the above finding implies that the inequality $1+\frac{2p}{X^2+2X-(p^2+3p)}>\frac{k}{l}$ never holds. Therefore, a balanced tax plan (τ_1,τ_2) with $\tau_1+\tau_2=\mathrm{X}$ must be chosen in this case.

(f) Next, consider a case where non-budget-balancing people are in the majority ($k<l$). If a no-tax plan $(0,0)$ is chosen, (2) is already proven. Therefore, we will focus on a case where tax plan (τ_1,τ_2) with $\tau_1+\tau_2=p\in(0,X]$ is chosen. In such a case, $\frac{(p+1)(p+2)}{2}k+\frac{(X+p+2)(X-p+1)}{2}l>k+\frac{(X+1)(X+2)}{2}l$ is necessary. Equivalently, $\frac{p^2+3p}{2}k>\frac{p^2+p}{2}l\Leftrightarrow\frac{l}{k}<1+\frac{2}{p+1}$ is necessary. Note that $\frac{2}{p+1}$ is a strictly decreasing function with regard to p. Let $\varepsilon_1\equiv\frac{l}{k}-1$ and $\varepsilon_2\equiv\frac{p}{X}$. Then, a necessary condition that a tax plan (τ_1,τ_2) with $\tau_1+\tau_2=p\in(0,X]$ is chosen is that $\varepsilon_1<\frac{2}{\varepsilon_2X+1}$ holds. This inequality shows that, whatever value $\varepsilon_1>0$ and $\varepsilon_2>0$ take, a sufficiently large X prevents this inequality from holding. In other words, whatever value $\varepsilon_1>0$ and $\varepsilon_2>0$ take, if X is sufficiently large, $\tau_1+\tau_2$ must be smaller than ε_2X to be chosen. ε_2 is decreasing on the right-hand side of $\varepsilon_1<$

$\frac{2}{\varepsilon_2 X+1}$ is and ε_1 is increasing on the left-hand side. Thus, (2) is proven and *an almost* no-tax plan is always *asymptotically* chosen if government debt per capita X is sufficiently large.

(g) Let's continue to consider a case where non-budget-balancing people are in the majority ($k < l$). (4) is proven because $\max_{p \in (0,X]} \left[1+\frac{2}{p+1}\right] = \left[1+\frac{2}{p+1}\right]_{p=1} = 2$ holds. If $\frac{l}{k} > 2$, the above finding implies that the inequality $\frac{l}{k} < 1+\frac{2}{p+1}$ never holds. Therefore, a no-tax plan $(0,0)$ must be chosen in this case.

(Q.E.D.)

Appendix 2 (Proof of Proposition 5)

Let p satisfy $p \in \mathbb{N}$ and $0 < p \leq X$. Let $T \subset \{(\tau_1, \tau_2) \in \mathbb{Z}^2 | \tau_1 \geq 0, \tau_2 \geq 0, \tau_1 + \tau_2 \leq X\}$ be N tax policies on the agenda and Δ_p be the number of tax policies on the agenda which satisfy $\tau_1 + \tau_2 = p$. Note that $\sum_{p=1}^{X} \Delta_p = N-1$ holds because there is a tax policy $(0,0)$ which is not included in $\sum_{p=1}^{X} \Delta_p = N-1$. It is easily proven that $\text{Prob}\{\exists p, \Delta_p \geq 2\} \xrightarrow[X\to\infty]{} +0.$[13] Therefore, it is almost certain that either $\Delta_p = 0$ or $\Delta_p = 1$ holds for all p $(0 < p \leq X)$.

The budget-balanced tax policy collects N points from budget-balancing people in total and 1 point from non-budget-balancing people in total. Therefore, the budget-balanced tax policy collects $Nk + l$ points.

Consider any tax policy with $\tau_1 + \tau_2 = p < X$. Since it is almost certain that either $\Delta_p = 0$ or $\Delta_p = 1$ holds for all p $(0 < p < X)$, if this tax policy is the t^{th} preferred by budget-balancing people, it is the $N+1-t^{\text{th}}$ preferred by non-budget-balancing people. Therefore, it collects $(N+1-t)k + tl$ points. It is obvious that $t \geq 2$ holds.

As a special case, no-tax policy collects $k + Nl$ points.

As $Nk + l > (N+1-t)k + tl$ always holds for $k > l$, statement (1) holds. Also, as $Nk + l < k + Nl$ as well as $(N+1-t)k + tl < k + Nl$ hold for $t \leq N-1$ when $k < l$, statement (2) holds.

(Q.E.D.)

Appendix 3 (Proof of Proposition 6)

Assume that N is sufficiently large to allow the law of large numbers to be applied. Then, it is *almost certain* that there are *approximately* $\frac{N}{\frac{(X+1)(X+2)}{2}}(Q+1) = \frac{2(Q+1)N}{(X+1)(X+2)}$ policies that satisfy $\tau_1 + \tau_2 = Q$. As a special case, substituting $Q = X$, we can show that there are approximately $\frac{2N}{X+2}$

---

[13] [sketch of proof] Consider a case where a budget-balanced tax policy (τ'_1, τ'_2) and no-tax policy $(0,0)$ are given *a priori* and other $N-2$ policy choices (τ_1^1, τ_2^1), (τ_1^2, τ_2^2), $\cdots$, $(\tau_1^{N-2}, \tau_2^{N-2})$ are allocated one by one. There are $E = \frac{(X+1)(X+2)}{2} - 2$ choices to be selected. Before allocating these choices, $\forall p \in (0,X)$; $\Delta_p = 0$ and $\Delta_X = 1$ hold. The first policy does not make any Δ_p greater than one with a probability of $\frac{E-X}{E}$. (Among E choices, only X budget-balanced choices make Δ_p greater than 1.) The second choice does not make any Δ_p greater than one with a probability of $\frac{(E-1)-X-(X-1)}{E-1}$ or more. The third choice does not make any Δ_p greater than one with a probability of $\frac{(E-2)-X-(X-1)-(X-2)}{E-2}$ or more.⋯ N-2th choice does not make any Δ_p greater than one with a probability of $\frac{\{E-(N-3)\}-X-(X-1)-(X-2)-\cdots-\{X-(N-3)\}}{E-(N-3)}$ or more. Therefore, as N is assumed to be sufficiently small, $1 - \text{Prob}\{\exists p, \Delta_p \geq 2\} \geq \frac{E-X}{E} \cdot \frac{(E-1)-X-(X-1)}{E-1} \cdot \frac{(E-2)-X-(X-1)-(X-2)}{E-2} \cdots \frac{\{E-(N-3)\}-X-(X-1)-(X-2)-\cdots-\{X-(N-3)\}}{E-(N-3)} \xrightarrow[N\ small,\ X\to\infty]{} 1$ and thus $\text{Prob}\{\exists p, \Delta_p \geq 2\}$ is asymptotically zero if X is sufficiently large.

budget- balanced policies.

(a) First, we calculate how many points the no-tax plan $(0,0)$ collects. Non-budget-balancing people give maximum points N to the no-tax plan $(0,0)$. Therefore, it collects Nl points from non-budget-balancing people. Budget-balancing people give a minimum 1 point to the no-tax plan $(0,0)$. Therefore, it collects k points from budget-balancing people. In sum, the no-tax plan $(0,0)$ collects $k+Nl$ points.

(b) Second, we calculate how many points budget-balanced tax plans (τ_1,τ_2) with $\tau_1+\tau_2=\mathrm{X}$ collect *on average*. Non-budget-balancing people give 1 point to $\frac{2N}{X+2}$ points, respectively, to budget-balanced tax plans. Therefore, they collect $\frac{\frac{2N}{X+2}\left(\frac{2N}{X+2}+1\right)}{2}l$ points in total from non-budget-balancing people. Budget-balancing people give N points to $\frac{\frac{2N}{X+2}\left(2N-\frac{2N}{X+2}+1\right)}{2}$ points, respectively, to budget balanced tax plans. Therefore, they collect $\frac{\frac{2N}{X+2}\left(2N-\frac{2N}{X+2}+1\right)}{2}k$ points in total from budget-balancing people. In sum, budget-balanced tax plans collect $\frac{N}{X+2}\left\{\left(N-\frac{N}{X+2}+\frac{1}{2}\right)k+\left(\frac{N}{X+2}+\frac{1}{2}\right)l\right\}$ points in total and $\left(N-\frac{N}{X+2}+\frac{1}{2}\right)k+\left(\frac{N}{X+2}+\frac{1}{2}\right)l$ points *on average*.

(c) Third, we calculate how many points a tax plan (τ_1,τ_2) with $\tau_1+\tau_2=p\in(0,X]$ can collect *at most*. Non-budget-balancing people strictly prefer any tax plan (τ_1,τ_2) with $\tau_1+\tau_2<p$ to a tax plan (τ_1,τ_2) with $\tau_1+\tau_2=p$. Therefore, a tax plan (τ_1,τ_2) with $\tau_1+\tau_2=p\in(0,X]$ collects *at most* $\sum_{Q'=Q}^{X}\frac{2(Q'+1)N}{(X+1)(X+2)}=\frac{(X+Q+2)(X-Q+1)N}{(X+1)(X+2)}$ points from a non-budget-balancing person. Budget-balancing people strictly prefer any tax plan (τ_1,τ_2) with $\tau_1+\tau_2>p$ to a tax plan (τ_1,τ_2) with $\tau_1+\tau_2=p$. Therefore, a tax plan (τ_1,τ_2) with $\tau_1+\tau_2=p\in(0,X]$ collects *at most* $\sum_{Q'=0}^{Q}\frac{2(Q'+1)N}{(X+1)(X+2)}=\frac{(Q+1)(Q+2)N}{(X+1)(X+2)}$ points from a budget-balancing person. In sum, a tax plan (τ_1,τ_2) with $\tau_1+\tau_2=p\in(0,X]$ can collect *at most* $\frac{(Q+1)(Q+2)N}{(X+1)(X+2)}k+\frac{(X+Q+2)(X-Q+1)N}{(X+1)(X+2)}l$ points.

(d) Consider a case where budget-balancing people are in the majority ($k>l$). As $\left(N-\frac{N}{X+2}+\frac{1}{2}\right)k+\left(\frac{N}{X+2}+\frac{1}{2}\right)l>k+Nl$ holds, a no-tax plan $(0,0)$ is never chosen. If a balanced tax plan (τ_1,τ_2) with $\tau_1+\tau_2=\mathrm{X}$ is chosen, (1) is already proven. Therefore, we will focus on a case where a tax plan (τ_1,τ_2) with $\tau_1+\tau_2=Q\in(0,X)$ is chosen. In such a case, $\frac{(Q+1)(Q+2)N}{(X+1)(X+2)}k+\frac{(X+Q+2)(X-Q+1)N}{(X+1)(X+2)}l>\left(N-\frac{N}{X+2}+\frac{1}{2}\right)k+\left(\frac{N}{X+2}+\frac{1}{2}\right)l$ is necessary. Equivalently, $\frac{k}{l}<\frac{2N(X+1)(X+2)-(X+1)(X+2)-2N(X+1)-2NQ(Q+1)}{2N(X+1)(X+2)+(X+1)(X+2)-2N(X+1)-2N(Q+1)(Q+2)}$ is necessary. Let $\varepsilon_1\equiv\frac{k}{l}-1$ and $\varepsilon_2\equiv1-\frac{Q}{X}$. Whatever value $\varepsilon_1>0$ and $\varepsilon_2>0$ take, for a sufficiently large N, a sufficiently large X prevents this inequality from holding.[14] In other words, whatever value $\varepsilon_1>0$ and $\varepsilon_2>0$ take, for a sufficiently large N, if X is sufficiently large, $\tau_1+\tau_2$ must be greater than $(1-\varepsilon_2)X$ to be chosen. Thus, (1) is proven.

(e) Next, consider a case where non-budget-balancing people are in a majority ($k<l$). If a no-tax plan $(0,0)$ is chosen, (2) is already proven. Therefore, we will focus on a case where a tax plan (τ_1,τ_2) with $\tau_1+\tau_2=Q\in(0,X]$ is chosen. In such a case, $\frac{(Q+1)(Q+2)N}{(X+1)(X+2)}k+\frac{(X+Q+2)(X-Q+1)N}{(X+1)(X+2)}l>k+Nl$ is necessary. Equivalently, $\frac{k}{l}>\frac{Q(Q+1)N}{(Q+1)(Q+2)N-(X+1)(X+2)}$ is

[14] Suppose N takes its maximum value $\frac{(X+1)(X+2)}{2}$. The inequality is then deduced to be $\frac{k}{l}<\frac{X^2+2X-Q^2-Q}{X^2+2X-Q^2-3Q}$. Since $\varepsilon_2\equiv1-\frac{Q}{X}$, this inequality is equivalent to $\frac{k}{l}<\frac{(2\varepsilon_2-\varepsilon_2^2)X+1+\varepsilon_2}{(2\varepsilon_2-\varepsilon_2^2)X-1+3\varepsilon_2}$. A sufficiently large X prevents this inequality from holding. An identical discussion holds for a sufficiently large N.

necessary. Let $\varepsilon_1 \equiv \frac{l}{k} - 1$ and $\varepsilon_2 \equiv \frac{Q}{X}$. Whatever value $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ take, for a sufficiently large N, a sufficiently large X prevents this inequality from holding.[15] In other words, whatever value $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ take, for a sufficiently large N, if X is sufficiently large, $\tau_1 + \tau_2$ must be smaller than $\varepsilon_2 X$ to be chosen. Thus, (2) is proven.

(f) The sufficient condition that a budget-balanced tax policy is adopted is $\forall Q \in (0,X); \left(N - \frac{N}{X+2} + \frac{1}{2}\right)k + \left(\frac{N}{X+2} + \frac{1}{2}\right)l > \frac{(Q+1)(Q+2)N}{(X+1)(X+2)}k + \frac{(X+Q+2)(X-Q+1)N}{(X+1)(X+2)}l \wedge \left(N - \frac{N}{X+2} + \frac{1}{2}\right)k + \left(\frac{N}{X+2} + \frac{1}{2}\right)l > k + Nl$. By simple calculation, this condition is equivalent to $\forall Q \in (0,X); \frac{k}{l} > \frac{2N(X+1)(X+2)-(X+1)(X+2)-2N(X+1)-2NQ(Q+1)}{2N(X+1)(X+2)+(X+1)(X+2)-2N(X+1)-2N(Q+1)(Q+2)} \wedge k > l$. For a sufficiently large N, this condition is equivalent to $\frac{k}{l} > \frac{2N(3X+1)-(X+1)(X+2)}{2N(X+1)+(X+1)(X+2)} \wedge k > l$ and thus $\frac{k}{l} \geq 3$ is a sufficient condition that a budget-balanced tax policy is chosen. Therefore, (3) is proven.

(g) The sufficient condition that a no-tax policy is adopted is $\forall Q \in (0,X]; \frac{(Q+1)(Q+2)N}{(X+1)(X+2)}k + \frac{(X+Q+2)(X-Q+1)N}{(X+1)(X+2)}l < k + Nl$. By simple calculation, this condition is equivalent to $\forall Q \in (0,X]; \frac{k}{l} < \frac{Q(Q+1)N}{(Q+1)(Q+2)N-(X+1)(X+2)}$. Since $\min_{Q\geq 1,N,X} \frac{Q(Q+1)N}{(Q+1)(Q+2)N-(X+1)(X+2)} = \frac{1}{2}$ holds, $\frac{k}{l} < \frac{1}{2}$ is a sufficient condition that a no-tax policy is chosen. Therefore, (4) is proven.

(Q.E.D.)

References

Alesina, Alberto and Allan Drazen (1991) "Why are Stabilizations Delayed?" *American Economic Review*, 81(5), 1170–1177.

Alesina, Alberto and Guido Tabellini (1990) "A Positive Theory of Fiscal Deficits and Government Debt." *The Review of Economic Studies*, 57(3), 403–414.

Barro, Robert J. (1979) "On the Determination of the Public Debt." *Journal of Political Economy*, 87(5), 940-971.

Borda, Jean-Charles de (1784) "Mémoire sur les élections au scrutin par M. de Borda" in *Mémoires de l'Académie Royale des Sciences année*, Paris, l'Imprimerie Royale; translated in Mclean and Urken (1995).

Buchanan, James M. (1987) "Budgetary Bias in Post-Keynesian Politics: The Erosion and Potential Replacement of Fiscal Norms." edited by James M. Buchanan, Charles K. Rowley, and Robert D. Tollison, *Deficits,* New York: Blackwell. Reprinted in "Debt and Taxes" (2000), vol.14 of The Collected Works of James M. Buchanan. Indianapolis: Liberty Fund.

Buchanan, James M. and Richard E. Wagner (1977) "Democracy in Deficit: The Political Legacy of Lord Keynes." New York: Academic Press.

Buchanan, James M. (1976). "Barro on the Ricardian Equivalence Theorem." *Journal of Political Economy* 84 (2), 337–342.

Caselli, Francesco (1997) "On the distribution of debt and taxes." *Journal of Public Economics* 65, 367-386.

Cogan, John (1993) "The Evolution of Congressional Budget Decision Making and the Emergence

15 Suppose N takes its maximum value $\frac{(X+1)(X+2)}{2}$. The inequality is then deduced to be $\frac{k}{l} > \frac{Q+1}{Q+3}$. Since $\varepsilon_2 \equiv \frac{Q}{X}$, this inequality is equivalent to $\frac{k}{l} > \frac{\varepsilon_2 X+1}{\varepsilon_2 X+3}$. A sufficiently large X prevents this inequality from holding. An identical discussion holds for a sufficiently large N.

of Federal Deficits." In *The Great Budget Puzzle*, edited by Cogan, Timothy Muris, and Allen Schick.

Condorcet, Marie Jean Antoine Nicolas de Caritat, marquis de (1785) "*Essai sur l'application de l'analyse à la probabilitié des décisions rendues à la pluralité des voix*." Paris, l'Imprimerie Royale; translated in Mclean and Urken (1995).

Cuadra, Gabriel, Juan M. Sanchez, and Horacio Sapriza (2010) "Fiscal policy and default risk in emerging markets." *Review of Economic Dynamics* 13(2), 452–469.

Fraenkel, Jon and Bernard Grofman (2014) "The Borda Count and its real-world alternatives: Comparing scoring rules in Nauru and Slovenia." *Australian Journal of Political Science* 49(2), 186–205.

Golder, Matt (2005) "Democratic systems round the world, 1946–2000." *Electoral Studies* 24, 103–121.

Hattori, Takahiro and Ryo Ishida (2020a) "Did the Introduction of Bitcoin Futures Crash the Bitcoin Market at the End of 2017?" *North American Journal of Economics and Finance*, 101322.

Hattori, Takahiro and Ryo Ishida (2020b) "The relationship between arbitrage in futures and spot markets and Bitcoin price movements: Evidence from the Bitcoin markets." Journal of Futures Markets 41(1), 105-114.

Ishida, Ryo (2015) "Vote with their donations: An explanation about crowding-in of government provision of public goods." PRI Discussion Paper Series No.15A-07.

Ishida, Ryo and Kazumasa Oguro (2018) "The Viability of a Voting System That Allocates Parliamentary Seats According to Life Expectancy: An Analysis Using OLG Models." Japan and The World Economy 47, 51-57.

Mochida, Nobuki (2016) "Medium-sized Social Spending with Low Tax Burden – Tax payer's attitude about Japan's fiscal conundrum." CIRJE Discussion Paper, CIRJE-J-283, Center for International Research on the Japanese Economy, the University of Tokyo, *in Japanese.*

Müller, Andreas, Kjetil Storesletten, and Fabrizio Zilibotti (2016) "Sovereign Debt and Structural Reforms." mimeo.

New York Times, The (2004) "The Next Nader Effect", March 9.

Oguro, Kazumasa, Ryo Ishida and Masaya Yasuoka (2018) "Voluntary Provision of Public Goods and Cryptocurrency." RIETI Discussion Paper Series 18-E-081.

Oguro, Kazumasa, Manabu Shimasawa, Reiko Aoki and Takashi Oshio (2013) "Demographic Change, Intergenerational Altruism, and Fiscal Policy - A Political Economy Approach -." Studies in Applied Economics 6, 1-15.

Romer, Thomas and Howard Rosenthal (1978) "Political Resource Allocation, Controlled Agendas, and the Status Quo." *Public Choice* 33, 27–43.

Weingast, Barry, Kenneth A. Shepsle, and Christopher Johnsen (1981) "The political economy of benefits and costs: A neoclassical approach to distributive politics." *Journal of Political Economy*, 84(4): 642–664.

Young, H.P. (1988) "Condorcet's Theory of Voting." *American Political Science Review* 82, 1231–1244.