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MIYAZAKI, Kenji / 宮崎, 憲治

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法政大学比較経済研究所 / Institute of Comparative Economic Studies, Hosei University

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Kenji Miyazaki

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Kenji Miyazaki[*][†]

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Abstract

This paper investigates whether a change in the growth rate of the money supply enhances the rate of capital accumulation in a cash-in-advance monetary model with recursive utility. Although money is superneutral in the steady state, the effect of the growth rate of money on the speed of capital accumulation depends not only on the curvature of the felicity but also on the slope and curvature of the discount rate function. We find that when the discount rate decreases with consumption and the elasticity of marginal utility is greater than unity, inflation worsens capital accumulation on the transition path.

Keywords: endogenous time preference, superneutrality, transition path, cash-in-advance, complementarity

JEL classifications: O42

[*]Faculty of Economics, Hosei University, 4342 Aihara, Machida, Tokyo, Japan, 194-0298; e-mail: miya_ken@hosei.ac.jp; tel: +81-42-783-2591; fax: +81-42-783-2611

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1 Introduction

Money is superneutral when a change in the growth rate of money supply does not affect any real variable except real balances. Superneutrality has already been discussed by many researchers. For instance, using the money-in-the-utility function with perfect foresight, Sidrauski (1967) demonstrated that money is superneutral at the steady state. Fishcher (1979) considered a special family of utility functions[1] to state that superneutrality does not generally hold on the transition path to the steady state and that higher inflation enhances faster capital accumulation when the degree of relative risk aversion is not zero. In response to the second assertion in Fishcher (1979), Asako (1983) provided a counterexample that when consumption and money are perfect complements, higher inflation yields a slower rate of capital accumulation when the degree of relative risk aversion is greater than one. The above-cited studies all assumed that the discount rate or time preference rate is constant, although many other empirical studies, as surveyed by Becker and Mulligan (1997), have supported a nonconstant rate of time preference.

Several studies have developed a monetary model with recursive utility; i.e., utility with an internally determined discount rate. For example, Epstein and Hynes (1983) pointed out that superneutrality in the steady state depends on whether the discount rate function is increasing. Likewise,

[1] To be specific, the utility function has constant relative risk aversion, and the elasticity of substitution between consumption and money is unity.

Hayakawa (1995) attached recursive utility to the model in Asako (1983) and found that money is still superneutral in the steady state, independent of the slope of the discount rate. More recently, Chen et al. (2008) and Miyazaki (2010) derived conditions for a qualitative equivalence in which the money-in-the-utility, the transaction-cost, and the cash-in-advance models predict the same negative relationship between the growth rates of the money supply and other variables, such as consumption, real balances, capital, and welfare. Nevertheless, none of these studies explored the effect of inflation on the transition path to the steady state.

This paper is the first attempt to investigate whether a change in the growth rate of the money supply enhances the rate of capital accumulation in a monetary model with recursive utility. The main reason why the extant literature is silent on the transition path is that a monetary model with endogenous time preferences generally has four dimensions and is generally considered to be too complicated to be analyzed. However, Obstfeld (1990), Chang (1994) and Miyazaki and Utsunomiya (2009) have succeeded in reducing one dimension of the dynamic system by exploiting the fact that the value function is the same as a costate variable of the dynamic system on the optimal path and that the derivative of the value function is the other costate variable. This paper employs their technique.

This paper employs the cash-in-advance model with endogenous time preference in which the cash-in-advance constraint applies only to the purchase of consumption goods. When the constraint is always binding, our model is exactly the same as Hayakawa (1995). We find that the effect of the growth rate of money on the speed of capital accumulation depends not only on the curvature of felicity but also on the slope and curvature of the discount rate function. Accordingly, when the discount rate decreases with consumption and the elasticity of marginal utility is greater than unity, inflation worsens capital accumulation on the transition path.

The remainder of this paper is organized as follows. Section 2 describes our model economy and discusses the uniqueness of the steady state. Section 3 conducts a dynamic analysis to demonstrate the extent to which monetary expansion affects the transition path of capital to the steady state, as well as providing a numerical example. Section 4 concludes.

2 Model Economy

In this section, we describe our model economy, define the monetary equilibrium and the steady state, and briefly discuss the uniqueness of the steady state and the comparative statics.

Consider a monetary economy, in which c_t, k_t, M_t, P_t, and $m_t = M_t/P_t$ respectively denote consumption, the capital stock, the money stock, the price level and real money balances in a continuous time period $t \geq 0$. Technology is characterized by an increasing, strictly concave function $f(k_t)$ with $f(0) = 0$.[2] Representative agents are infinitely lived with perfect foresight

[2]We can easily extend this to $\tilde{f}(k) = f(k) - \delta k - nk$, where δ and n are the constant depreciation and population growth rates, respectively. To show the uniqueness, we set

and complete access to capital markets. Their preference is characterized by a felicity function $u(c_t)$ and a discounting function $\rho(c_t)$. We assume that u is increasing and strictly concave and that ρ is positive and concave as standard assumptions. We also assume $uu_{cc} - u_c^2 > 0$ as well as $u < 0$ sufficiently to provide the optimal solution, as discussed in the appendix.

The homogeneous economic agents initially have $k_0 > 0$ and $M_0 > 0$, and have the following lifetime utility:

$$\int_0^\infty \exp(-\Delta_t) u(c_t) dt, \tag{1}$$

where Δ_t, determined by $\dot{\Delta}_t = \rho(c_t)$, represents the cumulative discount rate. The discount rate ρ, a function of consumption, represents the degree of impatience. When $\rho_c > 0$ (resp. $\rho_c < 0$), the agents become more (resp. less) impatient as they consume. When $\rho_c = 0$, the degree of impatience is constant.

The agents face two constraints. The first is the budget constraint:

$$\dot{k}_t + \dot{m}_t = f(k_t) + v_t - c_t - \pi_t m_t, \tag{2}$$

where v_t is a lump-sum transfer and $\pi_t = \dot{P}_t / P_t$ is the rate of inflation. The second is the budget constraint:

$$c_t \leq \gamma m_t, \tag{3}$$

$\delta = n = 0.$

where $0 < \gamma \leq 1$. When $\gamma = 1$, the cash-in-advance constraint applies to the purchase of all consumption goods.

The representative agent chooses c_t and m_t to maximize (1) subject to the budget and the cash-in-advance constraints (2) and (3), the initial conditions being $k_0 > 0$ and $M_0 > 0$. The necessary and sufficient conditions for optimization are discussed in the appendix.

We assume that the cash-in-advance constraint (3) is always binding. Then, the agent's problem is equivalent to maximizing:

$$\int_0^\infty \exp(-\Delta_t) u(\min[c_t/\gamma,\, m_t]) dt,$$

subject to (2) and the initial conditions. The parameter γ represents the degree of complementarity between consumption and money. This is exactly the same assumption as in the model in Hayakawa (1995).

The government behaves in a (monetary-theoretically) conventional way. It prints money at a constant rate $\mu = \dot{M}_t/M_t$ and runs a balanced budget by transferring seigniorage revenues to the consumers in a lump-sum manner: $v_t = \mu m_t$.

In equilibrium, the money and the goods markets clear:

$$\dot{m}_t = (\mu - \pi_t) m_t \quad (4)$$

$$\dot{k}_t = f(k_t) - c_t. \quad (5)$$

Let $R_t = f_k(k_t) + \pi_t$ be the nominal interest rate. A monetary equilibrium is then a positive set of paths $\{c_t, k_t, m_t, R_t\}_{t \in [0,\infty)}$, in which c_t, k_t, and m_t are positive; R_t is nonnegative; the representative agent maximizes (1) subject to (2) and (3) and the initial conditions, and the government behavior condition and the market clearing conditions hold. Note that the rate of inflation is determined by $\pi_t = R_t - f_k(k_t)$. In what follows, we omit the time index to ease the notational burden.

Employing (4) and (14) from the appendix, we obtain the following system dynamics:[3]

$$\begin{aligned} \dot{R} &= (\gamma + R)(f_k - \rho) - [\theta(\gamma + R) + c\rho_c]\frac{\dot{c}}{c} - \gamma\rho_c\dot{k}, \\ \frac{\dot{c}}{c} &= f_k + \mu - R, \end{aligned}$$

and (5), where $\theta = -c\{u_{cc} - V(m+k)\rho_{cc}\}/\{u_c - V(m+k)\rho_c\}$ is the elasticity of marginal utility,[4] and $V(m+k)$ is the indirect lifetime utility or value function.[5] The above dynamics describe R, c, and k in equilibrium. The other variable m is determined by $m = c/\gamma$.

The elasticity of marginal utility θ depends not only on the curvature of the felicity but also on the slope and curvature of the discount rate function.

[3]Asako (1983) construct a system dynamics using m, k, and a costate variable for $m+k$, whereas we use c, k, and R. As shown in the appendix, nominal interest rates are the ratio of the costate for money to the multiplier for the constraint (3). When $\rho_c = 0$, our dynamic system is essentially the same as Asako (1979).

[4]The elasticity of marginal utility differs from the elasticity of marginal felicity, $-cu_{cc}/u_c$.

[5]The value function is introduced in the appendix.

Given $u < 0$ and $\rho > 0$, V takes a negative value. The more concave both u and ρ are, the greater θ is. A steeper slope for a negative $\rho_c < 0$ raises θ. In contrast, a steeper slope for a positive $\rho_c > 0$ and a less concave u and ρ lower θ. We assume $\theta > 0$.[6]

In a steady state, $\dot{R} = \dot{c} = \dot{k} = 0$. The conditions in the steady state denoted by an asterisk are $\pi^* = \mu$, $c^* = \gamma m^* = f(k^*)$:

$$\begin{aligned} \rho(c^*) &= f_k(k^*) \\ R^* &= f_k(k^*) + \mu \\ V(k^* + m^*) &= u(c^*)/\rho(c^*). \end{aligned} \tag{6}$$

Combining $c^* = f(k^*)$ and (6) leads to:

$$f_k(k^*) = \rho(f(k^*)).$$

When:

$$f_{kk}(k) < \rho_c(f(k))f_k(k) \tag{7}$$

for all k, $\lim_{k\to 0} f_k = \infty$, and $\lim_{k\to\infty} f_k = 0$, then there exists a unique $k^* > 0$. Once k^* is determined, $c^* > 0$ and $m^* > 0$ are determined by $c^* = f(k^*)$ and $m^* = c^*/\gamma$. For $R^* \geq 0$, the government must set the growth rate of money supply μ to be equal to or greater than $-f_k(k^*)$. Equation (7) is known as the Correspondence Principle (CP),[7] in which an increase in

[6] When $\rho_c \geq 0$, then θ is always positive because $u < 0$ and $\rho > 0$ are both concave.

[7] Chen et al. (2008) used the CP condition to prove uniqueness only in the case where

the discount rate dominates the increase in the marginal product of capital. As we have assumed $f_k > 0$ and $f_{kk} < 0$, the condition $\rho_c \geq 0$ for all $c > 0$ always assures (7).

It is also obvious that any change in monetary growth at the steady state does not change the real variables: k^*, c^*, and m^*. That is, money is superneutral, even when the discount rate is a function of consumption, as discovered by Hayakawa (1995).[8]

3 Dynamic Analysis

In this section, we provide the main result of this note; i.e., an investigation of how monetary expansion affects the transition path of capital to the steady state. At the end of the section, we also provide a numerical example to confirm our result.

Linearizing around the steady state, we obtain:

$$\begin{bmatrix} \dot{R} \\ \dot{c} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} (R+\gamma)\theta + c\rho_c & -\rho_c R & \begin{matrix}(1-\theta)(R+\gamma)f_{kk} \\ -\rho_c(cf_{kk}+\gamma f_k)\end{matrix} \\ -c & 0 & cf_{kk} \\ 0 & -1 & f_k \end{bmatrix} \begin{bmatrix} R-R^* \\ c-c^* \\ k-k^* \end{bmatrix}, \tag{8}$$

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the discount rate is independent of real balances in the money-in-the-utility model. The uniqueness in other cases is only shown graphically without rigorous proof.

[8] As a change in the growth rate of money supply also has no effect on real balances, Hayakawa (1995) used the term "strong superneutral".

where the coefficients in the matrix are evaluated at the steady state.

The characteristic equation of the coefficient matrix of (8), say A, is written as:

$$\begin{aligned}\psi(\lambda,\mu) &= -\lambda^3+\{(R+\gamma)\theta+f_k+c\rho_c\}\lambda^2 \qquad (9)\\ &\quad -\{(R+\gamma)\theta f_k+cf_{kk}+c(f_k-R)\rho_c\}\lambda+c(R+\gamma)(f_{kk}-\rho_c f_k).\end{aligned}$$

When there is a unique negative root for (9), then the dynamic system has a saddle point and accordingly is locally stable in an economic sense. For a saddle point, as Fishcher (1979) has shown, it is sufficient to show that:

$$\begin{aligned}\det A &= c(\gamma+R)(f_{kk}-\rho_c f_k)<0\\ \mathrm{tr}A &= (\gamma+R)\theta+c\rho_c+f_k>0\end{aligned}$$

at the steady state. The determinant is negative as long as the CP condition (7) holds. The trace is positive when $\theta>0$ and $\rho_c\geq 0$. Even in the case of $\theta>0$ and $\rho_c<0$, higher rates of inflation or nominal interest rates make the trace more likely to be positive. Given $c=f(k)$ at the steady state, $\rho_c>-f_k/f$ leads to $\mathrm{tr}A>0$. In what follows, we assume that $\theta>0$ and $\rho_c>\max[f_{kk}/f_k,-f_k/f]$ at the steady state.

Now, we investigate whether the growth rate of money supply affects the rate of capital accumulation. When the unique negative root is denoted by

λ^*, the linearized dynamics of k is described as:

$$\dot{k} = -\lambda^*(k^* - k).$$

When $k < k^*$, the sign of $d\dot{k}/d\mu$ is the opposite sign of $d\lambda^*/d\mu$, indicating that an increasing $-\lambda^*$ accelerates the rate of capital accumulation.

We calculate the sign of $d\lambda^*/d\mu$. As shown in Fishcher (1979) and Asako (1983), $d\lambda^*/d\mu$ has the same sign as $\partial\psi/\partial\mu$.[9] Therefore, we investigate the sign of:

$$\frac{\partial\psi}{\partial\mu} = \frac{\partial R}{\partial\mu}\{\theta\lambda^2 - (\theta f_k - c\rho_c)\lambda + c(f_{kk} - \rho_c f_k)\}$$

in the neighborhood of the steady state. Given:

$$\begin{aligned}\psi &= (\gamma + R - \lambda/\theta)\{\theta\lambda^2 - (\theta f_k - c\rho_c)\lambda + c(f_{kk} - \rho_c f_k)\} \\ &\quad -(\lambda/\theta)[(\theta - 1)cf_{kk} + c\rho_c\{(1+\theta)f_k + \theta\gamma - (1+\theta)\lambda\}] = 0\end{aligned}$$

at the steady state, we obtain:

$$\frac{\partial\psi}{\partial\mu} = c\lambda\frac{(\theta - 1)f_{kk} + \rho_c\{(1+\theta)f_k + \theta\gamma - (1+\theta)\lambda\}}{\theta(\gamma + R) - \lambda}\frac{\partial R}{\partial\mu}$$

around the steady state.

As $\partial R^*/\partial\mu = 1$ and $\lambda^* < 0$, the sign of $d\lambda^*/d\mu$ is the opposite sign of $\zeta \equiv (\theta - 1)f_{kk} + \rho_c\{(1+\theta)f_k + \theta\gamma - (1+\theta)\lambda\}$. Therefore, when $k < k^*$

[9] This is because $d\lambda^*/d\mu = -(\partial\psi/\partial\mu)/(\partial\psi/\partial\lambda^*)$ and $\partial\psi/\partial\lambda^* < 0$ as $\lambda^* < 0$ and $\psi(0,\mu) = \det A < 0$.

and ζ is greater (resp. smaller) than zero, then expanding monetary growth raises (resp. lowers) the speed of capital accumulation. When $\rho_c = 0$ and $u(c) = c^{1-\sigma}/(1-\sigma)$, then $d\lambda^*/d\mu >$ (resp. $<$) 0 as $\theta = \sigma >$ (resp. $<$) 1. This case is consistent with Asako (1982).

Therefore, we summarize the following proposition.

> **Proposition:** Assume that a steady state exists and that $\theta > 0$ and $\rho_c > \max[f_{kk}/f_k, -f_k/f]$ at the steady state. When $\rho_c \leq 0$ and $\theta > 1$, then monetary expansion worsens capital accumulation on the transition path to the steady state. On the contrary, when $\rho_c \geq 0$ and $\theta < 1$, then monetary expansion enhances capital accumulation.

Intuitive explanations for the proposition are as follows. In a constant time preference framework, the elasticity of marginal utility $-cu_{cc}/u_c$ is the inverse of the elasticity of intertemporal substitution. Similarly, in an endogenous time preference framework, the agents with a larger elasticity of marginal utility θ prefer their flatter consumption profile. Such agents sacrifice their capital accumulation to achieve consumption smoothing. The consumers with $\rho_c < 0$ become more impatient as they consume less. Because higher inflation (or a higher holding cost of money) distorts the intertemporal allocation, a combination of $\theta > 1$ and $\rho_c \leq 0$ generates a negative relationship between inflation and capital accumulation. With small $\theta < 1$, they do not care about consumption smoothing and can sacrifice their current consumption for total welfare. With a combination of $\rho_c \geq 0$ and $\theta < 1$, a higher growth rate of the money supply or higher inflation increases the rate of capital accumulation.

The role of the slope of the discount function in this paper is consistent with the results of past comparative statics. Chen et al. (2008) showed a long-run negative (resp. positive) relationship between inflation and capital when the degree of impatience is decreasing (resp. increasing) in money in the money-in-the-utility approach and in consumption in the transaction cost approach. Miyazaki (2010) considered a general cash-in-advance economy in which the constraint is applied not only to consumption but also to some or all investment, and discovered that although inflation always provides a negative effect on capital at the steady state, such a negative effect is mitigated by a positive slope $\rho_c > 0$ and is aggravated by a negative slope $\rho_c < 0$. In our results, the slope of ρ does not change the steady state but affects the transition path to the steady state in the same direction as the above-cited literature.

Finally, we present a numerical example. We consider $f(k) = k^\alpha$, $u(c) = c^{1-\sigma}/(1-\sigma)$, and:

$$\rho(c) = \begin{cases} \alpha c^{\epsilon} & (0 \leq \epsilon < 1), \\ \alpha(2-c)^{-\epsilon} & (-1 < \epsilon < 0), \end{cases}$$

where $0 < \alpha < 1$, $\sigma > 1$, and $-1 < \epsilon < 1$. Note that ρ is increasing in c when $\epsilon > 0$, decreasing when $\epsilon < 0$, and always concave.[10] In this setup, the steady state establishes $k^* = c^* = \gamma m^* = 1$ for any rate of monetary supply $\mu \geq -f_k = -\alpha$. Furthermore, $\rho_c > \max[f_{kk}/f_k, -f_k/f]$ at the steady state leads to $\epsilon > \max[(\alpha - 1)/\alpha, -1]$. The elasticity of marginal utility is a continuous function of σ and ϵ and is represented by:

$$\theta = \begin{cases} \frac{\sigma(\sigma-1)-\epsilon(\epsilon-1)}{\sigma+\epsilon-1} & (0 \leq \epsilon < 1), \\ \frac{\sigma(\sigma-1)+\epsilon(\epsilon+1)}{\sigma+\epsilon-1} & (-1 < \epsilon < 0). \end{cases}$$

Note that $\theta > 0$ as long as $\sigma + \epsilon - 1 > 0$.

Starting with zero rates of money growth ($\mu = 0$), we quantitatively investigate whether a marginal increase in the growth rate of money supply enhances capital accumulation with various combinations of σ and ϵ. We set $\alpha = 0.3$ and $\gamma = 1$. In this specification, the steady state is locally stable.[11] The result is summarized in Figure 1. When $\rho_c = 0$ or the discount rate is constant, higher growth rates of money supply always have a negative effect on transition as long as $\sigma > 1$. This is consistent with Asako (1983). When the elasticity of marginal utility θ is not so large and the discount rate increases with consumption, the monetary expansion enhances the rate of capital accumulation. When the discount rate decreases with consumption,

[10]This functional form is $\rho < 0$ for $c > 2$. We focus our attention on the steady state. Generally, there exists no function globally satisfying $\rho > 0$, $\rho_c < 0$, and $\rho_{cc} < 0$.

[11]This is because $\epsilon > \max[-0.7/0.3, -1] = -1$.

it always worsens the rate of capital accumulation as long as a positive θ exists.

4 Conclusion

This paper has investigated the extent to which a change in the growth rate of monetary supply enhances the rate of capital accumulation in a cash-in-advance monetary model with recursive utility, in which money is required to purchase consumption goods and the discount rate is a function of consumption. In such an economy, this paper has discovered that money is superneutral at the steady state but it is not superneutral on the transition path to the steady state. We show that the effect of the growth rate of money on the speed of capital accumulation depends not only on the curvature of felicity but also on the slope and curvature of the discount rate function. When the discount rate decreases with consumption and the elasticity of marginal utility is greater than unity, then inflation worsens capital accumulation on the transition path.

This paper is the first attempt to investigate whether a change in the growth rate of monetary supply enhances the rate of capital accumulation in a monetary model with recursive utility. A natural future task must be to examine the properties of the transition path in other monetary models, such as the money-in-the-utility, the transaction cost, and the generalized cash-in-advance models in which the constraint applies not only to consumption but also some or all investment.

Appendix

This appendix provides the necessary and sufficient conditions for optimization in which the economic agent maximizes (1) subject to (2) and (3) and the initial conditions.[12]

Let $a = m + k$. We denote investment by $x = \dot{k}$.[13] The present-value Hamiltonian is:

$$H = \exp(-\Delta)[u(c) + q(f(k) + v - c - \pi m) + \xi(\gamma m - c) + \eta(a - k - m) - \phi\rho(c)],$$

where ξ and η are the Lagrange multipliers for $c = \gamma m$ and $a = m + k$, and q and ϕ are the costate variables for $\dot{a}$ and $\dot{\Delta}$, respectively. As shown in the next paragraph, ϕ is negative when $u < 0$.

[12] In this appendix, the time index is omitted to ease the burden of notation.

[13] Capital depreciation is assumed to be zero in order to compare our results with those in Wang and Yip (1992) and Chen et al. (2008). Introducing capital depreciation still produces the same comparative statics results.

The first-order conditions yield:

$$\begin{aligned} u_c - \phi\rho_c &= q + \xi \\ \eta &= \xi\gamma - \pi q \\ \eta &= f_k q \\ \dot{q} &= \rho q + \eta \\ \dot{\phi} &= -u + \phi\rho, \end{aligned} \tag{10}$$

and the initial conditions and the transversality conditions: $\lim_{t\to\infty} q_t a_t \exp(-\Delta_t) = 0$ and $\lim_{t\to\infty} \phi_t \Delta_t \exp(-\Delta_t) = 0$. Note that the last differential equation (10) with the last transversality condition has the solution (1), leading to $\phi < 0$ when $u < 0$.

For a sufficient condition for the maximization problem, we should assume that the Hamiltonian H is concave with respect to c, m, k, a, and Δ for any $\tilde{\xi} \geq 0$, $\tilde{\eta} \geq 0$, $\tilde{q} \geq 0$ and $\tilde{\phi} \leq 0$, where $\tilde{\xi} = \exp(-\Delta)\xi$, $\tilde{\eta} = \exp(-\Delta)\eta$, $\tilde{q}_k = \exp(-\Delta)q_k$ and $\tilde{\phi} = \exp(-\Delta)\phi$. A sufficient condition for concavity is that $\exp(-\Delta)u(c)$ is concave with respect to c and Δ. Such concavity holds if $u < 0$, $u_{cc} < 0$, and $uu_{cc} - u_c^2 > 0$ for all c, excluding the possibility of $u > 0$. An example satisfying the conditions is $u(c) = c^{1-\sigma}/(1-\sigma)$, $\sigma > 1$.

We define the nominal interest rates as $R = f_k + \pi$. Then, the first order-conditions are equivalent to (10), $\eta = f_k q$, $\xi = Rq/\gamma$ and:

$$u_c - \phi\rho_c = q(1 + R/\gamma) \quad (11)$$

$$\dot{q}/q = \rho - f_k. \quad (12)$$

The time differential of (11) demonstrates $(u_{cc} - \phi_c\rho_{cc})\dot{c} - \rho_c\dot{\phi} = (1 + R/\gamma)\dot{q} + q\dot{R}/\gamma$. Using (11) and (12), we obtain:[14]

$$\dot{c} = \frac{u_c - \phi\rho_c}{u_{cc} - \phi\rho_{cc}}\left[\rho + \frac{\dot{\phi}\rho_c}{q(1 + R/\gamma)} - f_k + \frac{\dot{R}}{\gamma + R}\right]. \quad (13)$$

For a simpler dynamic analysis, we introduce the value function $V(a)$, the maximized value of lifetime utility expressed as a function of the starting asset $a = m + k$. As Obstfeld (1990) discusses, if no parameters are expected to change, the dynamic system no longer requires (10) by imposing $\phi = V(a)$ on the optimal path. Given $\dot{\phi} = V'(a)(\dot{m} + \dot{k})$, $V'(a) = q$, and $c = \gamma m$, equation (13) is:

$$\frac{\dot{c}}{c} = \frac{1}{\theta}\left[f_k - \rho - \frac{\dot{c} + \gamma\dot{k}}{\gamma + R}\rho_c - \frac{\dot{R}}{\gamma + R}\right], \quad (14)$$

where $\theta = -c(u_{cc} - V(m + k)\rho_{cc})/(u_c - V(m + k)\rho_c)$ represents the elasticity of the marginal utility.

[14]Because of (10) and (11), the first and second terms in the square bracket in (13) become

$$\rho + \frac{\phi\rho - u}{u_c - \phi\rho_c}\rho_c = \frac{u_c\rho - u\rho_c}{u_c - \phi\rho_c}.$$

This is called the rate of time preference (Epstein, 1987, and Obstfeld, 1990).

References

[1] Asako, Kazumi (1983) "The Utility Function and the Superneutrality of Money on the Transition Path." *Econometrica*, Vol. 51, pp. 1593–1596.

[2] Becker, Gary S., and Casey B. Mulligan (1997) "The Endogenous Determination of Time Preference." *Quarterly Journal of Economics*, Vol. 122, pp. 729–758.

[3] Chang, Fwu-Ranq (1994) "Optimal Growth and Recursive Utility: Phase Diagram Analysis." *Journal of Optimization Theory and Applications*, Vol. 80, No. 3, pp. 425–439.

[4] Chen, Been-Lon., Mei Hsu, and Chia-Hui Lin (2008) "Inflation and Growth: Impatience and a Qualitative Equivalent." *Journal of Money, Credit, and Banking*, Vol. 40, pp. 1310–1323.

[5] Epstein, Larry. G., and J. Allan Hynes (1983) "The Rate of Time Preference and Dynamic Economic Analysis." *Journal of Political Economy*, Vol. 91, No. 4, pp. 611–635.

[6] Fishcher, Stanley (1979) "Capital Accumulation on the Transition Path in a Monetary Optimizing Model." *Econometrica*, Vol. 47, pp. 1433–1439.

[7] Hayakawa, Hiroaki (1995) "The Complete Complementarity of Consumption and Real Balances and the Strong Superneutrality of Money." *Economics Letters*, Vol. 48, pp. 91–97.

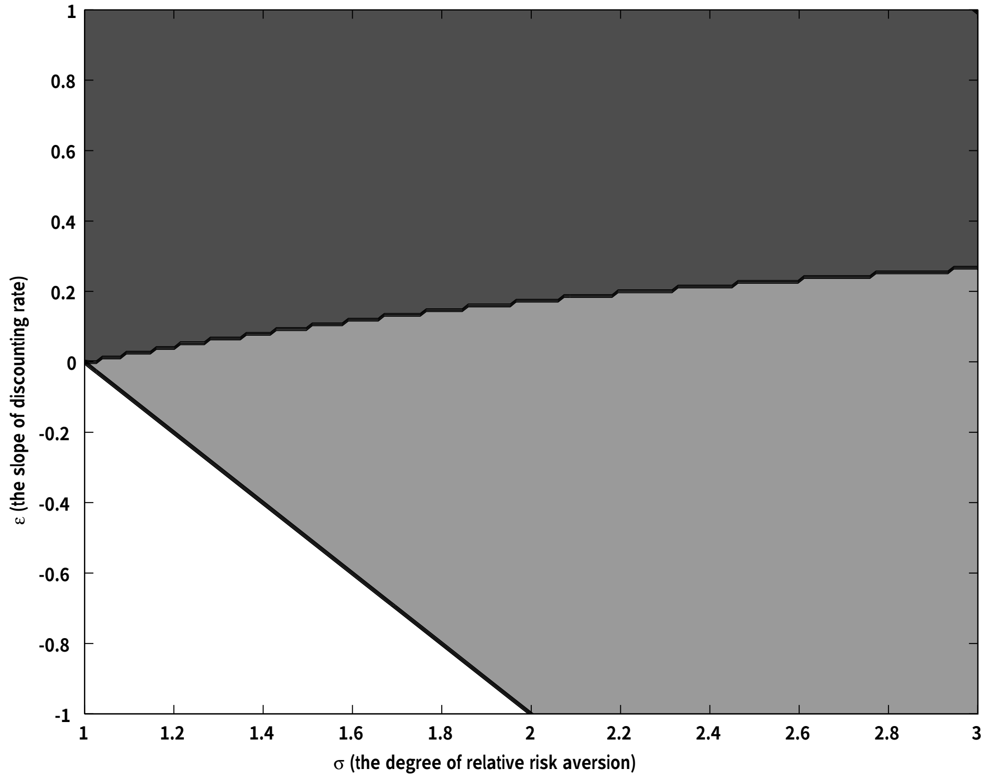
[8] Miyazaki, Kenji (2010) "Inflation, Growth, and Impatience in a Cash-in-advance Economy." ICES Working Paper, Hosei University.

[9] Miyazaki, Kenji and Hitoshi Utsunomiya (2009) "A Note on Local Stability Conditions for Two Types of Monetary Models with Recursive Utility," in: T.E. Simos, et al. (eds.), *Numerical Analysis and Applied Mathematics*, AIP Conference Proceedings 1168, American Institute of Physics, Melville, New York, Vol. 2, pp. 953–956.

[10] Obstfeld, Maurice (1990) "Intertemporal Dependence, Impatience, and Dynamics." *Journal of Monetary Economics*, Vol. 26, No. 1, pp. 45–75.

[11] Sidrauski, Miguel (1967) "Rational Choice and Patterns of Growth in a Monetary Economy." *American Economic Review, Papers and Proceedings*, Vol. 57, pp. 534–544.

Figure 1: The effect of monetary expansion on capital accumulation with several combinations of ϵ and σ



Note: The lightly shaded area depicts the case of a negative effect, whereas the heavily shaded area represents the case of a positive effect. The unshaded area represents the case where the elasticity of utility is negative ($\theta < 0$).