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The Impacts of Firms' Technology Choice on the Gender Differences in Wage and Time Allocation: A Cross-Country Analysis*

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Abstract

This paper investigates the impacts of firms' technology choice on the cross-country variations in gender gaps, especially those of the wage and time devoted to home production which vary from country to country. For this purpose, we construct a general equilibrium model with firms' technology choice as well as home production. The term technology includes labor market institutions, corporate culture and so on that would affect the labor productivity of each gender in different ways and reflects the relative labor abundance of each gender. The numerical results show that the cross-country variations in both gender wage and time gaps are considerably affected by the technology choice, suggesting the persistence of the gender gap; and that a convergence in the technology choice across countries does not imply smaller cross-country variations in all measure of the gender gaps.

Keywords: appropriate technology choice, gender wage gap, home production

JEL classification: E13, E24, D13, J22, J16, D58

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1 Introduction

There are still much variations in the gender gaps of wage rates and time spent for home production even among the developed countries, despite the passing equal pay act and equal opportunity laws and progressing higher education of female. What cause these differences in both the wage and time gaps across countries? Is there an unique mechanism that will explain the variations in both wage and time gaps?

This paper investigates a cross-country variation of the gender wage gap (hereafter *wage gap*) and the home production time gap (hereafter *time gap*) among a sample of eight industrialized nations.*¹ We focus on home production hours in spite of focusing on market work hours by many studies such as Olovsson (2004), Ohanian et al. (2008) and McDaniel (2011) because home production is more volatile than market hours between countries. In addition, recent works emphasize importance of relationship between market work and home production when comparing cross-country differences in time use.*² But, there is not a single work which provides a cross-country analysis of time gap except ours, to the best of our knowledge.

Meanwhile, since last century, increase in female labor supply is observed in many countries and this tendency will seem to continue. Many developed countries are promoting the participation of female in the labor market to achieve work-life balance and deal with the declining birthrate and aging society. Changing female relative labor supply can lead to technology and institution change more appropriate to female worker, e.g., directed technical change à la Acemoglu (2002). If labor market institution become equalized among countries, what happens to change the wage gap and the time gap?

In order to answer these questions, we first construct a general equilibrium model of the gender wage gap with firms' technology choice and home production of households consisting of two different marital status: single and couple. Firms can choose their production technologies as well as labor inputs. Depending on the factor abundance and relative cost of choosing different technologies, firms' technologies can be biased towards either male or female, resulting in the wage gap. Term technology in this context can be broadly interpreted, and it includes labor market institutions, corporate culture, personnel allocation, employment regulations and social norms that affects workers' productivities.

We then calibrate parameters in such a way that the equilibrium matches the data under the calibrated parameters. Except for technology choice, the specification of the model follows

*¹ These countries consist of Austria, Germany, Italy, Japan, Netherlands, Spain, the United Kingdom (U.K.), and the United States (U.S.).

*² Prescott (2004) stresses the role of the cross-country difference in tax rates in explaining the difference in market hours worked between U.S. and European countries using a simple neoclassical framework without home production. Later, Rogerson (2009) reports that home production drastically changes the relationship between the taxes and market hours worked.

the standard model in the literature and also focuses on the plainest form in order to make interpretations easiest. The advantages of this strategy are that given the limited availability of the time use data, especially on housework, we can still identify all the relevant parameters; and also that we can still identify the impacts of firms' technology choice on the gender gaps which is of our main interest and is clearly defined compared with other possible sources of the gender gaps which have multiple interpretations due to our calibration strategy.

The model is an application of the Caselli and Coleman (2006)'s framework to the gender gap context. They consider the relationship between the skill premium and the relative abundance of skilled workers in order to explain the cross-country differences in income per worker. We treat the gender gaps instead of the skill premium. Specifically, in our context, inputs consist of male and female labor, and the technology choice friction is interpreted as relative costs of choosing different technologies which affect the wage rate of each gender unevenly. The assumption that firms distinguish male labor and female labor is supported by the previous literature which suggests that the elasticity of substitution between male and female in market activities ranges from two to three (Olivetti and Petrongolo, 2011). One interpretation of relative costs is labor market discrimination such as the taste-based discrimination discussed by Becker (1971). General equilibrium approach then generates rich interactions between the wage and time gaps which are often neglected in the labor economics literatures.

Given this approach, we restate the previous questions as follows: What are the impacts of firms' technology choice on the cross-country variation in the observed gender differences in wage and time allocation. Are the sources of the variations the same for both wage and time gaps? In order to answer these questions, we conduct counterfactual simulations which compare equilibria under appropriate and inappropriate technology choice, where firms can and cannot choose their technology depending on their environments, respectively.

The main finding is that technology choice has considerable impacts on the cross-country variations in not only the wage gap but also the time gaps of both single and couple households in the sense that the observed cross-country variation in technology can affect the equilibria of countries and thus gender gaps significantly. Not surprisingly, technology choice reproduces a non-negligible part of the observed cross-country variation in the wage gap, and this is also the case for the time gap of the single households. What is, however, noteworthy is the contrasting result of the time gap of the couple households. That is, in the case of the couple time gap, technology choice contributes to a reduction in the cross-country variation. This is mainly because an important part of the observed cross-country variation in the couple time gap is due to the cross-country variation in the factors related to home production, the effect of which and that of technology choice on the cross-country variation offset each other.

Two policy implications are drawn from these results: The first is that there exists the major difficulties in narrowing the gender gaps. This is because these gaps arises, to a large extent, from technology choice which is broadly interpreted and thus includes the labor mar-

ket institutions, corporate culture and social norms which are difficult to change dramatically. The second is that the global policy coordination aiming to narrow the gender gaps by affecting firms' technology choice, even if succeeded to alter firms' behavior and make differences in technology choice across countries smaller, might not result in smaller gender gaps in all measures. Rather, while achieving smaller gaps in the wage gap and time gap of the single households, such a policy is associated with a widened cross-country variation in the time gap of the couple households, i.e., in some countries the couple time gap might shrink, but the other countries might experience higher time gap.

There are some empirical works that conduct an international comparison of gender wage gap, e.g., Blau and Kahn (1992, 1995, 1996a, 1996b, 2003) and Olivetti and Petrongolo (2008, 2011). In labor economics, institution is one of the main topics as comprehensively reviewed in, Blau and Kahn (1999), Nickell and Layard (1999) and Boeri (2011). Blau and Kahn also argued that institutions have a explanatory power of cross-country differences of the wage gap. However, due to their approach based on the traditional reduced form regression, they evaluate partial equilibrium effects while we overcome this limitation by using a general equilibrium model that is able to assess indirect effects of changing equilibria. Another difference is that while Blau and Kahn treat only observed exogenous effects of institution, e.g., parental leave and degree of occupational segregation by gender, we assume endogenous institutions which are included in TFP such as economic growth model, e.g., Jones and Romer (2010). These treatments can assess some unobserved technology and institution effect to productivities.

The structure of this paper is as follows: We first provide the model in Section 2. Then we calibrate the model and quantify the effects of firms' technology choice on the cross-country variations in the gender wage and time gaps for a benchmark case in Section 3, which is followed by the robustness analysis in Section 4. Finally, in Section 5, we conclude the paper.

2 The Model

We consider a closed economy with no capital stock.^{*3} Economic agents consist of firms, households and the government. Further, households are divided into two groups: single and couple. In addition to production activities of firms, home production take place in each type of households. The government conducts only an income redistribution policy. All the markets are competitive.

2.1 Firms

Competitive firms use male labor L_m and female labor L_f , which are measured in terms of efficiency unit. The production technology exhibits constant returns to scale (CRS) and is

^{*3} In Section 4, this assumption is relaxed in order to perform robustness checks of our main results.

specified by a constant elasticity of substitution (CES) form:

$$Y = [(A_m L_m)^\sigma + (A_f L_f)^\sigma]^{\frac{1}{\sigma}}, \quad \sigma < 1, \quad (1)$$

where Y is output, and A_s is sex- s labor augmenting technology. σ determines the elasticity $1/(1-\sigma)$ of substitution between male labor and female labor. This general form of production function is used in order to take account of the literature. The empirical studies on the elasticity of substitution is few, but these studies consistently suggest that the elasticity of substitution ranges from two to three (Olivetti and Petrongolo, 2011).

Like Caselli and Coleman (2006), the model differs from the standard one in that firms choose their technology levels appropriately, i.e., (A_m, A_f) :

$$A_m^\omega + v A_f^\omega \leq B, \quad (2)$$

where ω , v and B are all positive parameters. B is interpreted as the inverse measure of the barrier to the world technology frontier, which means a subset of production technologies the technologically most advanced country, the country with highest B , can access. The combination of ω and v governs the curvature of the country-specific technology frontier defined by the pair (A_m, A_f) implied by (2) at equality. As B increases, or the barrier diminishes, the technology frontier expands, and firms within a given country can access a wider subset of production technologies.

v can be interpreted as the relative cost of shifting to female labor augmenting technology choice from male one (hereafter *relative cost*), which reflects all the sources of the gender gap in the efficiency wage rates other than the relative labor abundance L_s of labor of each sex s . Assume that $L_m = L_f$. If $v = 1$, then firms choose $A_m = A_f$, i.e., there is no gender wage gap. However, if $v > 1$, i.e., the relative cost is higher, firms choose the production technology in such a way that $A_m > A_f$, which results in a gender wage gap. Of course, one possible interpretation of v is sex discrimination such as glass ceiling.^{*4} v could also reflect working regulations, the culture in firms and country, preference of managers as well as political or bargaining balance of each sex or asymmetric effects of government policies, e.g. labor law, on employment environment of each sex.

Formally, the firms' profit maximization problem is:

$$\max_{\{L_s, A_s\}_{s \in \{m, f\}}} \left\{ [(A_m L_m)^\sigma + (A_f L_f)^\sigma]^{\frac{1}{\sigma}} - w_m L_m - w_f L_f \right\} \quad s.t. \quad (2).$$

^{*4} Although Arrow (1971) criticized exogenously specified discrimination guessing that free entry of firms will expel prejudiced employers in the long run, and actually this decreasing trend in discrimination is estimated by Flabbi (2010), even though we do still observe discrimination. O'Neill (2003) shows that about 42% of the male-female gap in median earnings in 2000 could not be explained by gender differences in schooling, experience, and job characteristics. In addition, it is worthy to note that discrimination captured by v includes not only prejudice of employers, mentioned by Arrow (1971), Becker (1971) and among others, but also asymmetric effects of policies. In addition, the degree of and speed of decrease in discrimination are different across countries. These cross-country variation is more important when we conduct a cross-country analysis in Section 3.

In addition to the usual marginal productivity conditions used to obtain the wage gap equation:

$$\frac{w_m e_m}{w_f e_f} = \frac{e_m}{e_f} \left(\frac{A_m}{A_f} \right)^\sigma \left(\frac{L_m}{L_f} \right)^{-(1-\sigma)}, \quad (3)$$

we also have the optimality conditions for technology choice consolidated as^{*5}

$$\frac{A_m}{A_f} = v^{\frac{1}{\omega-\sigma}} \left(\frac{L_m}{L_f} \right)^{\frac{\sigma}{\omega-\sigma}}, \quad (4)$$

which suggests both endogenous and exogenous comparative advantages work in technology choice. That is, the relative sex- s augmenting productivity is determined by the relative abundance of sex- s labor and relative cost. Thus, the hourly wage gap $w_m e_m / (w_f e_f)$ depends on firms' technology choice A_m / A_f as well as the gender gap in the skill e_s and decreasing returns to scale, the latter of which is weakened by the technology choice due to the complementarity between the technology choice and labor supply under the empirically valid case, i.e., $\sigma / (\omega - \sigma)$.

2.2 Households

Households are divided into two different groups: single and couple households. The former consists of a single person, either male or female, while the latter includes both one male and one female. Unlike single households, members in each couple household can cooperate each other with respect to their time allocation, implying that the elasticities of labor supply are different across these two different groups in general (Jones et al., 2003). Thus, letting N_s^* and N denote the measure of the single households of sex s and that of the couple households, respectively, the total population \mathbb{N} of the economy is given by $\mathbb{N} = N_m^* + N_f^* + 2N$, which is normalized to unity without loss of generality.^{*6}

^{*5} Here, we assume the interior solution in a sense that all the firms choose the same positive pair of (A_m, A_f) . Specifically, we are assuming that the hypothesis that $\omega / \sigma > 1 / (1 - \sigma)$ of the proposition in Caselli and Coleman (2006) holds. Intuitively, this inequality says that the degree ω / σ of decreasing returns to scale (DRS) in technology choice dominates the degree $1 / (1 - \rho)$ of the positive circular causation in technology choice, and thus there is no benefit from perfect specialization, and the optimal technology choice becomes the interior solution. We verify that the inequality actually holds given the result of our calibration.

^{*6} In the quantitative analysis, we calibrate measures (N, N_m^*, N_f^*) under the assumption of this household structure in such a way that the model can match the ratio of the aggregate labor supply of each sex. Given this calibration procedure and the fact that the real world includes households with memberships other than those specified in the model, readers should not interpret the household consisting of a couple in the model literally. Instead, it should be simply interpreted as a virtually representative household members in which can cooperate each other. Similarly, the single households should be interpreted as those without cooperation. In what follows, however, we use the single or couple households for convenience.

2.2.1 Single Household

A sex- s single household considers home production as well as the standard consumption and time allocation problem:^{*7}

$$\begin{aligned}
& \max_{c_s^*, g_s^*, h_{M,s}^*, h_{N,s}^* \geq 0} \left\{ \alpha_s^* \ln(c_s^*) + (1 - \alpha_s^*) \frac{(1 - h_{M,s}^* - h_{N,s}^*)^{1-\gamma_s^*} - 1}{1 - \gamma_s^*} \right\} \\
& \text{s.t.} \\
& c_s^* = \mathcal{H}^s(g_s^*, e_s h_{N,s}^*) = [\xi_s^* g_s^{*\eta} + (1 - \xi_s^*)(e_s h_{N,s}^*)^\eta]^{\frac{1}{\eta}}, \xi_s^* \in (0, 1), \eta < 1, \\
& (1 + \tau_c)g_s^* \leq (1 - \tau_\ell)w_s e_s h_{M,s}^* + T, \\
& h_{M,s}^* + h_{N,s}^* \leq 1,
\end{aligned} \tag{5}$$

where c_s^* is consumption of home goods produced by means of a CRS technology $\mathcal{H}^s(g_s^*, e_s h_{N,s}^*)$ with elasticity of substitution of $1/(1 - \eta)$, of which inputs consist of market goods g_s^* and effective home production hours, i.e., skill e_s times home production hour $h_{N,s}^*$.^{*8} ξ_s^* is the weight of market goods in home production of sex- s single households. Letting $h_{M,s}^*$ denote market hours and normalizing the time endowment to unity, $1 - h_{M,s}^* - h_{N,s}^*$ becomes the leisure time. τ_c is the consumption tax, τ_ℓ the labor income tax, T the lump-sum transfer per person, w_s the wage rate of sex s , α_s^* the share parameter for consumption, and γ_s the inverse of the Frisch elasticity of leisure defined as the elasticity of leisure with respect to the wage rate holding the marginal utility of consumption constant, respectively.^{*9}

FOCs state that marginal utility from hours to each activity is balanced each other:^{*10}

$$\begin{aligned}
& \alpha_s^* \frac{\mathcal{H}_g^s}{\mathcal{H}^s} \frac{1 - \tau_\ell}{1 + \tau_c} w_s e_s = \alpha_s^* \frac{\mathcal{H}_N^s e_s}{\mathcal{H}^s}, \\
& \text{or } \frac{\mathcal{H}_N^s}{\mathcal{H}_g^s} = \frac{1 - \xi_s^*}{\xi_s^*} \left(\frac{g_s^*}{e_s h_{N,s}^*} \right)^{(1-\eta)} = \frac{1 - \tau_\ell}{1 + \tau_c} w_s, \quad \text{all } s \in \{m, f\},
\end{aligned} \tag{6}$$

where $\mathcal{H}_g^s \equiv \partial \mathcal{H}^s / \partial g_s^*$, and $\mathcal{H}_N^s \equiv \partial \mathcal{H}^s / \partial h_{N,s}^*$. The interpretation of the first equation above is as follows: An additional market hour increases labor income net of labor income tax by $(1 - \tau_\ell)w_s e_s$, which is equivalent to $(1 - \tau_\ell)/(1 + \tau_c)w_s e_s$ units of market goods. Multiplying this amount by marginal productivity \mathcal{H}_g^s of market goods in home production and marginal

^{*7} The input structure of home production is the same as Becker (1965), who was followed by Olovsson (2004), Ragan (2013), and Rogerson (2009) among others. For preference, we follow Gronau (1977) as in Chang and Schorfheide (2003a), and Rogerson (2009).

^{*8} The inclusion of skill e_s in the labor input is consistent with the arguments by Gronau (1980, 2008) that more educated people are more better at implementing their tasks. The assumption that efficiency in the home work is proportional to that in market activities seems less important when investigating the time gap which is related to the ratio of efficiencies e_m/e_f more than levels themselves given that the difference across sexes with respect to the impacts of education on the home productivity are not decisive (Table 7 in Gronau and Hamermesh (2008)).

^{*9} Frisch elasticity is usually derived in relation with the intertemporal labor supply elasticity in dynamic models. Although our model is static, Frisch elasticity of leisure in our static framework is equivalent to that in dynamic models when utility function specifies a separable leisure function and time separable.

^{*10} Here, we are assuming that the zero lower bound of $h_{M,s}^*$ does not bind, and this is the case of interest given that agents within the same group of households are identical.

utility of consumption α_s^*/\mathcal{H}^s , we obtain the left hand side (LHS), the marginal utility of an additional market hour. The right hand side (RHS), the marginal utility of an additional home hour, has a similar reasoning.

Taking the ratio of each sex results in the effective time gap:

$$\frac{e_m h_{N,m}^*}{e_f h_{N,f}^*} = \left\{ \frac{w_m / [(1 - \xi_m^*) / \xi_m^*]}{w_f / [(1 - \xi_f^*) / \xi_f^*]} \right\}^{-\frac{1}{1-\eta}} \frac{g_m^*}{g_f^*}. \quad (7)$$

The ratio is decreasing in the ratio of the efficiency wage w_s normalized by the relative weight $(1 - \xi_s^*) / \xi_s^*$ of home production due to the opportunity cost and increasing in the ratio of market goods g_s^* due to complementarity. The elasticity of the ratio with respect to the former is exactly the same as that between market goods and labor input in home production.

2.2.2 Couple Household

A typical couple household differs from a single one in that the budget constraint is consolidated; and that members in the household solve a common allocation problem:^{*11}

$$g, \{c_s, h_{M,s}, h_{N,s}, z_s\}_{s \in \{m,f\}} \geq 0 \left\{ \sum_{s \in \{m,f\}} \alpha_s \ln(c_s) + \ell(1 - h_{M,m} - h_{N,m}, 1 - h_{M,f} - h_{N,f}) \right\} \quad (8)$$

s.t.

$$\sum_{s \in \{m,f\}} c_s = \mathcal{H}(g, e_m h_{N,m}, e_f h_{N,f})$$

$$= \left\{ \xi g^\eta + (1 - \xi) [z_m (e_m h_{N,m})^\rho + z_f (e_f h_{N,f})^\rho]^\frac{\eta}{\rho} \right\}^\frac{1}{\eta}, \rho < 1, \quad (9)$$

$$(1 + \tau_c)g \leq (1 - \tau_\ell) \sum_{s \in \{m,f\}} w_s e_s h_{M,s} + 2T, \quad (10)$$

$$h_{M,s} + h_{N,s} \leq 1, \quad \text{all } s \in \{m, f\}, \quad (11)$$

where ℓ is a leisure function, which is strictly increasing, twice continuously differential and concave, and \mathcal{H} is the home production function of which inputs consist of market goods and the CRS composite of time of both members with elasticity of substitution of $1/(1 - \rho)$. The crucial difference of the above problem from the single one is that the members in the couple households can cooperate each other by choosing their time allocation $\{h_{M,s}, h_{N,s}\}_{s \in \{m,f\}}$ for given weights (z_m, z_f) , which we call z_s *home production effort*, or simply *effort*, of sex s hereafter. It is interpreted as human capital, the way how members in a couple household cooperate each other, and so on. Variables and parameters which have the same notation except for asterisk have the same meaning as in the case of the single households. The household with two members receives the lump-sum transfer equal to $2T$.

Solving the allocation problem of the home goods, i.e., $\{c_s\}_{s \in \{m,f\}}$, we obtain the reduced form problem, of which FOCs with respect to time allocation state that marginal utility from

^{*11} In this paper, we do not introduce any strategic behavior between members in the household. The input structure of the home production function is a direct extension of the single one to the case of couple households.

hours to each activity is balanced each other as in the case of single households:^{*12}

$$\frac{\mathcal{H}_g}{\mathcal{H}} \frac{1 - \tau_\ell}{1 + \tau_c} w_s e_s = \frac{\mathcal{H}_s e_s}{\mathcal{H}}, \quad \text{all } s \in \{m, f\},$$

where $\mathcal{H}_g \equiv \partial\mathcal{H}/\partial g$ and $\mathcal{H}_s \equiv \partial\mathcal{H}/\partial(e_s h_{N,s})$, and the LHS and RHS are the marginal utilities of an additional market and home hour, respectively.

Further, taking the ratio of this equation for each sex s gives the effective time gap for the couple households which is similar to the one in the case of single households:

$$\frac{\mathcal{H}_m}{\mathcal{H}_f} = \frac{z_m}{z_f} \left(\frac{e_m h_{N,m}}{e_f h_{N,f}} \right)^{-(1-\rho)} = \frac{w_m}{w_f}, \quad \text{or} \quad \frac{e_m h_{N,m}}{e_f h_{N,f}} = \left(\frac{w_m/z_m}{w_f/z_f} \right)^{-\frac{1}{1-\rho}}, \quad (12)$$

which says that the time gap depends on not only the efficiency wage gap representing the comparative advantage in market activities but also the *effort gap* z_m/z_f , the comparative advantage in the home production.

This corresponds to (7) in the case of single households, and effort gap z_m/z_f is the counterpart of the ratio of the relative weight $(1 - \xi_s^*)/\xi_s^*$. However, the crucial difference appears in the elasticity of the time gap with respect to the relative efficiency wage gap. The absolute elasticity of the single households is equal to the elasticity $1/(1 - \eta)$ of substitution between the market goods and time spent for the home production while that of the couple households is equivalent to the one $1/(1 - \rho)$ between the male and female in the home production. The cooperation between members in a couple household makes the market goods g public goods, and this is the reason why the above equation has no counterpart of g_m^*/g_f^* .

2.3 Government

The government levies consumption and proportional labor income taxes on households. The collected revenues are then used for redistribution through the lump-sum transfer T per person. Thus, the government budget constraint is

$$NT = N\tau_c g + \sum_{s \in \{m, f\}} N_s^* \tau_c g_s^* + \sum_{s \in \{m, f\}} N\tau_\ell w_s e_s h_{M,s} + \sum_{s \in \{m, f\}} N_s^* \tau_\ell w_s e_s h_{M,s}^*. \quad (13)$$

2.4 Equilibrium

Now, we can define a competitive equilibrium of the economy. We focus on a symmetric equilibrium where firms choose the same technology pair, i.e., (A_m, A_f) .

Definition. *Given a tax system (τ_c, τ_ℓ) , a symmetric competitive equilibrium of the economy is a set of a price system (w_m, w_f) , time allocation $\{h_{M,s}^*, h_{N,s}^*, h_{M,s}, h_{N,s}\}_{s \in \{m, f\}}$, quantities $(\{c_s^*, c_s, g_s^*\}_{s \in \{m, f\}}, g, \{L_s\}_{s \in \{m, f\}})$, technology choice $\{A_s\}_{s \in \{m, f\}}$, and a lump-sum transfer T such that*

^{*12} Again, we are assuming the interior solution.

1. given prices, households maximize their utility;
2. given prices and technology constraint, firms maximize their profit;
3. markets clear:

$$\sum_{s \in \{m, f\}} N_s^* g_s^* + Ng = Y, \quad (14)$$

$$L_s = N_s^* e_s h_{M,s}^* + N e_s h_{M,s} \quad \text{all } s \in \{m, f\}; \text{ and} \quad (15)$$

4. the government budget constraint (13) is satisfied.

3 Quantitative Analysis

In this section, conducting counterfactual simulations with the model described in the previous section, we ask what are the quantitative effects of technology choice on the cross-country variations in the gender gaps: hourly wage gap $w_m e_m / (w_f e_f)$, time gap $h_{N,m}^* / h_{N,f}^*$ of the single households, and that $h_{N,m} / h_{N,f}$ of the couple households. The results show that technology choice has large impacts on all of gender gaps. In addition, it is also shown that mechanisms determining the time gaps of the single and couple households are different, implying that the convergence in A_m / A_f is associated with a convergence in the single time gap $h_{N,m}^* / h_{N,f}^*$ but not in the couple one $h_{N,m} / h_{N,f}$.

In the following, we first calibrate the model and design the simulation method which allows us to quantify the effects of technology choice on the gender gaps. We then provide the results focusing on the importance of technology choice in subsections that follow. In our study we use cross-section data sets consisting mainly of the Multinational Time Use Study (MTUS), Survey on Time Use and Leisure Activities (Japan) and EU KLEMS. We will discuss the detail in AppendixA.

3.1 Calibration

We specify the leisure function ℓ in the couple households as follows:

$$\ell(1 - h_{M,m} - h_{N,m}, 1 - h_{M,f} - h_{N,f}) = \sum_{s \in \{m, f\}} (1 - \alpha_s) \frac{(1 - h_{M,s} - h_{N,s})^{1 - \gamma_s} - 1}{1 - \gamma_s}, \quad (16)$$

where $\alpha_s \in (0, 1)$ is the weight of consumption. Stated differently, we assume that within each couple household, members solve a Pareto problem with equal treatment where the actions of each member affect the partner's utility only indirectly.

Given this specification and those laid out in the previous section, we calibrate unknown variables such as productivities A_s and consumption (c_s^*, c_s) and parameters together by solving the simultaneous equations derived from the FOCs and, in some case, by an estimation. Intuitively, we assume that under the calibrated parameters, the equilibrium is equivalent to

the observed data.*¹³

There are twenty-four parameters, each of which is categorized into one of two types of parameters: household-side and firm-side parameters. Household-side parameters consist of preference $\{\alpha_s^*, \alpha_s, \gamma_s\}_{s \in \{m, f\}}$, home production ($\{\xi_s^*, z_s\}_{s \in \{m, f\}}, \xi, \eta, \rho$), household structure ($\{N_s^*\}_{s \in \{m, f\}}, N$) in workers, skill $\{e_s\}_{s \in \{m, f\}}$, and tax rates (τ_c, τ_ℓ) . Firm-side parameters consist of the elasticity $1/(1 - \sigma)$ of substitution between male and female and technology constraint (ω, v, B) . The result of the calibration is summarized in Table 16-17.

3.2 Simulation Method

In quantifying the effects of technology choice on the cross-country variations in the gender gaps, we first consider the effects of a change in the environments on the cross-country variations in the gender gaps: from the situation where each country are faced with their calibrated country-specific parameters to the one where all countries have the same U.S. equivalent level of parameters.

The effects of technology choice is then revealed by comparing two scenarios: In the first scenario, firms can optimally choose their technology (call this case *appropriate technology choice*), and thus there should be no difference across equilibria and thus gender gaps of countries. In contrast, the second scenario assumes that firms cannot choose their best technology and thus face with the calibrated A_m/A_f because of sufficiently high adjustment costs or, more broadly interpreted, history dependence (call this case *inappropriate technology choice*). In this case, we should observe the cross-country variations in the gender gaps which arise purely due to the cross-country variations in firms' technology choice before the change in the environments.

Thus, to the extent that the cross-country variations in the gender gaps observed in the data are reproduced by the inappropriate technology choice, we can say that the effects of technology choice on the cross-country variations in the gender gaps are large. More specifically, by measuring the correlation between the data and the counterfactual under the inappropriate technology choice (let $Corr(CF, Data)$ denote the correlation) and then by calculating the ratio of the cross-country variance $Var(CF)$ of some gender gap under the inappropriate technology choice to that $Var(Data)$ of the corresponding data, we can quantify the impacts of technology choice on the cross-country variations in the gender gaps. If $Corr(CF, Data) < 0$, then technology choice itself cannot explain the observed variation, and from a different perspective the larger $Var(CF)/Var(Data)$ is, the more the observed technology choice affected the variations in the gender gaps. Note that if both $Corr(CF, Data)$

*¹³ Since we take the values of elasticities from previous literatures, this calibration approach suggests that parameters except for elasticities are computed as residuals. This is the reason why we follow the previous literatures in the specification while keeping the model as plain as possible. Even with the limited availability of the time use data, this method together with the plain model allows us to identify the values of parameters. The more detailed procedure is described in AppendixB.

and $Var(CF)/Var(Data)$ are close to one, it can be said that technology choice itself can explain the observed variations in the gender gaps.

In what follows, we call this method comparing the inappropriate technology choice with the data the *independent experiment* of technology choice. A similar method can be applied to the other sources of the cross-country variation of the gender gaps such as effort z_s , skill e_s , preference (α_s^*, α_s) and so on. That is, in order to quantify the impacts of some factor, we assume counterfactually that countries are different only in this factor and compare the associated equilibrium with the data. We also call this experiment the independent experiment.

We also design another type of experiments which we call *conditional experiments* of technology choice. A conditional experiment of technology choice is a slight extension of the independent experiment of technology choice. Specifically, it compares the equilibrium with one or a few additional cross-country difference(s) in parameters as well as inappropriate technology choice. Intuitively, this experiment quantify the effects of the combination of several sources, including at least technology choice, of the cross-country variation in the gender gaps.

3.3 Wage Gap

The theoretical implication of inappropriate technology choice for the wage gap is understood by comparing the inappropriate technology choice with appropriate technology choice where all countries have the same parameter values as the U.S. and firms choose their technology optimally. Then, the inappropriate technology choice is characterized by a shift of (A_m, A_f) on the U.S.-equivalent technology frontier.

Here suppose, without loss of generality, that A_m and A_f moves from a northwest point US , representing the United States or the appropriate technology choice, to a southeast point i on the U.S.-equivalent technology frontier as shown in Figure 1, i.e., A_m and A_f increases and decreases, respectively. Due to the associated changes in the labor productivities, the wage rate of the males and that of the females increases and decreases, respectively, implying the efficiency wage gap w_m/w_f increases given the other things being equal in a way specified by (3). However, this increase seems to be weakened by the general equilibrium effect or the associated increase in the relative aggregate labor supply of the males and thus its negative effect on the wage gap due to the decreasing returns to scale. At least for the single household decision, the previous literature such as Rogerson (2009) suggests that the single male (female) household increases (decreases) his (her) time spent on market activities with its response to the wage rate strengthened by the substitution between market goods and time spent on his (her) home production. As for the couple households, the integrated budget constraint makes the sign of the associated change in the household's labor income ambiguous. Thus, the above magnification effects of the substitution between market goods and time devoted to home production on the response of the market hour of each sex are now ambiguous.

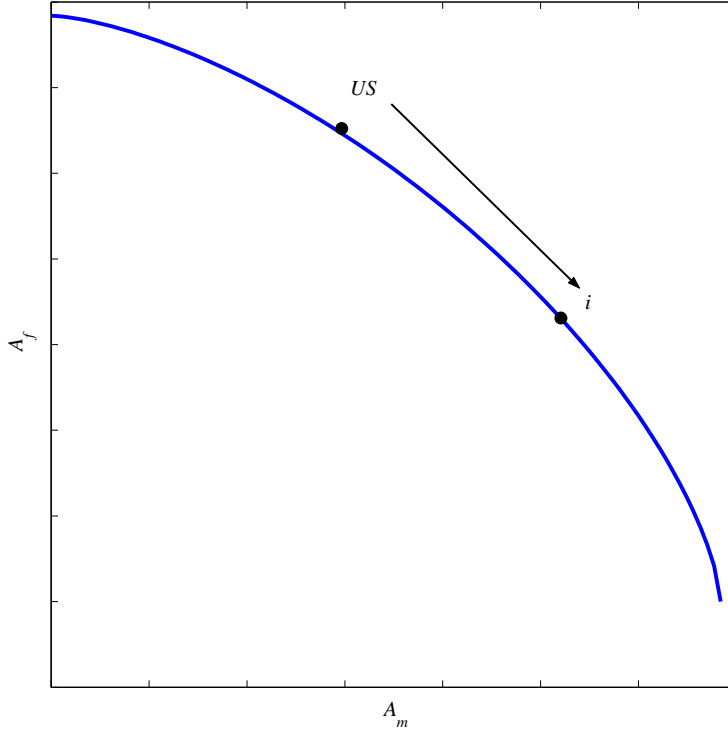


Figure1 Technology Shift on the U.S.-equivalent Technology Frontier

However, even with this ambiguous magnification effect, we might expect that an increase in the ratio $h_{M,m}/h_{M,f}$ of market hours is a natural consequence of comparative advantage, and this is actually the case as confirmed by our calculation.

This result is then compared with the observed cross-country variation in the hourly wage gap $w_m e_m / (w_f e_f)$ by the independent experiment, which suggests that technology choice contributes to the cross-country variation in the wage gap to a relatively large extent as shown by the left panel of Figure 2 and Table 1. The variance ratio $Var(CF)/Var(Data)$ of technology choice is largest among the sources of the gender gaps, 0.346. Not surprisingly, the correlation $Corr(Data, CF)$ between the data and counterfactual is positive and fairly close to one. Independent experiments also suggest that skill e_s and preference (α_s^*, α_s) are also other importance sources of the cross-country variation in the wage gap. The variance ratios of these are about 78% and 36% of that of technology choice, respectively. The former is consistent with the literature and, together with the latter, suggests that the importance of the general equilibrium analysis which can capture the effect of the latter and verified its relatively large impact on the cross-country variation in the wage gap.

Conditional experiments support the result of the independent experiment that technology choice is important in understanding the cross-country variation in the wage gap. Both the variance ratio $Var(CF)/Var(Data)$ and correlation $Corr(Data, CF)$ are robust even if, in addition to technology choice, we added another source of the cross-country variation of the gender gaps. Importantly, the pair of technology choice and preference explains the most of

| | Var | $\frac{Var(CF)}{Var(Data)}$ | $Corr(Data, CF)$ |
|--|-------|-----------------------------|------------------|
| Data | 0.076 | — | — |
| Independent Experiments | | | |
| – Technology Choice A_m/A_f | 0.026 | 0.346 | 0.840 |
| – Effort z_s | 0.002 | 0.022 | 0.289 |
| – Skill e_s | 0.020 | 0.270 | 0.359 |
| – Preference α_s, α_s^* | 0.009 | 0.125 | 0.958 |
| – Tax τ_ℓ, τ_c | 0.000 | 0.001 | 0.535 |
| – Population N, N_s^* | 0.001 | 0.011 | -0.627 |
| Conditional Experiments of Technology Choice A_m/A_f | | | |
| – Effort z_s | 0.032 | 0.423 | 0.796 |
| – Skill e_s | 0.039 | 0.510 | 0.929 |
| – Preference α_s, α_s^* | 0.068 | 0.893 | 0.927 |
| – Tax τ_ℓ, τ_c | 0.018 | 0.243 | 0.854 |
| – Population N, N_s^* | 0.021 | 0.275 | 0.798 |
| – Effort & Preference $z_s, \alpha_s, \alpha_s^*$ | 0.075 | 0.987 | 0.905 |
| – Skill & Preference $e_s, \alpha_s, \alpha_s^*$ | 0.084 | 1.110 | 0.966 |

Table1 Counterfactual Experiments: Wage Gap Variation

Notes: “Independent Experiments” shows the effect of the setting of simulated exogenous variable independently on cross-country variations by comparing to calculate variance and correlation. “Conditional Experiments of Technology Choice $A_m = A_f$ ” shows the effect of several combinations which all include technology choice. Other exogenous variables and parameters are set to equal U.S. calibrated values. The second column from the left shows variance between each sample countries by data and counterfactual simulations, respectively. The third column calculates variance ratio of data and counterfactual simulation that is defined as the second column each row divided by the second column of the first row. The forth column calculates correlation between data and simulation results.

the cross-country variation of the wage gap with the variance ratio of 0.893 and correlation of 0.927 as shown in Table 1, and it is noteworthy that the correlation of the combination is well above the summation of the variance ratios associated with the independent experiments of technology choice and preference. If we add either effort or skill as well as preference, both measures become closer to one, but compared with the combination of technology choice and preference, the improvements are relatively small.

3.4 Single Time Gap

Suppose again that technology (A_m, A_f) shifts towards southeast on the U.S.-equivalent technology frontier, and thus the efficiency wage gap w_m/w_f also increases as shown by the previous subsection. Each single household then takes these changes as given and chooses the time $h_{N,s}^*$ devoted to her or his own home production. According to (7), the associated change in the time gap $h_{N,m}^*/h_{N,f}^*$ is the sum of the two counteracting forces: The first derives from the associated increase in the relative opportunity costs, i.e., the change in $(w_m/w_f)^{-1/(1-\eta)}$, which is negative. The second is positive due to the complementarity between market goods and time devoted to home production, i.e., the change in g_m^*/g_f^* which seems to increase since g_m^* (g_f^*) is likely to increase (decrease) faced with an increase (decrease) in the wage rate w_m (w_f). It is confirmed that the resulting change in the time gap is a decrease.

Then, the question is to what extent this cross-country variation in the time gap induced by technology choice can explain the observed variation across countries. The independent experiment suggests that technology choice can explain not all but some non-negligible part of the cross-country variation in the time gap of the single households. A positive correlation $Corr(Data, CF)$ between the data and counterfactual, though much smaller than that in the case of the wage gap as shown by the center panel of Figure 2 or Table 2, implies that the cross-country variation induced by technology choice is consistent with the observed variation. In addition, the value of the variance ratio $Var(CF)/Var(Data)$, 0.228, shows that its impact is not negligible.

The importance of technology choice in understanding the cross-country variation in the time gap is also suggested by comparisons between the independent experiment of technology choice with those of the other sources of the cross-country variation. Skill e_s , which directly affects the time gap, has the highest variance ratio, 0.478, which is about twice larger than that of technology choice. However, a negative correlation, -0.164 , suggests that skill itself cannot explain the observed cross-country variation. Among the other sources impacting on the time gap only through general equilibrium effects, preference has comparable numbers for both the variance ratio and correlation, 0.216 and 0.242, respectively. Effort, tax and population, the first of which is closely related to the couple households, are all negligible impact on the time gap in the sense that the variance ratio is relatively small compared with that of technology choice.

This conclusion is robust in the sense that even if we allowed additional variations in the other sources of the gender gaps, both the correlation $Corr(Data, CF)$ and variance ratio $Var(CF)/Var(Data)$ do not change so much. As shown in Table 2 which reports the results of several conditional experiments, the correlation $Corr(Data, CF)$ between the data and counterfactual is still positive, ranging from 0.089 with skill gap to 0.371 with tax, and the variance ratio $Var(CF)/Var(Data)$ is also far from zero, ranging from 0.158 with tax to 1.184 with skill and preference.

| | Var | $\frac{Var(CF)}{Var(Data)}$ | $Corr(Data, CF)$ |
|--|-------|-----------------------------|------------------|
| Data | 0.032 | — | — |
| Independent Experiments | | | |
| – Technology Choice A_m/A_f | 0.007 | 0.228 | 0.330 |
| – Effort z_s | 0.001 | 0.017 | -0.715 |
| – Skill e_s | 0.015 | 0.478 | -0.164 |
| – Preference α_s, α_s^* | 0.007 | 0.216 | 0.242 |
| – Tax τ_ℓ, τ_c | 0.000 | 0.000 | -0.004 |
| – Population N, N_s^* | 0.000 | 0.008 | -0.779 |
| Conditional Experiments of Technology Choice A_m/A_f | | | |
| – Effort z_s | 0.010 | 0.299 | 0.127 |
| – Skill e_s | 0.026 | 0.808 | 0.089 |
| – Preference α_s, α_s^* | 0.026 | 0.807 | 0.285 |
| – Tax τ_ℓ, τ_c | 0.005 | 0.158 | 0.371 |
| – Population N, N_s^* | 0.006 | 0.187 | 0.257 |
| – Effort & Preference $z_s, \alpha_s, \alpha_s^*$ | 0.032 | 0.973 | 0.186 |
| – Skill & Preference $e_s, \alpha_s, \alpha_s^*$ | 0.038 | 1.184 | 0.198 |

Table2 Counterfactual Experiments: Time Gap Variation of Single Households

Notes: “Independent Experiments” shows the effect of the setting of simulated exogenous variable independently on cross-country variations by comparing to calculate variance and correlation. “Conditional Experiments of Technology Choice $A_m = A_f$ ” shows the effect of several combinations which all include technology choice. Other exogenous variables and parameters are set to equal U.S. calibrated values. The second column from the left shows variance between each sample countries by data and counterfactual simulations, respectively. The third column calculates variance ratio of data and counterfactual simulation that is defined as the second column each row divided by the second column of the first row. The fourth column calculates correlation between data and simulation results.

3.5 Couple Time Gap

Also take as given a southeast shift of technology (A_m/A_f) on the U.S.-equivalent frontier. Then, unlike the single household, we should observe a clear-cut relationship between the associated increase in the efficiency wage gap w_m/w_f and the time gap $h_{N,m}/h_{N,f}$. According to (12), the couple household chooses its members’ time devoted to home production in such a way that the female engages in home production more than the male does, or stated

differently, the time gap $h_{N,m}/h_{N,f}$ is negatively correlated with the efficiency wage gap w_m/w_f . Intuitively, market goods g are, though cooperation between members, shared like public goods within the households, and thus the effects of complementarity between market goods and time devoted to home production on the time gap cancel out across members, and only the effects of the opportunity costs prevail, resulting in a perfect log-linear relationship between the time gap and the wage gap.

Then, to what extent can this cross-country variation in the time gap induced by technology choice explain the actual variation? What is noteworthy is that the result is in contrast with the case of the single household. The independent experiment shows that the correlation $Corr(Data, CF)$ between the data and counterfactual is negative, about -0.242 as shown in Table 3 or observed from the right panel in Figure 2, suggesting that technology choice itself cannot explain the observed cross-country variation in the time gap. That is, the observed cross-country variation in the time gap of the couple household is driven by some factor(s) the effects of which are negatively correlated with that of technology choice.

This result, however, does not mean that technology choice is not an importance source of the cross-country variation in the time gap. In terms of the impact of technology choice on the cross-country variation in the time gap, which is measured by the variation ratio $Var(CF)/Var(Data)$, technology choice itself has a considerable impact on the time gap $h_{N,m}/h_{N,f}$ of the couple household. Table 3 reports that the variance ratio is about 0.492. This impact is robust in the sense that the variance ratio does not change so much and rather increases when combined with other sources of the cross-country variation of the gender gaps as shown by conditional experiments.

In addition, technology choice is also important in the sense that there is no single factor that can explain the actual cross-country variation in the time gap of the couple households. Although effort z_s has a correlation $Corr(Data, CF)$ between the data and counterfactual sufficiently close to one, its variance ratio $Var(CF)/Var(Data)$ is too large to explain the cross-country variation. Instead, the combination of technology choice and effort or the triplet of technology choice, effort and preference has the variance ratio and correlation closer to one compared with those of either technology choice or effort itself, implying that without technology choice it is difficult to explain the cross-country variation in the time gap. Among of these, the latter explains the most of the cross-country variation with the variance ratio of 1.145 and correlation of 0.984.

The above results thus suggest that the mechanisms determining the time devoted to home production are crucially different across different types of households not only in the sense that the cooperation between members makes the net effects of the opportunity costs larger but also in the sense that the actual cross-country variation in the time gap of the couple household deviates from the prediction with technology choice only to a large extent. An immediate implication of this result is that the global policy trend, which is expected to narrow the gender gaps by affecting technology choice and is characterized by the convergence in A_m/A_f ,

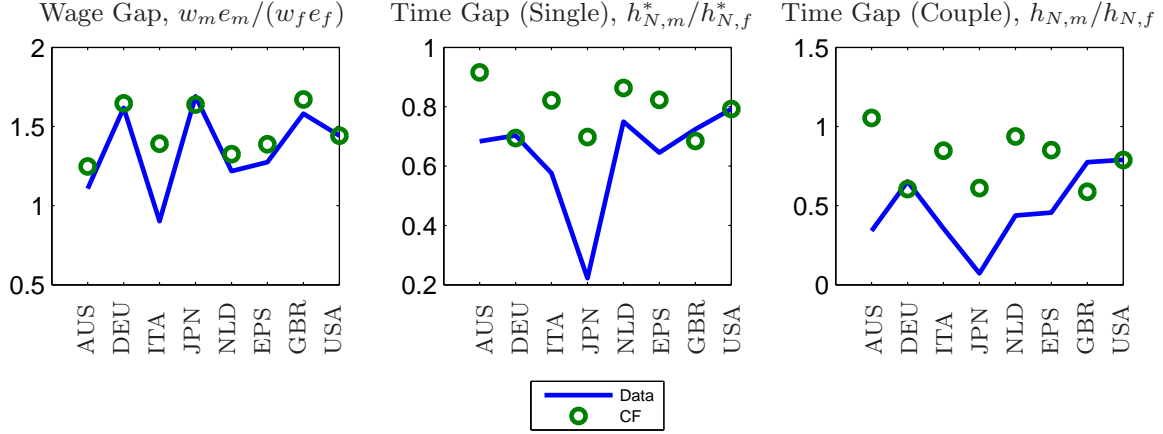


Figure2 Effects of Technology Choice on the Gender Gaps: Independent Experiment including Italy

Notes: Figure shows the male-female to ratio each variables. CF (green open circle) are represented counterfactual simulation results

might not achieve smaller wage and time gaps (at least for that of the couple households) simultaneously. As shown by independent and conditional experiments, the cross-country variation in technology choice A_m/A_f has an offsetting effect on the cross-country variation in the time gap of the couple households which is widened by the cross-country variation in effort z_s . Thus, if A_m/A_f 's of countries converge, the effect of effort becomes larger, resulting in a more wider cross-country variation in the time gap. This means that in some countries the time gap become narrower while other countries experience higher time gaps.*¹⁴

4 Robustness Analysis

We performed sensitivity checks by changing parameter values, assumptions and utility function specification within the context of the baseline. Table 4–6 compare the results when the main experiments are implemented under alternative assumptions. These results show that the firms' technology choice can explain the cross-country variance to some extent even under different assumptions, and thus we concluded that the firms' technology choice has a significant impact to the gender wage and time gaps.

Specifically, we conduct four types of sensitivity experiment:

1. Endogenous Home Production Effort
2. With Physical Capital Model
3. Composite Type Utility Function

*¹⁴ As shown in Section 4, the result that the correlation between couple time gaps of the data and counterfactual under inappropriate technology choice is negative is robust to different parameter values and specifications. Thus, stated differently, the implication that a convergence in A_m/A_f results in a divergence in the couple time gap $h_{N,m}/h_{N,f}$ is also a robust result.

| | Var | $\frac{Var(CF)}{Var(Data)}$ | $Corr(Data, CF)$ |
|--|-------|-----------------------------|------------------|
| Data | 0.059 | — | — |
| Independent Experiments | | | |
| – Technology Choice A_m/A_f | 0.029 | 0.492 | -0.242 |
| – Effort z_s | 0.175 | 2.943 | 0.887 |
| – Skill e_s | 0.006 | 0.099 | -0.263 |
| – Preference α_s, α_s^* | 0.013 | 0.219 | -0.111 |
| – Tax τ_ℓ, τ_c | 0.000 | 0.002 | -0.161 |
| – Population N, N_s^* | 0.001 | 0.018 | -0.497 |
| Conditional Experiments of Technology Choice A_m/A_f | | | |
| – Effort z_s | 0.090 | 1.515 | 0.964 |
| – Skill e_s | 0.039 | 0.653 | -0.276 |
| – Preference α_s, α_s^* | 0.079 | 1.324 | -0.249 |
| – Tax τ_ℓ, τ_c | 0.021 | 0.359 | -0.209 |
| – Population N, N_s^* | 0.024 | 0.405 | -0.303 |
| – Effort & Preference $z_s, \alpha_s, \alpha_s^*$ | 0.068 | 1.145 | 0.984 |
| – Skill & Preference $e_s, \alpha_s, \alpha_s^*$ | 0.080 | 1.351 | -0.275 |

Table3 Counterfactual Experiments: Time Gap Variation of Couple Households

Notes: “Independent Experiments” shows the effect of the setting of simulated exogenous variable independently on cross-country variations by comparing to calculate variance and correlation. “Conditional Experiments of Technology Choice $A_m = A_f$ ” shows the effect of several combinations which all include technology choice. Other exogenous variables and parameters are set to equal U.S. calibrated values. The second column from the left shows variance between each sample countries by data and counterfactual simulations, respectively. The third column calculates variance ratio of data and counterfactual simulation that is defined as the second column each row divided by the second column of the first row. The fourth column calculates correlation between data and simulation results.

4. Changing Elasticity of Substitution Values

Different from calibration forms and simulation algorithm of baseline model are discussed in AppendixD.

4.1 Endogenous Home Production Effort

The *home production effort*, or simply *effort*, z_s is given exogenously in main experiments, so even, when firm changes technology choice, home production effort doesn’t change. For

| $Var(CF)/Var(Data)$ | Independent Experiments | | | Conditional Experiments | | |
|-------------------------------------|-------------------------|-------|------------|-------------------------|-------|------------|
| | Technology Choice | Skill | Preference | Skill & Preference | Skill | Preference |
| Baseline | 0.35 | 0.27 | 0.12 | 1.11 | 0.51 | 0.89 |
| 4.1: Endogenous effort | 0.23 | 0.36 | 0.09 | 0.95 | 0.51 | 0.60 |
| 4.1 + 4.2: With capital | 0.53 | 0.56 | 0.34 | 1.03 | 0.70 | 0.87 |
| 4.1 + 4.2 + 4.3: Alt. Utility func. | 0.69 | 0.70 | 0.23 | 1.07 | 0.89 | 1.06 |
| 4.4: $1/(1-\rho) = 1.11$ | 0.37 | 0.24 | 0.13 | 1.14 | 0.51 | 0.96 |
| 4.4: $1/(1-\rho) = 3.33$ | 0.32 | 0.31 | 0.11 | 1.07 | 0.52 | 0.81 |
| 4.4: $1/(1-\sigma) = 1.4$ | 0.27 | 0.09 | 0.19 | 1.13 | 0.43 | 0.92 |
| 4.4: $1/(1-\sigma) = 2.6$ | 0.44 | 0.39 | 0.10 | 1.09 | 0.55 | 0.93 |

| $corr(Data, CF)$ | Independent Experiments | | | Conditional Experiments | | |
|-------------------------------------|-------------------------|-------|------------|-------------------------|-------|------------|
| | Technology Choice | Skill | Preference | Skill & Preference | Skill | Preference |
| Baseline | 0.84 | 0.36 | 0.96 | 0.97 | 0.93 | 0.93 |
| 4.1: Endogenous effort | 0.84 | 0.36 | 0.95 | 0.91 | 0.85 | 0.93 |
| 4.1 + 4.2: With capital | 0.84 | 0.35 | 0.96 | 0.95 | 0.90 | 0.93 |
| 4.1 + 4.2 + 4.3: Alt. Utility func. | 0.84 | 0.35 | 0.85 | 0.92 | 0.89 | 0.89 |
| 4.4: $1/(1-\rho) = 1.11$ | 0.84 | 0.36 | 0.96 | 0.97 | 0.94 | 0.93 |
| 4.4: $1/(1-\rho) = 3.33$ | 0.84 | 0.36 | 0.96 | 0.95 | 0.91 | 0.93 |
| 4.4: $1/(1-\sigma) = 1.4$ | 0.93 | 0.37 | 0.96 | 0.95 | 0.87 | 0.98 |
| 4.4: $1/(1-\sigma) = 2.6$ | 0.75 | 0.35 | 0.96 | 0.97 | 0.95 | 0.87 |

Table4 Robustness Analysis of Wage Gap Variation

example, if firm decides to enhance life work balance to work more easily for female workers, the couple household may change each spouses function and husband may work more home production. Then, male's home production effort will increase due to changing comparative advantage. In this subsection, we examine such a effect to effort. The couple household can choose the effort under constraint of technology frontier in home production with a similar fashion to firms' technology choice problem. The couple household maximizes the utility function including below constraint,

$$z_m^{\omega_H} + v_H z_f^{\omega_H} \leq B_H. \quad (17)$$

This constraint plays a role similar to the technology choice problem of the firm side. B_H is the inverse measure of the barrier to household technology frontier, v_H is the relative cost of shifting to spouse's home production productivity and ω_H governs the curvature of household technology frontier. If $\rho > 0$, which is the case we consider in this paper, that $\omega_H > 1$ guarantees an interior solution of the household.

The FOCs with respect to z_m and z_f and taking the ratio of this equation for each sex s ,

$$\frac{z_m}{z_f} = v_H^{\frac{1-\rho}{(1-\rho)\omega_H-1}} \left(\frac{w_m}{w_f} \right)^{-\frac{\rho}{(1-\rho)\omega_H-1}},$$

imply that the home production effort changes due to comparative advantage of market work.

When we calibrate z_m and z_f by data, we restrict to $z_m + z_f = 1$ as main experiments settings to identify parameters. But, when calculating simulation, we can identify it without

| $Var(CF)/Var(Data)$ | Independent Experiments | | | Conditional Experiments | | |
|-------------------------------------|-------------------------|-------|------------|-------------------------|-------|------------|
| | Technology Choice | Skill | Preference | Skill & Preference | Skill | Preference |
| Baseline | 0.49 | 0.10 | 0.22 | 1.35 | 0.65 | 1.32 |
| 4.1: Endogenous effort | 1.58 | 1.37 | 0.69 | 7.38 | 3.95 | 4.40 |
| 4.1 + 4.2: With capital | 1.29 | 1.24 | 0.97 | 2.84 | 2.01 | 2.27 |
| 4.1 + 4.2 + 4.3: Alt. Utility func. | 1.67 | 0.97 | 0.02 | 2.57 | 2.11 | 0.27 |
| 4.4: $1/(1-\rho) = 1.11$ | 0.16 | 0.07 | 0.07 | 0.60 | 0.21 | 0.43 |
| 4.4: $1/(1-\rho) = 3.33$ | 1.26 | 0.90 | 0.60 | 4.82 | 2.60 | 3.57 |
| 4.4: $1/(1-\sigma) = 1.4$ | 0.39 | 0.32 | 0.35 | 1.20 | 0.49 | 1.59 |
| 4.4: $1/(1-\sigma) = 2.6$ | 0.64 | 0.03 | 0.17 | 1.43 | 0.75 | 1.38 |

| $corr(Data, CF)$ | Independent Experiments | | | Conditional Experiments | | |
|-------------------------------------|-------------------------|-------|------------|-------------------------|-------|------------|
| | Technology Choice | Skill | Preference | Skill & Preference | Skill | Preference |
| Baseline | -0.24 | -0.26 | -0.11 | -0.27 | -0.28 | -0.25 |
| 4.1: Endogenous effort | -0.22 | -0.36 | -0.13 | -0.22 | -0.25 | -0.25 |
| 4.1 + 4.2: With capital | -0.25 | -0.36 | -0.16 | -0.24 | -0.27 | -0.28 |
| 4.1 + 4.2 + 4.3: Alt. Utility func. | -0.26 | -0.33 | 0.27 | -0.22 | -0.25 | 0.42 |
| 4.4: $1/(1-\rho) = 1.11$ | -0.24 | 0.20 | -0.10 | -0.17 | -0.13 | -0.23 |
| 4.4: $1/(1-\rho) = 3.33$ | -0.25 | -0.33 | -0.13 | -0.25 | -0.27 | -0.27 |
| 4.4: $1/(1-\sigma) = 1.4$ | -0.11 | -0.29 | -0.13 | -0.25 | -0.23 | -0.19 |
| 4.4: $1/(1-\sigma) = 2.6$ | -0.28 | -0.25 | -0.10 | -0.29 | -0.30 | -0.27 |

Table5 Robustness Analysis of Time Gap Variation of Couple Households

this restriction that does not need any more, i.e., $z_m + z_f \neq 1$.

4.2 With Physical Capital Model

In this subsection, the endogenous home production effort model is further extended to include capital stock that is given exogenously. Each household have one unit of capital stock k and rent it to firms to at a rental rate r . The total capital stock equals to $\mathbb{N}k = K$. The couple and the single household's budget constraint are added capital income,

$$\text{Couple household: } (1 + \tau_c)g \leq (1 - \tau_\ell)(w_m e_m h_{Mm} + w_f e_f h_{Mf}) + (1 - \tau_k)2rk + 2T, \quad (18)$$

$$\text{Single household: } (1 + \tau_c)g_s^* \leq (1 - \tau_\ell)w_s e_s h_{M,s}^* + (1 - \tau_k)rk + T, \quad (19)$$

where τ_k is the capital income tax, r is the rental rate of capital and k is a per capita physical capital, $k \equiv K/\mathbb{N}$.

The government's budget constraint also changes as including capital income tax revenue,

$$\mathbb{N}T = \mathbb{N}\tau_c g + \sum_{s \in \{m, f\}} \mathbb{N}_s^* \tau_c g_s^* + \sum_{s \in \{m, f\}} \mathbb{N} \tau_\ell w_s e_s h_{M,s} + \sum_{s \in \{m, f\}} \mathbb{N}_s^* \tau_\ell w_s e_s h_{M,s}^* + \tau_k K \quad (20)$$

FOCs of household are same as the main model.

Firms then use capital, labor and technology to produce output according to the two-tier production function,

$$\max_{K, \{L_s, A_s, K\}_{s \in \{m, f\}}} \{Y - w_m L_m - w_f L_f - rK\},$$

| $Var(CF)/Var(Data)$ | Independent Experiments | | | Conditional Experiments | | |
|-------------------------------------|-------------------------|-------|------------|-------------------------|-------|------------|
| | Technology Choice | Skill | Preference | Skill & Preference | Skill | Preference |
| Baseline | 0.23 | 0.48 | 0.22 | 1.18 | 0.81 | 0.81 |
| 4.1: Endogenous effort | 0.17 | 0.39 | 0.18 | 0.93 | 0.64 | 0.63 |
| 4.1 + 4.2: With capital | 0.53 | 0.62 | 0.59 | 1.16 | 0.87 | 1.07 |
| 4.1 + 4.2 + 4.3: Alt. Utility func. | 0.67 | 0.48 | 0.44 | 1.10 | 0.88 | 1.17 |
| 4.4: $1/(1 - \rho) = 1.11$ | 0.24 | 0.51 | 0.23 | 1.27 | 0.86 | 0.85 |
| 4.4: $1/(1 - \rho) = 3.33$ | 0.21 | 0.44 | 0.20 | 1.08 | 0.74 | 0.74 |
| 4.4: $1/(1 - \sigma) = 1.4$ | 0.18 | 0.73 | 0.30 | 1.08 | 0.71 | 0.91 |
| 4.4: $1/(1 - \sigma) = 2.6$ | 0.29 | 0.37 | 0.18 | 1.23 | 0.86 | 0.81 |

| $corr(Data, CF)$ | Independent Experiments | | | Conditional Experiments | | |
|-------------------------------------|-------------------------|-------|------------|-------------------------|-------|------------|
| | Technology Choice | Skill | Preference | Skill & Preference | Skill | Preference |
| Baseline | 0.33 | -0.16 | 0.24 | 0.20 | 0.09 | 0.29 |
| 4.1: Endogenous effort | 0.39 | -0.16 | 0.23 | 0.22 | 0.11 | 0.32 |
| 4.1 + 4.2: With capital | 0.33 | -0.16 | 0.28 | 0.27 | 0.13 | 0.31 |
| 4.1 + 4.2 + 4.3: Alt. Utility func. | 0.33 | -0.15 | 0.36 | 0.31 | 0.21 | 0.35 |
| 4.4: $1/(1 - \rho) = 1.11$ | 0.33 | -0.17 | 0.24 | 0.20 | 0.09 | 0.29 |
| 4.4: $1/(1 - \rho) = 3.33$ | 0.33 | -0.16 | 0.24 | 0.20 | 0.09 | 0.29 |
| 4.4: $1/(1 - \sigma) = 1.4$ | 0.38 | -0.17 | 0.24 | 0.21 | 0.08 | 0.27 |
| 4.4: $1/(1 - \sigma) = 2.6$ | 0.28 | -0.16 | 0.24 | 0.19 | 0.09 | 0.27 |

Table6 Robustness Analysis of Time Gap Variation of Single Households

$$Y = K^\theta [(A_m L_m)^\sigma + (A_f L_f)^\sigma]^{\frac{1-\theta}{\sigma}} \quad (21)$$

$$s.t. A_m^\omega + v A_f^\omega \leq B,$$

where θ is the capital share and $0 < \theta < 1$.

4.3 Composite Type Utility Function

The utility function in baseline model is separable between consumption, leisure and each spouses. We examine whether we would obtain the same results under the different specifications for the household utility function. In this subsection, we chose the following specification which deal with the composite hours of leisure between husband and wife:

$$\max \left\{ \sum_{s \in \{m, f\}} \alpha_s \ln(c_s) + b \ln \left\{ [a_m (1 - h_{M,m} - h_{N,m})^\epsilon + a_f (1 - h_{M,f} - h_{N,f})^\epsilon]^{\frac{1}{\epsilon}} \right\} \right\} \quad (22)$$

where $\epsilon < 1$ governs the elasticity $1/(1 - \epsilon)$ between the male and female in leisure activities.

4.4 Elasticity of Substitution

Unfortunately, to our knowledge, there are no empirical studies on the elasticity of substitution of home production between couples, i.e., $1/(1 - \rho)$. Some previous studies of gender gap give this elasticity with lack of foundation. However, sharing roles of home production

may be affected by this elasticity. So, we checked sensitivity of the value of elasticity.

And, there are few empirical works that estimated the elasticity of substitution between male labor and female labor, $1/(1 - \sigma)$. Our baseline simulation is based on mean value of those works and we checked sensitivity of this value, too.

4.5 Results

We conduct several alternative specification and parameters checks to verify the robustness of the findings reported above. We do not experiment about *effort*, because home production effort are determined endogenously in these models, except baseline model. Table 4–6 show that there are no significant difference among each specification and parameter settings. We can conclude that our results are robust.

5 Conclusion

To what extent and how does firms' technology choice affect the cross-country variations in the gender gap in wage and time allocation?

In order to answer the question, we build a general equilibrium model of the gender wage gap and time allocation with technology choice and home production of households with different marital status. Firms choose their production technology depending on the relative abundance of labor of each sex and relative costs of shifting their technology.

The main finding is that technology choice has considerable impacts on the cross-country variation in not only the gender wage gap but also the gender difference of time allocation, implying that effects of a policy aiming to narrow the gender gaps are gradual since it must face with firms' technology choice including the labor market institutions, corporate culture and social norms which are difficult to change dramatically. It is also shown that there is no single mechanism determining the observed cross-country variations in the gender gaps. Therefore, the convergence in the technology choice across countries itself does not result the convergence in all measures of the gender gaps in general, suggesting that policy makers should set multiple targets when intending to narrow all measures of the gender gaps.

Possible extension is of course to introduce the bargaining in the household problem taking into account of the literature of the collective model (cf. Bourguignon et al. (2009)).

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AppendixA Data

AppendixA provides details of the data which we use for calibration and simulation.

A.1 Gross Domestic Product

We use per worker GDP denoted by y . GDP data is based on value added in the EU KLEMS. We convert national currency-measure GDP into 1997-basis PPP value and exclude government expenditures. The government expenditures data are obtained from OECD statistics. The numbers of workers (number of persons engaged) are also obtained from EU KLEMS.

A.2 Time Allocation

The data source of time allocation differs depending on countries. For countries except for Japan, we use *Multinational Time Use Study* (MTUS), and *Survey on Time Use and Leisure Activities* for Japan. The procedure of the construction of time allocation consistent with the model is discussed below for each statistics.

A.2.1 Multinational Time Use Study (MTUS)

The time allocation data for market working hours and home production hours are obtained from the Multi-National Time Use Study (MTUS). This data set contains the time allocation of individuals among countries. There are several versions of the data available, such as World 5.53, World 5.8 and World 6.0. Difference among these three versions are, that latter two versions include participants aged less than 18 and time allocation data are in more detailed way, while World 5.53 are categorized in broader way. In order to divide time allocation of a day into three blocks, market work, home production and leisure, respectively, the World 5.53 fully satisfies our aim. The countries included in World 5.53 are listed in the Table 7.

MTUS time use data is given in a form of a diary collected from the individuals. Records of

| Country | Survey Years |
|----------------|------------------------|
| Austria | 1992 |
| Germany | 1991-92, 2001-02 |
| Italy | 2002-03 |
| Netherlands | 1990, 1995, 2000, 2005 |
| Spain | 2002-03 |
| United Kingdom | 1995, 2000-01, 2005 |
| United States | 1992-94 , 2003 |

Table7 MTUS: Countries and Survey Years

one's behaviors are divided into harmonized 41 activities and for each activity, the amount of time allocated measured in the unit of minute are available. Therefore we have constructed the definition of time allocation for market work, home production and leisure, and reallocated former 41 activities into each category. Specifically we choose 4 variables to indicate the market work and 5 variables for home production and all the others as leisure. Details are shown in the Table 8 and Table 9.

Next we describe the methodology for constructing the time use data consistently for our analysis. We have dropped the individuals who are not employed (including the retired person) and the ages are restricted to the range of 20 to 60. Both students and samples with military duty has been omitted. Also, we ignored the diaries recorded on weekends and people working approximately less than 25 hours a week, or working more than 70 hours a week. The upper bound for home production hours are set to 10 hours a day. After filtering out the noisy samples, we left with a following countries that has sufficient sample size for constructing our time allocation data.

Construction method for time allocation variables are fairly simple. We have aggregated all the individuals' time allocation from their diaries which satisfies our requirements and employed the mean value as the representing time allocation for the economy. The basic statistics are shown in the Table 10 and 11.

A.2.2 Survey on Time Use and Leisure Activities

We obtained the time allocation data for Japan from the aggregated data of *Survey on Time Use and Leisure Activities* (Ministry of Internal Affairs and Communications, Statistics Bureau of Japan.). Construction methods for our variables are almost the same as MTUS. We defined worked hours as the market working hours $h_{M,s} \{s \in m, f\}$, and housework as the home production $h_{N,s} \{s \in m, f\}$, respectively. The data are shown in the Table 12.

| Variable Name | Variable Label | Variable Name | Variable Label |
|---------------|------------------------------|---------------|-------------------------------|
| AV1 | Paid work | AV21 | Walking |
| AV2 | Paid work at home | AV22 | Religious activities |
| AV3 | Paid work, second job | AV23 | Civic activities |
| AV4 | School, classes | AV24 | Cinema or theatre |
| AV5 | Travel to/from work | AV25 | Dances or parties |
| AV6 | Cook, wash up | AV26 | Social clubs |
| AV7 | Housework | AV27 | Pubs |
| AV8 | Odd jobs | AV28 | Restaurants |
| AV9 | Gardening | AV29 | Visit friends at their homes |
| AV10 | Shopping | AV30 | Listen to radio |
| AV11 | Childcare | AV31 | Watch television or video |
| AV12 | Domestic travel | AV32 | Listen to records, tapes, cds |
| AV13 | Dress/personal care | AV33 | Study, homework |
| AV14 | Consume personal services | AV34 | Read books |
| AV15 | Meals and snacks | AV35 | Read papers, magazines |
| AV16 | Sleep | AV36 | Relax |
| AV17 | Free time travel | AV37 | Conversation |
| AV18 | Excursions | AV38 | Entertain friends at home |
| AV19 | Active sports participation | AV39 | Knit, sew |
| AV20 | Passive sports participation | AV40 | Other leisure |
| | | AV41 | Unclassified or missing |

Table8 Definition of harmonized activities in MTUS

| Variable | MTUS Variables |
|-----------------|--------------------------|
| Market Work | AV1, AV2, AV3, AV5 |
| Home Production | AV6, AV7, AV8, AV9, AV10 |
| Leisure | All the others |

Table9 Definition of time allocation for market work, home production and leisure

AppendixB Calibration

In this section, we describe the detailed procedure of our calibration. Table 13 shows all variables in the baseline model. Variables are classified three types. First, “Data” represents that these variables are given by data directly. Second, “Exogenous parameters” are mainly taken from the previous studies. Third, “Calibrated parameters” are given by equations presented below.

We first calibrate the household structure $(\{N_s^*\}_{s \in \{m, f\}}, N)$, skill $\{e_s\}_{s \in \{m, f\}}$, and tax

| Variable | Obs. | Mean | S.D. | Min | Max |
|----------------|------|-------|-------|-------|-------|
| Austria | | | | | |
| h_{M_m} | 696 | 0.357 | 0.157 | 0.010 | 0.573 |
| h_{M_f} | 696 | 0.327 | 0.244 | 0.042 | 0.573 |
| h_{N_m} | 696 | 0.048 | 0.074 | 0.000 | 0.396 |
| h_{N_f} | 696 | 0.142 | 0.093 | 0.000 | 0.365 |
| Germany | | | | | |
| h_{M_m} | 1767 | 0.334 | 0.157 | 0.003 | 0.580 |
| h_{M_f} | 1767 | 0.317 | 0.277 | 0.035 | 0.580 |
| h_{N_m} | 1767 | 0.070 | 0.078 | 0.000 | 0.368 |
| h_{N_f} | 1767 | 0.107 | 0.090 | 0.000 | 0.309 |
| Italy | | | | | |
| h_{M_m} | 368 | 0.343 | 0.140 | 0.063 | 0.576 |
| h_{M_f} | 368 | 0.300 | 0.236 | 0.139 | 0.549 |
| h_{N_m} | 368 | 0.042 | 0.054 | 0.000 | 0.319 |
| h_{N_f} | 368 | 0.119 | 0.088 | 0.000 | 0.264 |
| Netherlands | | | | | |
| h_{M_m} | 2855 | 0.358 | 0.160 | 0.010 | 0.573 |
| h_{M_f} | 2855 | 0.234 | 0.212 | 0.010 | 0.542 |
| h_{N_m} | 2855 | 0.052 | 0.066 | 0.000 | 0.396 |
| h_{N_f} | 2855 | 0.118 | 0.107 | 0.000 | 0.354 |
| Spain | | | | | |
| h_{M_m} | 1016 | 0.356 | 0.155 | 0.014 | 0.569 |
| h_{M_f} | 1016 | 0.331 | 0.270 | 0.014 | 0.576 |
| h_{N_m} | 1016 | 0.048 | 0.061 | 0.000 | 0.438 |
| h_{N_f} | 1016 | 0.106 | 0.081 | 0.000 | 0.271 |
| United Kingdom | | | | | |
| h_{M_m} | 963 | 0.335 | 0.169 | 0.014 | 0.576 |
| h_{M_f} | 963 | 0.320 | 0.253 | 0.007 | 0.552 |
| h_{N_m} | 963 | 0.055 | 0.069 | 0.000 | 0.431 |
| h_{N_f} | 963 | 0.071 | 0.046 | 0.000 | 0.365 |
| United States | | | | | |
| h_{M_m} | 2474 | 0.348 | 0.166 | 0.003 | 0.580 |
| h_{M_f} | 2474 | 0.333 | 0.308 | 0.007 | 0.580 |
| h_{N_m} | 2474 | 0.052 | 0.077 | 0.000 | 0.417 |
| h_{N_f} | 2474 | 0.069 | 0.059 | 0.000 | 0.299 |

Table10 Basic Statistics (Couples)

rates (τ_c, τ_L) , which are independently calibrated, in the household side. Then, given the result and also fixed exogenous parameters, we calibrate the firm side parameters. Finally, we calibrate the rest of parameters in the household side.

| Variable | Obs. | Mean | S.D. | Min | Max |
|----------------|------|-------|-------|-------|-------|
| Austria | | | | | |
| $h_{M,m}^*$ | 269 | 0.355 | 0.174 | 0.021 | 0.552 |
| $h_{M,f}^*$ | 269 | 0.338 | 0.118 | 0.073 | 0.573 |
| $h_{N,m}^*$ | 269 | 0.067 | 0.086 | 0.000 | 0.406 |
| $h_{N,f}^*$ | 269 | 0.097 | 0.078 | 0.000 | 0.365 |
| Germany | | | | | |
| $h_{M,m}^*$ | 676 | 0.345 | 0.163 | 0.007 | 0.576 |
| $h_{M,f}^*$ | 676 | 0.329 | 0.168 | 0.014 | 0.569 |
| $h_{N,m}^*$ | 676 | 0.062 | 0.060 | 0.000 | 0.326 |
| $h_{N,f}^*$ | 676 | 0.088 | 0.076 | 0.000 | 0.347 |
| Italy | | | | | |
| $h_{M,m}^*$ | 179 | 0.338 | 0.187 | 0.132 | 0.569 |
| $h_{M,f}^*$ | 179 | 0.304 | 0.024 | 0.014 | 0.542 |
| $h_{N,m}^*$ | 179 | 0.053 | 0.056 | 0.000 | 0.292 |
| $h_{N,f}^*$ | 179 | 0.092 | 0.044 | 0.000 | 0.243 |
| Netherlands | | | | | |
| $h_{M,m}^*$ | 1815 | 0.345 | 0.194 | 0.010 | 0.573 |
| $h_{M,f}^*$ | 1815 | 0.309 | 0.013 | 0.010 | 0.573 |
| $h_{N,m}^*$ | 1815 | 0.057 | 0.062 | 0.000 | 0.365 |
| $h_{N,f}^*$ | 1815 | 0.077 | 0.051 | 0.000 | 0.281 |
| Spain | | | | | |
| $h_{M,m}^*$ | 282 | 0.324 | 0.169 | 0.014 | 0.576 |
| $h_{M,f}^*$ | 282 | 0.313 | 0.127 | 0.007 | 0.576 |
| $h_{N,m}^*$ | 282 | 0.063 | 0.062 | 0.000 | 0.368 |
| $h_{N,f}^*$ | 282 | 0.098 | 0.034 | 0.000 | 0.340 |
| United Kingdom | | | | | |
| $h_{M,m}^*$ | 507 | 0.337 | 0.197 | 0.007 | 0.569 |
| $h_{M,f}^*$ | 507 | 0.295 | 0.032 | 0.007 | 0.573 |
| $h_{N,m}^*$ | 507 | 0.056 | 0.069 | 0.000 | 0.361 |
| $h_{N,f}^*$ | 507 | 0.077 | 0.054 | 0.000 | 0.438 |
| United States | | | | | |
| $h_{M,m}^*$ | 2002 | 0.352 | 0.181 | 0.001 | 0.578 |
| $h_{M,f}^*$ | 2002 | 0.335 | 0.122 | 0.002 | 0.580 |
| $h_{N,m}^*$ | 2002 | 0.052 | 0.077 | 0.000 | 0.410 |
| $h_{N,f}^*$ | 2002 | 0.066 | 0.069 | 0.000 | 0.451 |

Table11 Basic Statistics (Singles)

B.1 Independently Calibrated Parameters

Household Structure

Main purpose of our paper is to investigate the aggregate gender gap. This requires that the male-female ratio of labor supply in our model should match the data. To do this, we calibrate the household structure to fit the male-female ratio of labor supply data. Our

| | Austria | Germany | Italy | Japan | Netherlands | Spain | UK | US |
|-----------------------|---------|---------|-------|-------|-------------|-------|------|------|
| L_m/L_f | 1.37 | 1.04 | 0.69 | 2.49 | 2.49 | 1.39 | 1.12 | 1.40 |
| L_f | 0.07 | 0.08 | 0.09 | 0.04 | 0.05 | 0.07 | 0.08 | 0.07 |
| L_m | 0.10 | 0.08 | 0.07 | 0.11 | 0.11 | 0.10 | 0.09 | 0.10 |
| N | 0.35 | 0.37 | 0.34 | 0.29 | 0.38 | 0.41 | 0.43 | 0.34 |
| N_f^* | 0.18 | 0.16 | 0.19 | 0.19 | 0.14 | 0.10 | 0.08 | 0.18 |
| N_m^* | 0.12 | 0.10 | 0.13 | 0.24 | 0.11 | 0.07 | 0.07 | 0.14 |
| e_m/e_f | 1.43 | 1.11 | 0.69 | 1.77 | 1.88 | 1.37 | 1.09 | 1.45 |
| e_f | 0.41 | 0.48 | 0.59 | 0.36 | 0.35 | 0.42 | 0.48 | 0.41 |
| e_m | 0.59 | 0.52 | 0.41 | 0.64 | 0.65 | 0.58 | 0.52 | 0.59 |
| $h_{M,m}^*/h_{M,f}^*$ | 1.05 | 1.05 | 1.11 | 1.23 | 1.12 | 1.04 | 1.14 | 1.05 |
| $h_{M,f}^*$ | 0.34 | 0.33 | 0.30 | 0.28 | 0.31 | 0.31 | 0.29 | 0.34 |
| $h_{M,m}^*$ | 0.36 | 0.35 | 0.34 | 0.34 | 0.34 | 0.32 | 0.34 | 0.35 |
| $h_{N,m}^*/h_{N,f}^*$ | 0.68 | 0.70 | 0.58 | 0.22 | 0.75 | 0.65 | 0.72 | 0.79 |
| $h_{N,f}^*$ | 0.10 | 0.09 | 0.09 | 0.06 | 0.08 | 0.10 | 0.08 | 0.07 |
| $h_{N,m}^*$ | 0.07 | 0.06 | 0.05 | 0.01 | 0.06 | 0.06 | 0.06 | 0.05 |
| $h_{M,m}/h_{M,f}$ | 1.09 | 1.05 | 1.14 | 1.30 | 1.53 | 1.08 | 1.05 | 1.05 |
| $h_{M,f}$ | 0.33 | 0.32 | 0.30 | 0.25 | 0.23 | 0.33 | 0.32 | 0.33 |
| $h_{M,m}$ | 0.36 | 0.33 | 0.34 | 0.32 | 0.36 | 0.36 | 0.34 | 0.35 |
| $h_{N,m}/h_{N,f}$ | 0.34 | 0.65 | 0.36 | 0.07 | 0.44 | 0.46 | 0.77 | 0.79 |
| $h_{N,f}$ | 0.14 | 0.11 | 0.12 | 0.13 | 0.12 | 0.11 | 0.07 | 0.07 |
| $h_{N,m}$ | 0.05 | 0.07 | 0.04 | 0.01 | 0.05 | 0.05 | 0.06 | 0.05 |
| τ_c | 0.20 | 0.19 | 0.23 | 0.13 | 0.23 | 0.17 | 0.17 | 0.07 |
| τ_ℓ | 0.41 | 0.41 | 0.38 | 0.25 | 0.37 | 0.30 | 0.29 | 0.21 |
| w_m/w_f | 0.77 | 1.46 | 1.30 | 0.96 | 0.65 | 0.93 | 1.45 | 0.99 |
| w_f | 0.18 | 0.15 | 0.13 | 0.29 | 0.23 | 0.13 | 0.13 | 0.24 |
| w_m | 0.14 | 0.21 | 0.17 | 0.28 | 0.15 | 0.12 | 0.19 | 0.24 |
| $w_m e_m/(w_f e_f)$ | 1.11 | 1.62 | 0.90 | 1.69 | 1.22 | 1.28 | 1.58 | 1.44 |
| y | 0.03 | 0.03 | 0.02 | 0.04 | 0.03 | 0.02 | 0.03 | 0.04 |

Table12 Data

| | |
|-------------------------|--|
| Data : | $\mathbb{N}, N, N_s^*, \tau_c, \tau_\ell, w_s, h_{M,s}, h_{M,s}^*, h_{N,s}, h_{N,s}^*, \forall s \in \{m, f\}$ |
| Exogenous parameters : | $\sigma, \rho, \eta, \gamma_s, \gamma_s^*, \forall s \in \{m, f\}$ |
| Calibrated parameters : | $A_s, B, \omega, \alpha_s^*, \alpha_s, v, \xi, \xi_s^*, z_s, v, c_s, c_s^*, g, g_s^*, T, \forall s \in \{m, f\}$ |

Table13 Variable list

model's population consist of three groups that couple household N with a male and a female, male single household N_m^* and female single household N_f^* , the members of which consist of only a male and a female, and so we calibrate three parameters N , N_m^* , N_f^* . Matched labor supply ratio, we also use Census by each countries to calibrate as much as possible to fit the Census household structure. The reason why we need additional target, not only labor supply ratio but also household structure, is because household structure system in our model requires two calibration target to satisfy rank conditions.

Except for Japan and U.S., we use *EU statistics on income and living conditions* which reports the distribution of population by household types. This database contains no information about the age profile and presence or absence of children by gender for single person. So, we assume that single person with dependent children has the same ratio by gender. We calculated,

$$N_s^* = \frac{\text{Single person ratio} - \text{Single person with dependent children ratio}}{\text{Single person ratio}} \\ \times \text{Single person by sex ratio}, \\ N = \text{Two adults younger than 65 years.}$$

Japan's household structure data are obtained from Ministry of Internal Affairs and Communications, *Census*, 2005 and U.S. one are obtained from Census Bureau, *Statistical Abstract of the United States*, 2009. We use below figures,

$$N_s^* = \text{Household living alone by sex}, \\ N = \text{Married Couple without children.}$$

Finally, we normalize total number of households $\mathbb{N} = \sum_{s \in \{m, f\}} N_s^* + N$ to unity.

Skill

Skill e_s is calibrated by human capital accumulated in schooling. Specifically, we employ the similar methodology stated in Caselli and Coleman (2006) to construct the skill data using EU KLEMS (Release March 2008). As mentioned earlier, the skill is defined as a sum of daily working hour ratio per worker where workers are divided into three groups for their respective schooling; low, medium and high education. We set low educated group as a baseline and take a weighted sum of the medium and high educated workers relative to low educated workers. The weights for accumulation is a relative labor income per unit of working hours to the baseline group. The skill measure is independently constructed for male and female. Then, each skill is normalized by total sum of both efficiency unit.

Tax Rates

Both consumption and labor income tax rates are taken from McDaniel (2007). She provides these tax rates as well as taxes on investment and capital for fifteen OECD countries.

B.2 Firm-side Parameters

For the firm side parameters, we first calibrate the hourly gender wage gap $w_m e_m / (w_f e_f)$, and fix the value of the elasticity of substitution between market hours of male and female, $1/(1-\sigma)$. Then, using these results as well as those of the independently calibrated parameters and MTUS, we calibrate (A_m, A_f) for eight countries. Finally, we conduct a regression which salvages the values of (ω, v, B) under a certain assumption.

Hourly Gender Wage Gap

Hourly gender wage gap $w_m e_m / (w_f e_f)$ is calculated from real labor compensation level and total hours worked by male and female workers. Both variables are obtained from the EU KLEMS data. Note that the skill ratio can be obtained from the result of B.1. Hourly wage rate can be defined as the real labor compensation level divided by the total hours worked by each groups of workers.

Elasticity of Substitution between Market Hours of the Male and Female Labor

We choose $\sigma = 0.52$, which implies that the elasticity of substitution between market hours worked of males and females is 2.08. This is included in the empirically plausible range, from 2 to 3. Olivetti and Petrongolo (2011) surveys studies of the elasticity of substitution between market hours of males and females. Layard (1982) reports the value of 2 for the United Kingdom. Lewis (1985) reports 2.3 for Australia. For the United States, Weinberg (2000) and Acemoglu et al. (2004) report 2.4 and 3, respectively.

Labor Augmenting Technologies

The values of (A_m, A_f) are given by the following equations:

$$A_m = \frac{Y}{L_m} \left[\frac{\left(\frac{w_m e_m}{w_f e_f} \right) L_m}{\left(\frac{w_m e_m}{w_f e_f} \right) L_m + \frac{e_m}{e_f} L_f} \right]^{\frac{1}{\sigma}}, \quad A_f = \frac{Y}{L_f} \left[\frac{\frac{e_m}{e_f} L_f}{\left(\frac{w_m e_m}{w_f e_f} \right) L_m + \frac{e_m}{e_f} L_f} \right]^{\frac{1}{\sigma}}.$$

These are obtained from the hourly wage gap equation (3) and the production function (1). We have already obtained the hourly wage gap $w_m e_m / (w_f e_f)$ and the skill ratio e_m / e_f . For output of the market goods, we use the GDP net of the government expenditure. The data source and calculation are explained in AppendixA. For L_s , we use the labor market clearing condition (15) with data of market hours obtained from MTUS and the household structure calibrated previously. Note that Y in the above equation corresponds to the GDP per capita if we normalize the total population \mathbb{N} of the economy to unity.

Estimation of ω

The parameter ω of technology frontier can be estimated through the following equation derived from the firms' optimality conditions. We recall from equation (4),

$$\frac{A_m}{A_f} = v^{\frac{1}{\omega-\sigma}} \left(\frac{L_m}{L_f} \right)^{\frac{\sigma}{\omega-\sigma}}.$$

Assuming that ω and σ are constant across all countries, we then built a fixed-effect model. Taking logarithm and first-difference of both sides leads to the following specification,

$$\text{dlog} \left(\frac{A_{m,i,t}}{A_{f,i,t}} \right) = \frac{\sigma}{\omega - \sigma} \text{dlog} \left(\frac{L_{m,i,t}}{L_{f,i,t}} \right) + FE,$$

where $FE = \frac{1}{\omega-\sigma} \text{dlog}(v_t)$. Note that this specification implicitly assumes the time trend of v , which cannot appear in our static model but in the data.

In order to perform the estimation of the above equation, we have constructed the (unbalanced) panel data from 1981 to 2005 for 14 countries. That is, Australia, Austria, Belgium, Czech, Denmark, Finland, Germany, Italy, Japan, Korea, Netherlands, Spain, U.K. and U.S. The descriptive statistics are shown in Table 14 and the estimated results are in Table 15. Since the estimated values correspond to the coefficients of the first term in the right-hand side of the equation, the parameters $\omega = 1.12$ can be calculated easily for a given σ . These estimation results are consistent with our assumption that the solution to the firm's problems is interior, i.e., $\omega > \sigma/(1 - \sigma)$.

| Variable | Obs. | Mean | Std. Dev. | Min | Max |
|------------------------|------|--------|-----------|--------|-------|
| $\text{dlog}(A_m/A_f)$ | 322 | -0.031 | 0.047 | -0.290 | 0.099 |
| $\text{dlog}(L_m/L_f)$ | 322 | -0.024 | 0.040 | -0.304 | 0.103 |

Table14 Descriptive statistics

| Variables | $\text{dlog} \left(\frac{A_m}{A_f} \right)$ |
|--------------------------------|--|
| $\text{dlog}(L_m/L_f)$ | 0.866*** (0.047) |
| Observations | 322 |
| Adjusted R^2 | 0.69 |
| Implied Parameter (ω) | 1.12 |

Notes: Reported in the table are the results from fixed effects panel regressions. Standard errors in parentheses. *** significant at the 1% level.

Table15 Estimation Results

Relative cost v and Technology Frontier B

After estimating ω , we can calculate the relative costs v and shift parameter B analytically. v is computed from firm side FOCs of A_s ,

$$v = \left(\frac{A_m}{A_f} \right)^{\omega - \sigma} \left(\frac{L_m}{L_f} \right)^{-\sigma},$$

and B is computed by using the technology constraint (2),

$$B = A_m^\omega + v A_f^\omega.$$

B.3 Household-side Parameters

For the rest of the household-side parameters, we first choose the values of elasticities. Then, using MTUS and FOCs of the households' problem, we calibrate $\{\xi_s^*, \alpha_s^*\}_{s \in \{m, f\}}$ related to the single household and $(\{z_s\}_{s \in \{m, f\}}, \xi, \{\alpha_s\}_{s \in \{m, f\}})$ related to the couple household in order.

The Inverse of the Frisch Elasticity of Leisure

We set $\gamma_s = \gamma_s^* = 0.9$ which is close to the value, 1, chosen by Prescott (2004). According to Rogerson (2009), who studies a model of time allocation with home production which has the same specification as our model, time allocation does not depend on the value of the Frisch elasticity of leisure so much.

Elasticity of Substitution between Home Goods and Composite Time

We conduct our quantitative analysis with several values of η in the range of 0.4 to 0.6, which is the empirically plausible range the literature suggests. As a study using macro data, McGrattan et al. (1997) report the range of 0.40 to 0.45. Instead, Chang and Schorfheide (2003b) report the range of 0.55 to 0.60. Micro studies report similar ranges. Rupert et al. (1995) report the range of 0.40 to 0.45. Aguiar and Hurst (2007) report the range of 0.50 to 0.60.

Elasticity of Substitution between Male's and Female's Time devoted to Home Production

We set $\rho = 0.5$ which implies that the value of the elasticity of substitution between time devoted to home production of a male and a female is 2. We also consider other values for ρ in order to check the robustness of our result in Section 4.

Wage Rates and Lump-sum Transfer

In order to calibrate the rest of the household-side parameters, we use FOCs of the households' problem. However, we need the values of wage rates $\{w_s\}_{s \in \{m, f\}}$ and the lump-sum transfer T , which are consistent with the model and previous calibration.

The wage rate w_s is given by the marginal productivity condition.

The lump-sum transfer T is given by

$$T = (\tau_c + \tau_\ell) \left[\left(\frac{N}{\bar{N}} \right) \sum_{s \in \{m, f\}} w_s e_s h_{M, s} + \sum_{s \in \{m, f\}} \left(\frac{N_s^*}{\bar{N}} \right) w_s e_s h_{M, s}^* \right],$$

which is obtained by substituting the budget constraints of households, (5) and (10), into the government budget constraint and solving the result for T .

Single Household

For the single household, we first calibrate ξ_s^* by

$$\xi_s^* = \frac{(e_s h_{N, s}^*)^{\eta-1}}{g_s^{*\eta-1} \frac{1-\tau_\ell}{1+\tau_c} w_s + (e_s h_{N, s}^*)^{\eta-1}}, \quad \text{all } s \in \{m, f\},$$

which is obtained from (6). g_s^* is computed by using the budget constraint (5). We use MTUS for time allocation, i.e., $h_{M, s}^*$ and $h_{N, s}^*$.

Given the value of ξ_s^* , we then calibrate α_s^* using one of FOCs:

$$\alpha_s^* = \frac{(1 - h_{M, s}^* - h_{N, s}^*)^{-\gamma_s^*}}{(1 - h_{M, s}^* - h_{N, s}^*)^{-\gamma_s^*} + \frac{(1 - \xi_s^*)(e_s h_{N, s}^*)^{\eta-1} e_s}{\xi_s^* g_s^{*\eta} + (1 - \xi_s^*)(e_s h_{N, s}^*)^\eta}}, \quad \text{all } s \in \{m, f\}.$$

Couple Household

For the couple household, we first calibrate $\{z_s\}_{s \in \{m, f\}}$ by

$$z_f = \frac{1}{\left(\frac{w_m}{w_f} \right) \left(\frac{e_m h_{N, m}}{e_f h_{N, f}} \right)^{1-\rho} + 1}, \quad z_m = 1 - z_f,$$

the former of which is given by substituting $z_m = 1 - z_f$ into (12) and solving the result for z_f .

Then, given this result, we obtain the value of ξ by

$$\xi = \frac{[z_m (e_m h_{N, m})^\rho + z_f (e_f h_{N, f})^\rho]^{\frac{\eta}{\rho}-1} z_s (e_s h_{N, s})^{\rho-1}}{g^{\eta-1} \frac{1-\tau_\ell}{1+\tau_c} w_s + [z_m (e_m h_{N, m})^\rho + z_f (e_f h_{N, f})^\rho]^{\frac{\eta}{\rho}-1} z_s (e_s h_{N, s})^{\rho-1}},$$

which is obtained from (12) with $s = m$.

Finally, we obtain $\{\alpha_s\}_{s \in \{m, f\}}$ by

$$\alpha_m = \frac{\frac{D_2 D_3}{D_1} + D_4 - D_3}{\frac{D_2 D_3}{D_1} + D_4 + D_3}, \quad \alpha_f = 1 - \frac{D_2}{D_1} (1 - \alpha_m),$$

where

$$D_1 \equiv \frac{w_m e_m}{w_f e_f}, \quad D_2 \equiv \frac{(1 - h_{M, m} - h_{N, m})^{-\gamma_m}}{(1 - h_{M, f} - h_{N, f})^{-\gamma_f}},$$

$$D_3 \equiv \frac{\xi g^{\eta-1}}{\xi g^\eta + (1 - \xi)(z_m (e_m h_{N, m})^\rho + z_f (e_f h_{N, f})^\rho)^{\frac{\eta}{\rho}}} \frac{1 - \tau_\ell}{1 + \tau_c} w_m e_m, \quad D_4 \equiv (1 - h_{M, m} - h_{N, m})^{-\gamma_m}.$$

| Parameter | Value | Description |
|---|-------|---|
| $1/(1 - \eta)$ | 2.00 | EOS b/w g and $h_{N,s}$ |
| $\gamma_s = \gamma_s^*, s \in \{m, f\}$ | 0.90 | the inverse of the Frisch elasticity of leisure |
| ω | 1.12 | firm production technology frontier curvature |
| $1/(1 - \rho)$ | 2.00 | EOS b/w $h_{N,m}$ and $h_{N,f}$ |
| $1/(1 - \sigma)$ | 2.10 | EOS b/w L_m and L_f |
| ω_H | 3.00 | home production technology frontier curvature |

Note: EOS = elasticity of substitution

Table16 Exogenous parameters

This system of equations is obtained by solving

$$D_1 = \frac{1 - \alpha_m}{1 - \alpha_f} D_2,$$

$$D_3 = \frac{1 - \alpha_m}{\alpha_m + \alpha_f} D_4,$$

which are obtained from FOCs, for (α_m, α_f) .

AppendixC Algorithm for Computing a Competitive Equilibrium

Exogenous parameters: $v, \omega, B, v_H, \omega_H, B_H, \xi, \alpha_m, \alpha_f, e_m, e_f, z_m, z_f, \sigma, \eta, \tau_c, \tau_\ell, \rho, N, N^*, \mathbb{N}$

Endogenous variables: $A_m, A_f, w_m, w_f, h_{Mm}, h_{Mf}, h_{Nm}, h_{Nf}, h_N, L_m, L_f, y, T, g, c_m, c_f, U$

To compute a competitive equilibrium, we use the following algorithm to obtain endogenous variables:

Step 1: Make an initial guess: $\{w_m = w_m^0, w_f = w_f^0, T = T^0\}$ and $h_{M,s,1} = 0.1, h_{M,s,n} = 0.7$ are given.

Step 2: For the given lower and upper bounds, $h_{M,s,1}$ and $h_{M,s,n}$, generate the equidistant grid, i.e., $h_{M,m,i} \in \{h_{M,m,1}, \dots, h_{M,m,n}\}, h_{M,f,i} \in \{h_{M,f,1}, \dots, h_{M,f,n}\}, i = 1, 2, \dots, n$.

Step 3: Compute the system: $\forall i = 1, \dots, n$ set

$$(10) : g_i = \frac{1 - \tau_\ell}{1 + \tau_c} (w_m e_m h_{M,m,i} + w_f e_f h_{M,f,i}) + \frac{2T}{1 + \tau_c}$$

| | Austria | Germany | Italy | Japan | Netherlands | Spain | UK | US |
|--------------|---------|---------|-------|-------|-------------|-------|------|------|
| A_m/A_f | 1.20 | 2.63 | 1.67 | 2.82 | 1.38 | 1.76 | 2.66 | 1.88 |
| A_f | 0.07 | 0.05 | 0.06 | 0.07 | 0.07 | 0.05 | 0.05 | 0.08 |
| A_m | 0.09 | 0.14 | 0.10 | 0.21 | 0.10 | 0.08 | 0.13 | 0.16 |
| B | 0.11 | 0.17 | 0.14 | 0.23 | 0.11 | 0.09 | 0.15 | 0.19 |
| α_f^* | 0.58 | 0.59 | 0.52 | 0.45 | 0.53 | 0.52 | 0.50 | 0.47 |
| α_m^* | 0.55 | 0.52 | 0.53 | 0.42 | 0.52 | 0.48 | 0.47 | 0.45 |
| α_f | 0.61 | 0.66 | 0.52 | 0.59 | 0.54 | 0.59 | 0.60 | 0.56 |
| α_m | 0.53 | 0.44 | 0.55 | 0.27 | 0.49 | 0.44 | 0.37 | 0.36 |
| c_f^* | 0.33 | 0.33 | 0.32 | 0.36 | 0.32 | 0.29 | 0.30 | 0.36 |
| c_m^* | 0.34 | 0.36 | 0.31 | 0.44 | 0.35 | 0.30 | 0.35 | 0.42 |
| c_f | 0.03 | 0.03 | 0.02 | 0.05 | 0.03 | 0.02 | 0.03 | 0.04 |
| c_m | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.03 |
| g | 0.05 | 0.06 | 0.05 | 0.08 | 0.05 | 0.04 | 0.05 | 0.08 |
| g_f^* | 0.03 | 0.03 | 0.02 | 0.03 | 0.03 | 0.02 | 0.02 | 0.03 |
| g_m^* | 0.03 | 0.03 | 0.02 | 0.05 | 0.03 | 0.02 | 0.03 | 0.05 |
| v | 0.76 | 1.55 | 1.35 | 0.97 | 0.62 | 0.94 | 1.54 | 1.01 |
| ξ | 0.88 | 0.87 | 0.88 | 0.84 | 0.86 | 0.88 | 0.87 | 0.83 |
| ξ_f^* | 0.90 | 0.92 | 0.91 | 0.87 | 0.90 | 0.90 | 0.91 | 0.87 |
| ξ_m^* | 0.93 | 0.91 | 0.92 | 0.93 | 0.92 | 0.92 | 0.90 | 0.88 |
| z_f | 0.65 | 0.45 | 0.61 | 0.75 | 0.63 | 0.58 | 0.43 | 0.48 |
| z_m | 0.35 | 0.55 | 0.39 | 0.25 | 0.37 | 0.42 | 0.57 | 0.52 |
| v_H | 0.42 | 1.82 | 0.84 | 0.33 | 0.38 | 0.69 | 1.93 | 1.06 |
| B_H | 0.16 | 0.33 | 0.25 | 0.15 | 0.15 | 0.21 | 0.34 | 0.26 |

Table17 Calibrated parameters

$$\begin{aligned}
(h_{N,m,i}, h_{N,f,i}) & \left\{ \begin{aligned} & \frac{(1-\xi)[z_m(e_m h_{N,m,i})^\rho + z_f(e_f h_{N,f,i})^\rho]^{\frac{\eta}{\rho}-1}}{\xi g_i^\eta + (1-\xi)[z_m(e_m h_{N,m,i})^\rho + z_f(e_f h_{N,f,i})^\rho]^{\frac{\eta}{\rho}}} z_m(e_m h_{N,m,i})^{\rho-1} e_m \\ & = \frac{1-\alpha_m}{\alpha_m + \alpha_f} (1 - h_{M,m,i} - h_{N,m,i})^{-\gamma_m} \\ & \frac{(1-\xi)[z_m(e_m h_{N,m,i})^\rho + z_f(e_f h_{N,f,i})^\rho]^{\frac{\eta}{\rho}-1}}{\xi g_i^\eta + (1-\xi)[z_m(e_m h_{N,m,i})^\rho + z_f(e_f h_{N,f,i})^\rho]^{\frac{\eta}{\rho}}} z_f(e_f h_{N,f,i})^{\rho-1} e_f \\ & = \frac{1-\alpha_f}{\alpha_m + \alpha_f} (1 - h_{M,f,i} - h_{N,f,i})^{-\gamma_f}. \end{aligned} \right. \quad (23) \\
& \implies \text{Solve the simultaneous equation for } (h_{N,m,i}, h_{N,f,i}) \\
(9) : c_{m,i} & = \frac{\alpha_m}{\alpha_m + \alpha_f} \left\{ \xi g_i^\eta + (1-\xi)[z_m(e_m h_{N,m,i})^\rho + z_f(e_f h_{N,f,i})^\rho]^{\frac{\eta}{\rho}} \right\}^{\frac{1}{\eta}}, \\
\text{FOC of } c_{s,i} : c_{f,i} & = \frac{\alpha_f}{\alpha_m} c_{m,i}
\end{aligned}$$

$$(8) : U_i = \sum_{s \in \{m, f\}} \left\{ \alpha_s \ln(c_{s,i}) + (1 - \alpha_s) \frac{(1 - h_{M,s,i} - h_{N,s,i})^{1-\gamma_s} - 1}{1 - \gamma_s} \right\},$$

The above two equations of (23) are the FOC of h_{N_s} on couple household problem .

Step 4: Compute $(h_{M,m,opt}, h_{M,f,opt}) = \operatorname{argmax}_{h_{M,m,i}, h_{M,f,i}} U_i, \quad i = 1, \dots, n$

Step 5: If $|h_{M,m,opt} - h_{M,m,opt+1}| < \epsilon$, then set $h_{M,m} = h_{M,m,opt}$ and $h_{M,f} = h_{M,f,opt}$ and proceed to *Step 6*.

If $|h_{M,m,opt} - h_{M,m,opt+1}| > \epsilon$, then set

$$\begin{aligned} h_{M,s,1} &= h_{M,s,opt} - 0.1d, \\ h_{M,s,n} &= h_{M,s,opt} + 0.1d, \quad s \in \{m, f\} \end{aligned}$$

where $d \equiv |h_{M,s,1} - h_{M,s,n}|$ and return to *Step 2*.

Step 6: Solve the equation for $h_{M,s}^*$, $s \in \{m, f\}$

$$\frac{\alpha_s^* \xi_s^* \frac{1-\tau_\ell}{1+\tau_c} w_s e_s}{g_s^* \left[\xi_s^* + (1 - \xi_s^*) \left(\frac{\xi_s^*}{1-\xi_s^*} \frac{1-\tau_\ell}{1+\tau_c} w_s \right)^{\frac{\eta}{\eta-1}} \right]} = (1 - \alpha_s^*) \left[1 - h_{M,s}^* - \left(\frac{\xi_s^*}{1-\xi_s^*} \frac{1-\tau_\ell}{1+\tau_c} w_s \right)^{\frac{1}{\eta-1}} \frac{g_s^*}{e_s} \right]^{-\gamma_s^*} \quad (24)$$

Step 7: Compute $(g_s^*, h_{N,s}^*, L_s)$ by

$$\begin{aligned} (5) : \quad g_s^* &= \frac{1 - \tau_\ell}{1 + \tau_c} w_s e_s h_{M,s}^* + \frac{T}{1 + \tau_c}, \\ (6) : \quad h_{N,s}^* &= \frac{g_s^*}{e_s} \left(\frac{\xi_s^*}{1 - \xi_s^*} \frac{1 - \tau_\ell}{1 + \tau_c} w_s \right)^{\frac{1}{\eta-1}} \\ (15) : \quad L_s &= N_s^* e_s h_{M,s}^* + N e_s h_{M,s}. \end{aligned}$$

In addition, compute (A_m, A_f) as follows: In the case of appropriate technology choice, then

$$\begin{aligned} (2) + (4) : \quad A_f &= \frac{B^{\frac{1}{\omega}}}{(v + v^{\frac{\omega}{\omega-\sigma}}) \left(\frac{L_m}{L_f} \right)^{\frac{\omega\sigma}{\omega-\sigma}}} \\ (2) : \quad A_m &= (B - v A_f^\omega)^{\frac{1}{\omega}}, \end{aligned}$$

In the case of *inappropriate* technology choice, then

$$\begin{aligned} A_f &= \left(\frac{B}{\bar{A}^\omega + v} \right)^{\frac{1}{\omega}}, \\ A_m &= \bar{A} A_f, \end{aligned}$$

where \bar{A} is a exogenous technology parameter.

After calculating A_m and A_f , compute (w_s, T) by

$$\begin{aligned} \text{FOC of } L_s : \quad w_s &= [(A_m L_m)^\sigma + (A_f L_f)^\sigma]^{\frac{1}{\sigma}-1} (A_s L_s)^{\sigma-1} A_s, \\ (5) + (10) + (13) : \quad T &= \frac{\tau_c + \tau_\ell}{N} \{ N(w_m e_m h_{Mm} + w_f e_f h_{Mf}) \\ &\quad + N_m^* (w_m e_m h_{Mm}^*) + N_f^* (w_f e_f h_{Mf}^*) \} \end{aligned}$$

Step 8: Set $\Lambda = 0.5$ and compute

$$\begin{aligned} w_s^1 &= \Lambda w_s^0 + (1 - \Lambda)w_m \\ T^1 &= \Lambda T^0 + (1 - \Lambda)T \end{aligned}$$

Step 9: If $\sqrt{(w_m^1 - w_m^0)^2 + (w_f^1 - w_f^0)^2 + (T^1 - T^0)^2} > \epsilon$, then set $w_s^0 = w_s^1$, $T^0 = T^1$ and return to Step 1.

If $\sqrt{(w_m^1 - w_m^0)^2 + (w_f^1 - w_f^0)^2 + (T^1 - T^0)^2} < \epsilon$, then stop.

AppendixD Robustness

In this section, we present calibration forms and simulation algorithm that are different from benchmark model. The results of calibration and simulation are available upon request.

D.1 Endogenous Home Production Effort

D.1.1 Calibration Forms

In this endogenous home production model, the only difference from the benchmark model is including home production technology frontier that has three unknown parameters (ω_H, ν_H, B_H) . We set $\omega_H = 3$ exogenously to avoid corner solutions. For the remaining two parameters ν_H and B_H , we derive analytically solutions,

$$\nu_H = \left(\frac{z_m}{z_f} \right)^{\omega_H - 1} \left(\frac{e_m h_{Nm}}{e_f h_{Nf}} \right)^{-\rho}, \quad (25)$$

$$B_H = z_m^{\omega_H} + \nu_H z_f^{\omega_H}, \quad (26)$$

the former of which is obtained from the FOCs with respect to z_s .

D.1.2 Simulation Algorithm

We substitute

$$\begin{aligned} z_f &= \left[\frac{B_H}{\nu_H + \left\{ \nu_H \left(\frac{e_m h_{Nm}}{e_f h_{Nf}} \right)^\rho \right\}^{\frac{\omega_H}{\omega_H - 1}}} \right]^{\frac{1}{\omega_H}}, \\ z_m &= (B_H - \nu_H z_f^{\omega_H})^{\frac{1}{\omega_H}}, \end{aligned}$$

(23) in Step 3 of the simulation algorithm in AppendixC.

D.2 With Physical Capital Model

D.2.1 Calibration Forms

Couple Household:

$$k = \frac{K}{N}$$

| | Austria | Germany | Italy | Japan | Netherlands | Spain | UK | US |
|----------|---------|---------|-------|-------|-------------|-------|------|------|
| k | 0.19 | 0.21 | 0.14 | 0.28 | 0.15 | 0.10 | 0.11 | 0.14 |
| θ | 0.37 | 0.35 | 0.36 | 0.44 | 0.35 | 0.39 | 0.28 | 0.36 |
| τ_k | 0.18 | 0.13 | 0.18 | 0.17 | 0.18 | 0.20 | 0.31 | 0.27 |

Table18 Capital stock data

$$(18) + (19) + (20) : T = \frac{\tau_c + \tau_\ell}{\mathbb{N}} \{N(w_m e_m h_{Mm} + w_f e_f h_{Mf}) + N_m^*(w_m e_m h_{Mm}^*) + N_f^*(w_f e_f h_{Mf}^*)\} + (\tau_c + \tau_k)rk$$

$$(18) : g = \frac{1 - \tau_\ell}{1 + \tau_c} (w_m e_m h_{Mm} + w_f e_f h_{Mf}) + \frac{(1 - \tau_k)2rk}{1 + \tau_c} + \frac{2T}{1 + \tau_c}$$

Single Household:

$$(19) : g_s^* = \frac{1 - \tau_\ell}{1 + \tau_c} w_s e_s h_{M,s}^* + \frac{(1 - \tau_k)rk}{1 + \tau_c} + \frac{T}{1 + \tau_c},$$

Firm:

$$\text{FOC of } K : \theta = \frac{rK}{Y}$$

$$(21) + \text{FOC of } L_s : A_s = \frac{1}{L_s} \left(\frac{Y}{K^\theta} \right)^{\frac{1}{1-\theta}} \left[\frac{w_s L_s}{w_m L_m + w_f L_f} \right]^{\frac{1}{\sigma}}$$

$$\text{FOC of } L_s : w_s = (1 - \theta)K^\theta [(A_m L_m)^\sigma + (A_f L_f)^\sigma]^{\frac{1-\theta}{\sigma}-1} (A_s L_s)^{\sigma-1} A_s$$

Remaining variables and parameters are the same as benchmark model.

D.2.2 Data

This model requires a real capital stock data k , capital compensation to GDP ratio θ and capital income tax rate τ_k . Capital stock and capital compensation to GDP ratio are obtained from the EU KLEMS 2009 version and capital income tax rate are obtained from McDaniel (2007) (see Table 12). EU KLEMS 2009 is the newest version, but this does not include detail labor statistics, such as labor compensation by gender and by skill. So, we also use EU KLEMS 2008 version to use labor data.

D.2.3 Simulation Algorithm

1. In *Step 1* of algorithm in AppendixC, add “ $r = r^0$ and r^0 is given”.
2. In *Step 3*, use

$$g = \frac{1 - \tau_\ell}{1 + \tau_c} (w_m e_m h_{Mm} + w_f e_f h_{Mf}) + \frac{1 - \tau_k}{1 + \tau_c} 2rk + \frac{2T}{1 + \tau_c}.$$

3. In *Step 7*, use

$$g_s^* = \frac{1 - \tau_\ell}{1 + \tau_c} w_s e_s h_{M,s}^* + \frac{1 - \tau_k}{1 + \tau_c} rk + \frac{T}{1 + \tau_c},$$

$$w_s = (1 - \theta)K^\theta [(A_m L_m)^\sigma + (A_f L_f)^\sigma]^{\frac{1}{\sigma}-1} (A_s L_s)^{\sigma-1} A_s,$$

$$T = \frac{\tau_c + \tau_\ell}{N} \{N(w_m e_m h_{Mm} + w_f e_f h_{Mf}) + N_m^*(w_m e_m h_{Mm}^*) + N_f^*(w_f e_f h_{Mf}^*)\} + (\tau_c + \tau_k)rk,$$

and add the following equations,

$$(21) : Y = K^\theta [(A_m L_m)^\sigma + (A_f L_f)^\sigma]^{\frac{1-\theta}{\sigma}},$$

$$\text{FOC of } K : r = \frac{\theta Y}{K}.$$

4. In Step 8, add

$$r^1 = \Lambda r^0 + (1 - \Lambda)r.$$

5. In Step 9, modify the convergence criterion,

$$\sqrt{(r^1 - r^0)^2 + (w_m^1 - w_m^0)^2 + (w_f^1 - w_f^0)^2 + (T^1 - T^0)^2} < \epsilon.$$

D.3 Composite Leisure Function

With this specification, we calibrate ϵ in such a way that ϵ is consistent with the Frisch elasticity of labor supply reported in the previous studies. Thus we first derive the form of the Frisch elasticity of labor supply. We use the reduced couple household's problem,

$$\begin{aligned} & \max_{g, \{h_{M,s}, h_{N,s}, z_s\}} \left\{ \ln[\mathcal{H}(\cdot)] + \tilde{b} \ln \left([a_m(1 - h_{M,m} - h_{N,m})^\epsilon + a_f(1 - h_{M,f} - h_{N,f})^\epsilon]^{\frac{1}{\epsilon}} \right) \right\} \\ \text{s.t. } & \mathcal{H}(g, e_m h_{N,m}, e_f h_{N,f}) = \left\{ \xi g^\eta + (1 - \xi) [z_m(e_m h_{N,m})^\rho + z_f(e_f h_{N,f})^\rho]^{\frac{\eta}{\rho}} \right\}^{\frac{1}{\eta}}, \\ & (1 + \tau_c)g \leq (1 - \tau_\ell) \sum_{s \in \{m, f\}} w_s e_s h_{M,s} + 2(1 - \tau_k)rk + 2T, \\ & h_{M,s} + h_{N,s} \leq 1, \quad \text{all } s \in \{m, f\}, \\ & z_m^{\omega_H} + \nu_H z_f^{\omega_H} \leq B_H, \\ & a_m + a_f = 1 \end{aligned} \quad (27)$$

where $\tilde{b} \equiv b/(\alpha_m + \alpha_f)$.

From FOCs of h_{Ms} ,

$$\tilde{b} \frac{a_s \ell_s^{\epsilon-1}}{a_m \ell_m^\epsilon + a_f \ell_f^\epsilon} = \chi(1 - \tau_\ell)w_s e_s, \quad \forall s, \quad (28)$$

where χ is the Lagrange multiplier of the budget constraint. We further take the total differentiation of this equation and suppose $d\chi = 0$,

$$\begin{aligned} & -\tilde{b} \left[\epsilon(a_m \ell_m^\epsilon + a_f \ell_f^\epsilon)^{-2} a_m \ell_m^{\epsilon-1} d\ell_m + \epsilon(a_m \ell_m^\epsilon + a_f \ell_f^\epsilon)^{-2} a_f \ell_f^{\epsilon-1} d\ell_f \right] a_s \ell_s^{\epsilon-1} \\ & + \tilde{b}(\epsilon - 1) (a_m \ell_m^\epsilon + a_f \ell_f^\epsilon)^{-1} a_s \ell_s^{\epsilon-2} d\ell_s = \chi(1 - \tau_\ell)e_s dw_s, \quad \forall s. \end{aligned} \quad (29)$$

Using (28), we obtain,

$$-\epsilon \frac{a_m \ell_m^\epsilon}{a_m \ell_m^\epsilon + a_f \ell_f^\epsilon} \frac{d\ell_m}{\ell_m} - \epsilon \frac{a_f \ell_f^\epsilon}{a_m \ell_m^\epsilon + a_f \ell_f^\epsilon} \frac{d\ell_f}{\ell_f} - (1 - \epsilon) \frac{d\ell_s}{\ell_s} = \frac{dw_s}{w_s}, \quad \forall s \in \{m, f\}. \quad (30)$$

Finally, substituting $dw_f/w_f = 0$ for (30) and solving for $\frac{d\ell_m/\ell_m}{dw_m/w_m}$, we obtain the Frisch elasticity of labor supply for male,

$$\phi_m \equiv \left. \frac{d\ell_m/\ell_m}{dw_m/w_m} \right|_{dw_f=d\chi=0} = - \left(1 + \frac{\epsilon}{1 - \epsilon} \frac{a_f \ell_f^\epsilon}{a_m \ell_m^\epsilon + a_f \ell_f^\epsilon} \right). \quad (31)$$

D.3.1 Calibration Forms

Couple Household: We solve the following equation for ϵ numerically,

$$(31): \quad \phi_m = - \left(1 + \frac{\epsilon}{1 - \epsilon} \frac{a_f \ell_f^\epsilon}{a_m \ell_m^\epsilon + a_f \ell_f^\epsilon} \right),$$

where $1/\phi_m$ is set to the value, 2, many macroeconomic studies use. a_m and a_f are computed by

$$\begin{aligned} \text{FOC of } h_{Mm}/h_{Mf}: \quad a_m &= \frac{\frac{w_m e_m}{w_f e_f} \left(\frac{1-h_{M,m}-h_{N,m}}{1-h_{M,f}-h_{N,f}} \right)^{1-\epsilon}}{1 + \frac{w_m e_m}{w_f e_f} \left(\frac{1-h_{M,m}-h_{N,m}}{1-h_{M,f}-h_{N,f}} \right)^{1-\epsilon}} \\ a_f &= 1 - a_m. \end{aligned}$$

\tilde{b} is obtained from

$$\text{FOC of } h_{Mm}: \quad \tilde{b} = \frac{\xi g^{\eta-1}}{\xi g^\eta + (1-\xi)[z_m(e_m h_{N,m})^\rho + z_f(e_f h_{N,f})^\rho]^{\frac{\eta}{\rho}}} \frac{1 - \tau_\ell}{1 + \tau_c} w_m e_m \frac{a_m \ell_m^\epsilon + a_f \ell_f^\epsilon}{a_m \ell_m^{\epsilon-1}}.$$

The other parameters are computed in the same way as in the benchmark case.

D.3.2 Simulation Algorithm

In *Step 3* of the simultaneous equation of Algorithm in section D.2.3, replace the FOC of h_{Mm} and h_{Mf} with

$$\begin{cases} \frac{(1-\xi)}{\Phi} [z_m(e_m h_{N,m})^\rho + z_f(e_f h_{N,f})^\rho]^{\frac{\eta}{\rho}-1} z_m e_m^\rho h_{N,m}^{\rho-1} = \tilde{b} \frac{a_m \ell_m^{\epsilon-1}}{a_m \ell_m^\epsilon + a_f \ell_f^\epsilon} \\ \frac{(1-\xi)}{\Phi} [z_m(e_m h_{N,m})^\rho + z_f(e_f h_{N,f})^\rho]^{\frac{\eta}{\rho}-1} z_f e_f^\rho h_{N,f}^{\rho-1} = \tilde{b} \frac{a_f \ell_f^{\epsilon-1}}{a_m \ell_m^\epsilon + a_f \ell_f^\epsilon} \end{cases},$$

and also replace the utility with

$$U = \left\{ \ln[\mathcal{H}(\cdot)] + \tilde{b} \ln \left([a_m(1 - h_{M,m} - h_{N,m})^\epsilon + a_f(1 - h_{M,f} - h_{N,f})^\epsilon]^{\frac{1}{\epsilon}} \right) \right\}.$$