重区分線形特性を有する超拡大カオス発生回路の解析と実装

KIMURA, Kazuyuki / 木村, 一行

(出版者 / Publisher)
法政大学大学院理工学・工学研究科
(雑誌名 / Journal or Publication Title)
法政大学大学院紀要. 理工学・工学研究科編
(巻 / Volume)
56
(開始ページ / Start Page)
1
(終了ページ / End Page)
4
(発行年 / Year)
2015-03-24
(URL)
https://doi.org/10.15002/00011130

重区分特性を有する超拡大カオス発生回路の解析と実装

ANAYSIS AND IMPLEMENTATION OF SUPER-EXPANDONG CHAOS GENERATING MANIFOLD PIECEWISE LINEAR CIRCUITS

木村 一行
Kazuyuki KIMURA

指導教員 斎藤利通

法政大学大学院理工学研究科電気電子工学専攻修士課程

This paper presents a novel autonomous chaotic system: the manifold piecewise linear system on the cylinder. This system is defined by second order continuous flow on the cylinder with hysteresis switching and the trajectories do not diverge. The system on the cylinder is equivalent to the systems having infinite equilibria. This system can exhibit super expanding chaos characterized by very large positive Lyapunov exponent. Presenting a simple test circuit, the super expanding chaos is confirmed experimentally.

Key Words : *chaos, manifold piecewise linear systems, return map*

1. Introduction

The manifold piecewise linear system (MPL) is an autonomous chaotic system that consists of second-order continuous system and hysteresis switching of the equilibrium points [1]-[3]. As the most important characteristics of the MPL, it should be noted that the dynamics is integrated into a piecewise linear one-dimensional return map and chaotic generation is guaranteed theoretically. The MPL is realized by a simple electric circuit and chaotic behavior is confirmed in the laboratory. It should also be noted that the chaotic behavior is applicable to engineering systems such as chaos-based communication and radar systems [4][5]. Up to the present, a variety of autonomous chaotic systems have been presented and the dynamics have been analyzed [6]-[7]. These systems have been contributed to development of nonlinear dynamical system theory and its engineering applications.

This paper presents a novel autonomous chaotic system: the manifold piecewise linear system on the cylinder (CMPL). The CMPL is defined by second order continuous flow on the cylinder with hysteresis switching and the trajectories do not diverge. The system dynamics on the cylinder is equivalent to systems having infinite equilibria. Especially, the CMPL can exhibit super expanding chaos characterized by very large positive Lyapunov exponent. In the MPL, the divergent trajectory is inevitable and the super expanding chaos is impossible. Presenting a simple test circuit, the super expanding chaos is confirmed experimentally.

2. Manifold Piecewise Linear System

Here, we introduce the MPL as preparation to present the CMPL. The MPL is defined by the following second-order piecewise linear system with hysteresis switching.

$$\ddot{x} - 2\delta\dot{x} + x = \begin{cases} p & (A) \\ -p & (B) \end{cases}, \quad \mathrm{x} \equiv (x, \dot{x}) \tag{1}$$

This system has two equilibria $\pm p$. In order to define the switching rule, we define the following two segments

$$\begin{aligned} L_{2+} &\equiv \{\mathrm{x} \mid x \geq 0,\ \dot{x} = 0\} \\ L_{2-} &\equiv \{\mathrm{x} \mid x < 0,\ \dot{x} = 0\}. \end{aligned} \tag{2}$$

The right hand side is switched from (A) to (B) if the trajectory hits L_{2+} and is switched from (B) to (A) if the trajectory hits L_{2-}. The MPL is characterized by two parameters: damping δ, equilibrium point p. For simplicity, we assume:

$$0 < \delta < 1,\ (\omega \equiv \sqrt{1-\delta^2}),\ p > 0 \tag{3}$$

The system has unstable complex characteristic roots $\delta \pm j\omega$. The trajectory rotates divergently around the equilibrium point p or $-p$. If the trajectory hits L_{2+},

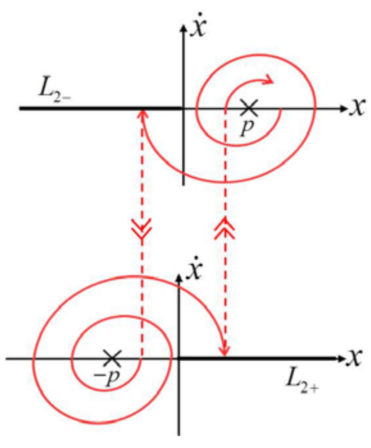

図 1 Switching of the MPL

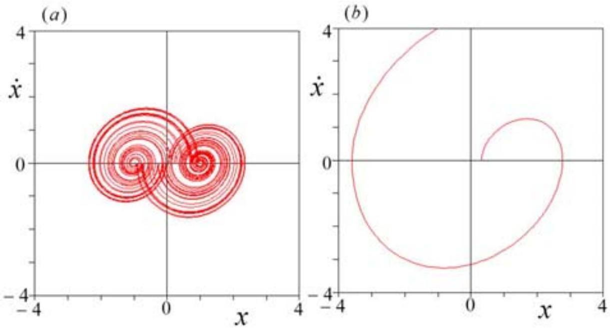

図 2 Trajectories. (a) Double-screw chaos for $\beta = \sqrt{2}$, (b) Divergence for $\beta = 2.6$

the equilibrium point is switched from p to $-p$. If the trajectory hits L_{2-}, the equilibrium point is switched from $-p$ to p. Note that the switching occur only on the x-axis. This system repeats in this manner. As shown in Fig. 2 (a), the MPL exhibits double-screw chaotic attractor. For convenience, we use the following parameter β hereafter.

$$\beta \equiv \mathrm{e}^{\frac{\delta\pi}{\omega}} > 1 \tag{4}$$

If the trajectory does not diverge the MPL has a positive Lyapunov exponent $\ln\beta$. Chaos generation is guaranteed if

$$1 < \beta < 2 \tag{5}$$

Fig. 2 shows typical trajectories where β is used as a parameter instead of δ, for convenience.

3. MPL on Cylinder

We propose the CMPL based on the switching rule of the MPL. As a preparation, we define the infinite-screw MPL with infinite equilibria on x-axis. In order to define switching rule, we define the following two segment

$$\begin{aligned} L_{n+} &\equiv \{\mathrm{x} \mid 2nT < x \le (2n+1)T,\ \dot{x} = 0\} \\ L_{n-} &\equiv \{\mathrm{x} \mid (2n-1)T < x \le 2nT,\ \dot{x} = 0\} \\ L_n &\equiv L_{n+} \cup L_{n-}. \end{aligned} \tag{6}$$

where $n \in$. The dynamics of the infinite-screw MPL on L_n is described by

$$\ddot{x} - 2\delta\dot{x} + x = \begin{cases} p + 2nT & (A_n) \\ -p + 2nT & (B_n) \end{cases}, \quad \mathrm{x} \in L_n \tag{7}$$

We consider the case: $0 < p < T$. The right hand side is switched to (A_n) if the trajectory hits L_{n+} and is switched to (B_n) if the trajectory hits L_{n-}. The trajectory rotates divergently around the equilibrium point $p+2nT$ or $-p+2nT$. If the trajectory hits L_{n+}, the equilibrium point is switched to $p+2nT$. If the trajectory hits L_{n-}, the equilibrium point is switched to $-p+2nT$. The system repeats this switching. The trajectory does not diverge in this system if β is finite. The system can exhibit multi-screw chaotic attractor as shown in Fig. 3(a).

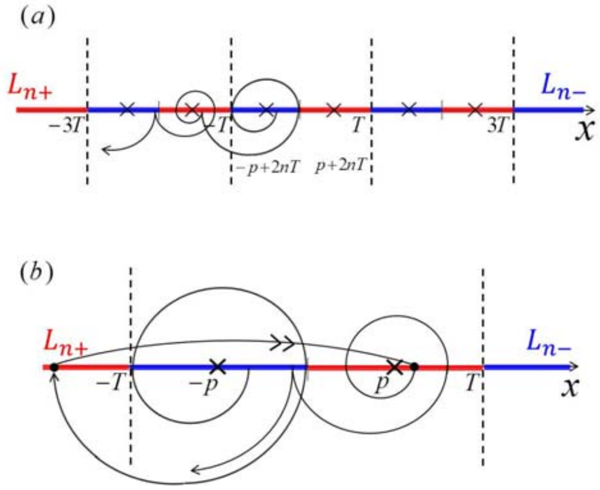

図 3 Switching rules. (a) Infinite-screw MPL (b) CMPL

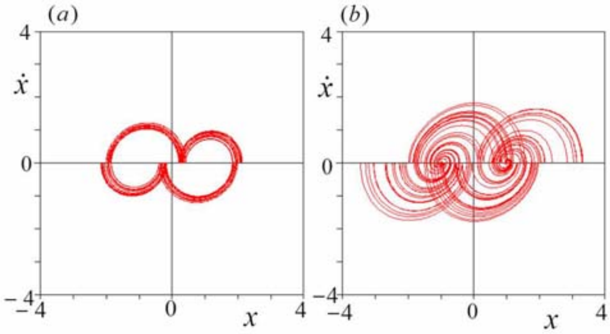

図 4 Trajectories. (a) Chaos for $\beta = \sqrt{2}$, (b) Super expanding chaos for $\beta = 2.6$

We propose the MPL on cylinder (CMPL). The dynamics of the CMPL is equivalent to the dynamics of the infinite-screw MPL.

If x hits L_{n+}, the right hand side is switched to (A_n) and x jumps to x_0 on L_{0+} where $\mathrm{x}_0 \equiv (\mathrm{x} + T) \bmod (2T) - T$. If x hits L_{n-}, the right hand side is switched to (B_n) and x jumps to x_0 on L_{0+}. The CMPL repeats this manner. Fig. 3 (b) illustrates switching of the CMPL. Note that the CMPL is characterized by three parameters: damping δ, the equilibrium point p and the circumference $2T$. For simplicity, we consider on the following parameter range:

$$1 < \beta < 3,\ p = \tfrac{T}{2} = 1 \tag{8}$$

The CMPL exhibits various chaotic phenomena as shown in Fig. 4. The MPL for $\beta \ge 2$ as shown in Fig. 2(b). For $\beta \ge 2$, the MPL exhibits divergent trajectory (Fig. 2 (b)), whereas the CMPL can exhibit chaos (Fig. 4 (b)), this is the super-expanding chaos characterized by very large Lyapunov exponent.

4. Laboratory Experiments

In order to observe the super expanding chaotic behavior, we have fabricated a simple test circuit as shown in Fig. 5. The test circuit of an equivalent to the CMPL. The test circuit is implemented by an OpAmp, a monostable multivibrator (M.M.), a flip-flop, and an analog switch. Here, v_1 and v_2 are capacitor voltages, V_{th} is a threshold voltage, and E is an equilibrium point voltage. If $v_1 < 0$ and $v_2 = 0$, an equilibrium point is switched from E to $-E$. If $v_1 > V_{th}$ and $v_2 = 0$, S_1 is closed. The capacitor voltage v_1 jumps to $v_1 - 2V_{th}$ and equilibrium point is switched from E to $-E$ simultaneously. Repeating in this manner, we can define the switching from $-E$ to E. This switching is equivalent to the CMPL. Figure 6 shows laboratory measurements. The data corresponds to Fig.4. We have verified the super expanding chaotic attractor for $\beta \geq 2$ in the laboratory experiment as shown in Fig. 6(b).

5. Conclusions

In this paper, we have presented the CMPL. For $\beta \geq 2$, the trajectory diverges in the MPL. For $\beta \geq 2$, the CMPL exhibits super expanding chaos. The trajectory does not diverge on the cylinder if β is finite. The system can exhibit various chaotic attractor. Using a simple test circuit, super expanding chaos attractor can be verified in the laboratory experiment.

Future problems include analysis in wider parameter range, development into higher order systems and engineering applications.

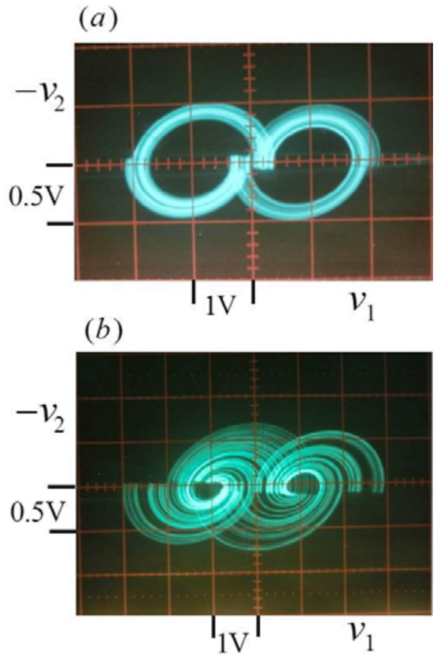

図 6 Laboratory measurements. $r_1 \simeq 1k\Omega$, $r_2 \simeq 1k\Omega$, $R_3 \simeq 1k\Omega$, $R_4 \simeq 5k\Omega$, $R_5 \simeq 5k\Omega$, $R_6 \simeq 1k\Omega$, $R_s \simeq 10k\Omega$, $C_1 \simeq 0.033\mu$F, $C_2 \simeq 0.033\mu$F, $V_{th} \simeq 2$V, $E \simeq 1$V (a)Chaos for $\beta \simeq \sqrt{2}$, $R_1 \simeq 4.9k\Omega$, $R_2 \simeq 10k\Omega$ (b)Super expanding chaos for $\beta \simeq 0.26$, $R_1 \simeq 5.1k\Omega$, $R_2 \simeq 3.9k\Omega$

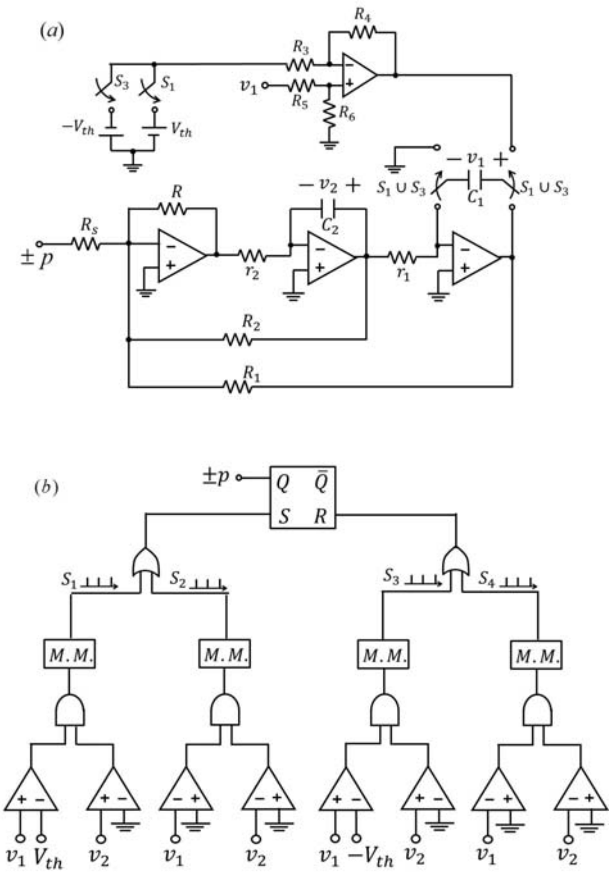

図 5 An implementation example. (a) Test circuit. (b) Switching circuit

参考文献

1) H. Fujuta and T. Saito, Continuous chaos represented by a nonlinear ordinary differential equation with manifold piecewise linear characteristics, in Proc. Int. Wiss. Koll., A-1, Ilmenau, pp. 11-14, 1981.
2) T. Saito and H. Fujuta, Chaos in a Manifold Piecewise Liner System, Trans. IECE, 64-A, 10, pp. 827-834, 1981.
3) T. Tsubone and T. Saito, Stabilizing and Destabilizing Control for a Piecewise Linear Circuit. IEEE Trans., CAS-I, 45, 2, pp. 172-177, 1998.
4) N. Corron and J. Blakely, Chaos for Communication and Radar, in Proc. NOLTA, pp. 322-325, 2011.
5) N. Corron, M. Stahl, J. Blakely, Experimental Ranging System Using Exactly Solvable Chaos. in Proc. NOLTA, pp. 454-457, 2012.
6) E. N. Lorenz, Deterministic nonperiodic flow. J. Atom. Sci., 20, pp. 130-141, 1963.
7) T. Matsumoto, L. O. Chua, M. Komuro, The Double Scroll, IEEE Trans. CAS. 32, 8, pp. 798-818, 1985.