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Bargaining for an Information Good with Externalities*

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Abstract

An information good has the property that it is replicable without costs, and it often involves an external diseconomy as it diffuses. Under the assumption of free resales of the information, we consider its trade by a bargaining model based on the argument of the bargaining set $\mathfrak{M}$ in cooperative game theory, and completely characterize the stable outcome of the bargaining.

1. Introduction

One of the remarkable characteristics of an information good is its free replicability. Once an agent acquires the good he can replicate it with a negligible cost and then resell it to other agents. Thus, under no special legal protection, such an information good will not be traded in the market (Arrow, 1962).

In general, however, an information good often has the effect of external diseconomies as exemplified by the case of new technologies and know-hows. Noguchi (1974) has argued that there are cases in which an information good with the externality can be traded. That is, when the external effect grows rapidly enough as the information diffuses, it is possible for a sole seller

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to make a contract with a group of buyers. More recently, Muto (1986) has investigated the problem of how far such an information good is diffused from a sole seller when its resale is allowed. Depending on the intensity of the external effects, there are cases in which the diffusion stops, and cases in which the diffusion goes through to the set of buyers along a sequential process of trading.

The purpose of the present paper is to reconsider the trade of such an information good. Under the condition that the resale is allowed, the problem arises from the fact that while each buyer has an incentive to resell the information after he acquired it, the external diseconomy induced by the diffusion of the information may deteriorate his own welfare. So, if the traders wish stable outcomes, how can the information good be traded?

The approach to this problem taken by Noguchi (1974) is one that resembles to the core theory of n-person games. Specifically, a contract was called *blocked* if for some group of the participants in the contract there existed a prospect to gain by violating the contract and reselling the information to some of the buyers who did not participate in the contract. If a contract is not blocked, then it is called a *self-binding* contract. This approach, however, precludes the possibility that, when the group threatens to resell, there will be a reaction from the opponent group of the participants. The possibility to react is real in the problem and should be embodied in the model because, due to the externality, the rest of the participants will be made worse off if the threat were carried out, and may also try to resell the information to protect its profits.

In what follows, we shall reformulate the trade as a bargaining model in which agents seek to make a binding agreement as to the allocation of an information good. The model corresponds to that of the bargaining set $\mathfrak{M}$ due to Aumann and Maschler (1963), modified appropreately to take the possibility to resell the information into the formalization of bargaining. We assume, in particular, that when a coalition of participants raises an objection by showing the possibility to resell, *all* the rest of the participants together counter this by stating that they can also resell the information to some buyers while keeping their own original profits unchanged. This constitutes the counter-objection in our bargaining model. The definition of the objection, on the other hand, is an appropreate reformulation of what Noguchi called "block". Then, an outcome of the bargaining will be called *stable* if for each objection there is a counter-objection against it.

The main result is a complete characterization of a stable outcome of the bargaining. If the number of the participants in the trade is 2, i. e., the seller and one buyer, then any outcome of the bargaining is stable. The buyer may have an objection, but any objection can be countered by the seller, and *vice versa.* On the other hand, if the number of the participants is greater than 2, then an outcome is stable if and only if there exists no objection. If there exist objections, then at least one of them cannot be countered. The condition for the nonexistence of an objection is essentially the same to that given by Noguchi (1974) for the existence of a self-binding contract, requiring that the externality be sufficiently strong. Thus, in this case, the intensity of the externality is a crucial factor of the bargaining.

2. *The Model*

Let $N=\{1, 2, \ldots\ldots, n\}$ be the set of all agents. Agent 1 is the sole seller of an information good, and the rest are the buyers. We assume that the information good is freely replicable without costs. Any buyer demands at most one unit of the information good. The information good produces a profit to each of the possessors. The profit is measured in terms of money and assumed same for all the possessors. The profit to any non-possessor is assumed zero. Also, the profit decreases as the number of the possessors increases. This is the assumption of the external diseconomy induced by the diffusion of the information. Typical examples of such an information good are new technologies or new technical know-hows. Thus, denoting by $E(k)$ the profit to any possessor when there are k possessors, the externality may by represented by the following monotonicity assumption:

Assumption 2.1 $E(1)>E(2)>,\ldots\ldots,>E(n)\geqq 0$.

It will turn out that the weak monotonicity is not sufficient for our purpose. It is also assumed that n is sufficiently large and that the profit when there are n possessors is negligibly small. We express this by a stronger form as follows:

Assumption 2.2 $E(n)=0$.

Let M be a subset of N, containing agent 1, and let m be the cardinality of M. $M-\{1\}$ is the set of buyers that the seller 1 is going to trade with. The buyers in $N-M$ will be referred to as the potential buyers. Under Assumptioh 2.2, we may assume that m is not too large, so that there will be always some potential buyers.

Let $p_i\geqq 0$ be the price of the information that the seller 1 offers to buyer i in M. Assume that $p_i\leqq E(m)$ for all i in $M-\{1\}$. Then, consider the n-vector $x=(x_1,\ldots\ldots, x_n)$ such that

$$x_1=E(m)+\sum_{i\epsilon M-\{1\}} p_i>E(1)$$

$$x_i=E(m)-p_i\geqq 0 \quad \text{for all } i\epsilon M-\{1\}, \text{ and}$$

$$x_j=0 \quad \text{for all } j\epsilon N-M.$$

Clearly, x is the vector of net profits when each of the buyers in M gets the information at the named price. The seller 1 can obtain the profit $E(1)$ by not selling the information; so that we may assume x_1 to be greater than $E(1)$ if the trade is to be carried out. On the other hand, we allow the trade to be effected with $x_i=0$ for buyers. When $M=\{1\}$, the vector x means no trade. Thus, the actual outcome of the trade can be represented by the following n-vector x.

Definition 2.1 An n-vector x is called an *M-imputation* if $2 \leqq m < n$, and if

$$\sum_{i \in M} x_i = mE(m),\ x_1 > E(1),\ 0 \leqq x_i \leqq E(m) \quad \text{for all } i \in M - \{1\}$$
$$\text{and } x_j = 0 \quad \text{for all } j \in N - M.$$

For M-imputations to exist, it will be necessary to assume:

Assumption 2.3 $mE(m) > E(1)$ for some integer m with $2 \leqq m < n$.

The question to be considered here is whether it is always possible to restrict the possessors of the information to M after they each have obtained the information. Since resales of the information are allowed, every possessor, on the one hand, may have an incentive to resell it to non-possessors and get additional profits thereby. But, on the other, the resale inficts the external diseconomy on every possessor, so that in some cases the payoffs of some possessors may result in a worse state than the original M-imputation. Taking this into account, we now formulate the trade as a bargaining model over M-imputations.

Any n-vector $y = (y_1, \ldots\ldots, y_n)$ is called a *payoff vector*. When S is any subset of N, we denote by $s = |S|$ the cardinality of S, and by $y(S)$ the summation of payoffs y_i over S.

Definition 2.2 A payoff vector y is an *objection* against an M-imputation x if for some nonempty $K \subset M$, and some nonempty $R \subset N - M$,

$$y_j > x_j \quad \text{for all } j \in K \cup R, \text{ and}$$
$$y(K \cup R) = (k+r)E(m+r) - kE(m) + x(K). \qquad (1)$$

Thus, in the objection, the members in K show the possibility that they can obtain more than x by reselling to R. The quantity given by (1) is the total gain to $K \cup R$ if the buyers j in M each get the information at price $E(m) - x_j$ and the possessors in K resell it to the potential buyers in R. This can be seen from the equalities:

$$y(K \cup R) = (k+r)E(m+r) - \sum_{j \in K} (E(m) - x_j), \quad \text{if } 1 \notin K,$$
$$= (k+r)E(m+r) + \sum_{j \in M-K} (E(m) - x_j), \quad \text{if } 1 \in K.$$

If the resale in the objection is to be executed, the share of all the remaining possessors in M becomes worse, and for some of them it may become even negative, i. e., $y_i = x_i - (E(m) - E(m+r)) < 0$. Hence, the agents in $M-K$ have enough reason to take a joint action against K.

Definition 2.3 A payoff vector z is a *counter-objection* against the objection y if for $L \equiv M-K$ and for some nonempty $T \subset N-M$,

$$z_j \geqq x_j, \text{ for all } j \in L \cup (T-R),$$
$$z_j \geqq y_j, \text{ for all } j \in T \cap R, \text{ and}$$
$$z(L \cup T) = (l+t)E(m+t) + kE(m) - x(K). \qquad (2)$$

Thus, the members in L counter the objection by showing against K the possibility that they can hold original shares by reselling to the potential buyers in T, guaranteeing each of the members in $T \cap R$ at least the amount K is going to offer.

The quantity given by (2) is the total gain to $L \cup T$ if the possessors in L resell the information to T. This can be verified by the equalities:

$$z(L \cup T) = |L \cup T| E(|M \cup T|) + \sum_{j \in K} (E(m) - x_j), \text{ if } 1 \in L,$$
$$= |L \cup T| E(|M \cup T|) - \sum_{j \in L} (E(m) - x_j), \text{ if } 1 \notin L.$$

Finally, we define:

Definition 2.4 An M-imputation x is said to be *stable* if any objection against x can be countered.

The outcome of the bargaining is thus the M-imputation for which either no objection exists, or if an objection exists it can be countered.

3. *The Result*

In this section, we state and prove the results. The first lemma is an immediate consequence of assumption 2.1.

Lemma 3.1 Let m and r be integers such that $1 \leqq m < n$, $1 \leqq r \leqq n-m$. If $E(m) \geqq (1+r)E(m+r)$, then

$$hE(m) \geqq (h+r)E(m+r), \text{ for all numbers } h \geqq 1.$$

Lemma 3.2 Let x be an M-imputation. Then, there is no objection against x if and only if

$$E(m) \geqq (1+r)E(m+r) \text{ for all integers } r=1, \ldots\ldots, n-m.$$

Proof. (necessity) Suppose that there is an integer $r(1 \leqq r \leqq n-m)$ such that $E(m) < (1+r)E(m+r)$. Let $K=\{i\}$ for any $i \in M$, and define a payoff vector y by

$$y_j = x_j + ((1+r)E(m+r) - E(m))/(1+r) \text{ for all } j \in K \cup R$$

where $R \subset N-M$, $|R|=r$. Then, $y_j > x_j$ for all $j \in K \cup R$ and

$$y(K \cup R) = (1+r)E(m+r) - E(m) + x(K)$$

But, by definition, the payoff vector y is an objection against x.

(sufficiency) Suppose there is an objection y against x. Then, there are $K \subset M$, and nonempty $R \subset N-M$ such that

$$y(K \cup R) = (k+r)E(m+r) - kE(m) + x(K) > x(K).$$

Hence, $(k+r)E(m+r) > kE(m)$, which, by Lemma 3.1, implies

$$(1+r)E(m+r) > E(m).$$

This completes the proof.

Lemma 3.3 Let x be an M-imputation, and suppose that $K \subset M$ has an objection against x. If every objection by K can be countered, then $m \leqq 2k$.

Proof. Let y be an objection by K against x such that $|R| = r$, where r is taken to satisfy

$$1 \leqq r \leqq n-m, \text{ and}$$
$$(k+r)E(m+r) \geqq (k+s)E(m+s)$$
$$\text{for all integers } s=1, \ldots\ldots, n-m.$$

Since y can be countered, it follows from Definition 2.3 that there must exist a payoff vector z such that for $L=M-K$ and some nonempty $T \subset N-M$,

$$\begin{aligned} z(L \cup T) &= (l+t)E(m+t) + kE(m) - x(K) \\ &\geqq x(L) + y(R \cap T) \\ &= x(L) + y(K \cup R) - y(K) - y(R-T) \\ &= x(M-K) + (k+r)E(m+r) - kE(m) + x(K) \\ &\quad - y(K \cup (R-T)). \end{aligned}$$

Hence,

$$y(K \cup (R-T)) - x(K \cup (R-T))$$

$$\geqq (m-2k)E(m)+(k+r)E(m+r)-(m-k+t)E(m+t)$$
$$\geqq (m-2k)E(m)+(k+t)E(m+t)-(m-k+t)E(m+t)$$
$$=(m-2k)(E(m)-E(m+t)). \qquad (3)$$

But, since any objection by K with $|R|=r$ can be countered, (3) must be true for any objection y' with y'_j greater than, but sufficiently close to x_j for each $j\epsilon K\cup(R-T)$. Hence, we obtain

$$0\geqq (m-2k)(E(m)-E(m+t)).$$

But, since $E(m)>E(m+t)$ by Assumption 2.1,

$$m\leqq 2k.$$

This completes the proof.

We can now state our theorem.

Theorem 3.1 Let x be any M-imputation.

(i) If $m=2$, then x is stable.

(ii) If $m>2$, then x is stable if and only if

$$E(m)\geqq (1+r)E(m+r) \text{ for all } r=1,\ldots\ldots, n-m.$$

Proof. (i). Put $M=\{1, i\}$. If there exists an objection y against x, then for some $k\epsilon M$, $y_k>x_k$. Define z by

$$z_{k'}=x_{k'}+(y_k-x_k) \text{ for } k'\epsilon M,\ k'\neq k.$$
$$z_j=y_j \quad \text{for all } j\epsilon R$$

Then,

$$z(L\cup R)=z(\{k'\}\cup R)$$
$$=x_k'+(y_k-x_k)+y(\{k\}\cup R)-y_k$$
$$=x_k'-x_k+(1+r)E(2+r)-E(2)+x_k$$
$$=(1+r)E(2+r)+E(2)-x_k.$$

The last equality follows from the fact that $2E(2)=x_k+x_k'$. Hence, z is a counter objection against y with $T=R$.

(ii). (necessity). Assume that x is stable. If there exist objections, then Lemma 3.2 implies that for some $r\epsilon\{1,\ldots\ldots, n-m\}$,

$$(1+r)E(m+r)>E(m).$$

Then, for any $i\epsilon M$, we can find y such that

$$y(\{i\}\cup R)=(1+r)E(m+r)-E(m)+x_i>x_i,$$
$$y_j>x_j \quad \text{for all} \quad j\epsilon\{i\}\cup R,$$

where $|R|=r$. Hence, $K\equiv\{i\}$ in fact has an objection against x.

But, since any objection must be countered, it follows from Lemma 3.3 that for $k=1$, $m \leqq 2k=2$. But, since $m>2$, this is a contradiction. Hence there must exist no objection, and the conclusion follows from Lemma 3.2.

(sufficiency). This follows immediately from Lemma 3.2; hence, completes the proof.

When $m=2$, any M-imputation is stable. Any objection by the buyer can be countered by the seller 1, and *vice versa.* This is so because in this setting the agent can do anything what the opponent can do.

When $m>2$, (ii) states that an M-imputation x is stable if and only if there exists no objection against x. If the external diseconomy is strong enough so that no single agent can raise objections, then no coalition of agents has objections, either. If, on the contrary, there exists an objection, then any one-person coalition has a justified objection, i. e., an objection which cannot be countered. In this case, the M-imputation x cannot be stable, which therefore means that no trade will occur.

The case $m=2$ will be unlikely if for some M with $m>2$ there is a stable M-imputation x with $x_1>2E(2)$. The seller will then choose M that is most profitable. On the other hand, if for any M with $m>2$ there is no stable M-imputation x, then the case $m=2$ would occur; only in this case the seller can expect a profit from the trade.

4. Concluding Remarks

The conclusion depends on the basic assumption that agents will seek to make a binding agreement in order to ensure stable profits from the information good. Under this assumption, an outcome can be thought of as stable if for each objection there is a counter objection against it. When $m>2$, the stability of an M-imputation is equivalent to the nonexistence of an objection. The stable M-imputation x is self-binding in Noguchi's terminology. In case $m=2$, any objection can be countered, so that any trade becomes stable.

If the resale can be carried out covertly without being detected, then each of the agents may have an incentive to violate the agreement, and the bargaining such as the one formalized here would become improper. In this case, a noncooperative-game model such as the one developed by Muto (1986) may provide insights as to how the information will be traded.

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