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(出版者 / Publisher)
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(雑誌名 / Journal or Publication Title)
比較経済研究所ワーキングペーパー

(巻 / Volume)
157

(開始ページ / Start Page)
1

(終了ページ / End Page)
14

(発行年 / Year)
2010-04-20

逐次的効用関数・曖昧さ・時間不整合性を考慮した国際マクロ分析　シリーズ No.8

Inflation, growth, and impatience in a cash-in-advance economy

Kenji Miyazaki

Inflation, growth, and impatience in a cash-in-advance economy

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April 20, 2010

Abstract

This note considers a cash-in-advance (CIA) economy in which the CIA constraint is applied not only to consumption but also to all or a part of investment and the discounting rate is a function of consumption. It investigates the effect of monetary growth on capital, money, consumption, and welfare. It demonstrates that as long as the condition assuring the uniqueness of steady state holds, the effect on the above variables is all-negative, although a positive slope of the discounting function mitigates the negative effect. This result can establish a qualitative equivalence among the money-in-the-utility model, the transaction-costs model, and the CIA model.

Keywords: endogenous time preference, superneutrality, qualitative equivalence, cash-in-advance

JEL classifications: O42

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[†]The author would like to thank Hitoshi Utsunomiya for excellent research assistance. The author is grateful for a grant-in-aid from the Ministry of Education and Science, the Government of Japan (21530277).

1 Introduction

As Wang and Yip (1992) and Chen et al. (2008) stated, the relationship between inflation and capital stock has been one of the central issues in monetary macroeconomic theory. Modern macroeconomics includes many approaches in terms of introducing money. Standard approaches are the money-in-the-utility (MIU) approach, the cash-in-advance (CIA) approach, and the transaction-cost (TC) approach. Money is demanded because money makes consumers happy (MIU), because they cannot buy goods without money (CIA), or because more money saves transaction costs, including time (TC). Predicting the effect of anticipated inflation on capital accumulation generally depends on what approach is adopted.

Feenstra (1986) established a functional equivalence between the three approaches. The CIA constraint on consumption only (Lucas, 1980) is a special case of a utility function[1] in the MIU approach or of a transaction cost function[2] in the TC approach. Feenstra demonstrated a duality between the MIU and TC approaches, but required a monetary model without capital and labor decisions and a redefinition of choice variables. The utility function is also too special to be used in a usual economic analysis. With such specification, Feenstra showed, money is superneutral in that monetary expansion does not affect any real variable except real balance holdings.

[1] Specifically, $w(c, m) = \min[c, m]$, where w is an instantaneous utility function of money m and consumption c, and m and c are perfect complements.

[2] Specifically, $T(c, m) = T_0 I(0 < c \leq m)$, where T represents the transaction-cost function, I is an indicator function and T_0 is a sufficiently large constant.

Wang and Yip (1992) established a qualitative equivalence; they discovered conditions in which all three approaches predict the same comparative static results in sign. They considered a monetary model with endogenous capital and labor decisions. Especially, in the CIA approach, they considered the generalized CIA constraints (Stockman, 1980), in which the constraint is applied not only to consumption but also to all or part of investments. When the real balance effect of money growth is weakly dominated by the consumption effect and some conditions[3] are satisfied, higher monetary growth lowers steady state capital, labor, real balances, consumption, and welfare.

Chen et al. (2008) shed new light on the qualitative equivalence between the MIU and TC approaches using an endogenous time preference. Even when labor supply is inelastic, they showed a long-run negative (resp. positive) relationship between inflation and capital when the degree of impatience is decreasing (resp. increasing) in money in the MIU approach and in consumption in the TC approach. However, unlike Wang and Yip (1992), they did not explore the CIA approach. To complete their qualitative equivalence, this note examines the effect of monetary growth on capital using the CIA model with endogenous time preference.

This note is not the first to employ the CIA model with endogenous time preference. Hayakawa (1995) investigated the relationship between inflation and capital when consumption and money are perfect complements in the

[3]The conditions are Pareto complementarity between consumption and money, Pareto complementarity between consumption and leisure, and Pareto substitutability between money and leisure.

MIU approaches. He found that money is strong superneutral,[4] independent of the nature of time preference. His functional specification is essentially the same as the model with the CIA constraint only on consumption.[5] Nevertheless, he did not examine the case of generalized CIA constraints. As Stockman (1985) showed, when the CIA constraint applies to both consumption and investment, higher growth rates of money supply lower capital stock in the case of constant time preference. Whether this property holds even in the case of endogenous time preference must be an interesting research question.

In the following, this note employs a CIA economy in which not only is consumption constrained but so is part or all of investment and the discounting rate is a function of consumption. We examine the effect of monetary expansion on capital, money, consumption, and welfare and demonstrate that as long as a condition assuring the uniqueness of steady state holds, the effect on all these variable is negative. The sign of the effect is independent of whether the discounting rate is increasing in consumption or not, which is different from the results of the MIU and TC models in Chen et al. (2008).

Furthermore, adding the comparative statics results to those in Chen et al. (2008), we establish a qualitative equivalence among the three approaches. Higher inflation lowers steady state capital, money, consumption, and welfare

[4] Hayakawa (1995) uses the term superneutral when monetary expansion does not affect any real variable except real balance holdings, and strong superneutral when it also affects real balances.

[5] To be exact, the CIA constraint should be binding.

when the degree of impatience is decreasing in money in the MIU approach and in consumption in the TC approach and when money is required not only to for consumption purchases but also for part or all of investment in the CIA approach.

2 Model and Results

This section describes our model economy, provides the monetary equilibrium and the steady state, discusses the uniqueness of that steady state, and finally conducts comparative statics.

Consider a monetary economy, in which c_t, m_t, and k_t respectively denotes consumption, real money balance, and capital at period t. Technology is characterized by an increasing, concave function $f(k_t)$ with $f(0) = 0$. Representative agents are infinitely long-lived with perfect foresight and complete access to the capital market. Their preference is characterized by a felicity function $u(c_t)$ and a discounting function $\rho(c_t)$. We assume u is increasing and strictly concave and ρ is positive and concave.

The homogeneous economic agents face two constraints. The first is the budget constraint:

$$\dot{k}_t + \dot{m}_t = f(k_t) + v_t - c_t - \pi_t m_t \tag{1}$$

where v_t is the lump-sum transfer, and the π_t is the rate of inflation.[6] The

[6]For a price level P_t at period t, the rate of inflation is determined by $\pi_t = \dot{P}_t / P_t$.

second is the budget constraint[7]:

$$c_t + \Gamma \dot{k}_t \leq m_t \tag{2}$$

for $0 \leq \Gamma \leq 1$. The cash-in-advance constraint with $\Gamma = 0$ applies only to the purchase of consumption, whereas the constraint with $\Gamma = 1$ indicates that money is also needed for investment. The former and the latter are respectively continuous versions of Lucas (1980) and Stockman (1981). The parameter Γ represents the degree of credit tightness.

Agents initially have capital stock $k_0 > 0$ and money stock $M_0 > 0$.[8] They have the following lifetime-utility:

$$\int_0^\infty u(c_t) \exp\{-\Delta_t\} dt, \tag{3}$$

where Δ_t, determined by $\dot{\Delta}_t = \rho(c_t)$, represents the cumulative discounting rate. The discounting rate ρ represents the degree of impatience and is a function of consumption.[9] When $\rho_c > 0$ (resp. $\rho_c < 0$), the agents become more (resp. less) impatience as they consume more. When $\rho_c = 0$, the degree of impatience is constant.

[7]Another type of constraint is proposed by Palivos et al. (1993). Their continuous version is

$$c + \Gamma(\pi, \kappa) k \leq m.$$

In their model, Γ is a function of the inflation rate π and a measure of credit looseness κ.

[8]For any initial price level P_0, real balances $m_0 = M_0/P_0$. Real balances m_t at period $t > 0$ are choice variables.

[9]See Chen et al. (2008) for justifying the consideration of endogenous time preference into a monetary model.

The representative agent chooses c_t and m_t to maximize (3) subject to the budget and cash-in-advance constraints (1) and (2), the initial conditions $k_0 > 0$ and $M_0 > 0$, and the transversality conditions. The necessary and sufficient conditions for optimization are discussed in the appendix.

The government behaves in a (monetary-theoretically) conventional way. It prints money at a constant rate μ and runs a balanced budget by transferring seigniorage revenues to the consumers in a lump-sum faishon: $v_t = \mu m_t$.

At equilibrium, the money and the goods markets are clear:

$$\dot{m}_t = (\mu - \pi_t) m_t \tag{4}$$

$$\dot{k}_t = f(k_t) - c_t. \tag{5}$$

A monetary equilibrium is a set of the path $\{c_t, k_t, m_t, \pi_t, \xi_t, \eta_t, \lambda_t, \phi_t\}_{t=0}^{\infty}$ that maximizes (3) subject to (1) and (2) as welll as the initial conditions, and where the government behavior condition and the market equilibrium conditions hold. Note that ξ_t, η_t, and λ_t are the shadow prices of money, capital, and the CIA constraints, respectively, and ϕ_t represents the indirect lifetime utlity or welfare from the period of t. These variables are defined in the appendix.

At steady state, $\dot{c} = \dot{m} = \dot{k} = \dot{\xi} = \dot{\eta} = \dot{\lambda} = \dot{\phi} = 0$. The steady state conditions (denoted by asterisks) are $\pi^* = \mu$, $c^* = m^* = f(k^*)$,

$$u_c(c^*) - \rho_c(c^*)\phi^* = \xi^* + \lambda^* \tag{6}$$

$$
\begin{aligned}
\eta^*/\xi^* &= 1+\Gamma\lambda^*/\xi^* && (7)\\
\lambda^*/\xi^* &= \mu+\rho(c^*) && (8)\\
f_k(k^*) &= \rho(c^*)\eta^*/\xi^* && (9)\\
\phi^* &= u(c^*)/\rho(c^*).
\end{aligned}
$$

Combining $c^* = f(k^*)$, (7), (8), and (9) leads to:

$$f_k(k^*) = \rho(f(k^*)) + \Gamma\rho(f(k^*))\{\mu + \rho(f(k^*))\}. \tag{10}$$

It is evident that there exists a unique $k^* > 0$ when $\mu + \rho(f(k)) > 0$[10] for all $k > 0$,

$$f_{kk} < \rho_c f_k\{1 + \Gamma(\mu + 2\rho)\} \tag{11}$$

for all k, $\lim_{k\to 0} f_k = \infty$, and $\lim_{k\to\infty} f_k = 0$. For $\xi^* > 0$, $\eta^* > 0$, and $\lambda^* > 0$, we should additionally assume $u_c - \rho_c u/\rho > 0$ for all $c > 0$. The additional assumption as well as (11) are assured by the condition that $\rho_c \geq 0$ for all $c > 0$. When $\Gamma = 0$, in which the cash-in-advance constraint applies only to consumption, then (11) simplifies to $f_{kk} < \rho_c f_k$. This is exactly the Correspondence Principle (CP), discussed in Chen et al. (2008).[11]

Next, we conduct comparative statics.[12] First, we examine the effect of

[10]It follows from (8) that $\lambda^*/\xi^* > 0$.

[11]Chen et al. proved the uniqueness only in the case where the discount rate is independent of real balances in the MIU model. The uniqueness in other cases is just shown graphically without rigorous proof.

[12]The comparative statics are evaluated at the steady state, and asterisks are omitted to ease the burden of notation.

rates of money supply μ on capital k at the steady state. It follows from (10) that

$$\frac{dk}{d\mu} = \frac{\Gamma\rho}{f_{kk} - \rho_c f_k\{1 + \Gamma(\mu + 2\rho)\}}. \tag{12}$$

at steady state. When $\Gamma = 0$, then $dk/d\mu = 0$. That is, money is superneutral, which is consistent with Hayakawa (1995). In the case of $\Gamma > 0$, when we assume (11), then $dk/d\mu < 0$. That is, money is not neutral.

Then, we investigate the effect of μ on other variables: consumption c, real balances m, and indirect lifetime-utility ϕ. Since $c = m = f(k)$ at the steady state, $dc/d\mu = dm/d\mu = f_k dk/d\mu$. Since $\phi = u/\rho$ at the steady state,

$$\begin{aligned} \frac{d\phi}{d\mu} &= \frac{d\phi}{dc}\frac{dc}{d\mu} = \frac{u}{\rho}\left(\frac{u_c}{u} - \frac{\rho_c}{\rho}\right)\frac{dc}{d\mu} \\ &= (\xi + \lambda)\frac{f_k}{\rho}\frac{dk}{d\mu}. \end{aligned}$$

From (6), $\xi + \lambda > 0$ when $u_c - \rho_c u/\rho > 0$ at the steady state. Thus, as long as $\xi + \lambda > 0$ and $\Gamma > 0$, the inflation effect on consumption, real balance, and welfare is negative, independent of the sign of ρ_c.

3 Discussion

This section discusses the comparative statics results. First, we explain the mechanism of the comparative statics and discuss the role of the discounting rate function. Then, comparing our results to past literature, we establish a qualitative equivalence. Finally, we suggest a future task to further this line of research.

The mechanism of the comparative statics (12) is as follows. From (8), increasing the growth rate of money raises the shadow price ratio of cash-in-advance constraints to money λ^*/ξ^*. From (7), increasing λ^*/ξ^* affects the shadow price ratio of capital to money η^*/ξ^* when $\Gamma > 0$. Since f_k decreases with k and $c^* = f(k^*)$, equation (9) indicates that increasing η^*/ξ^* has a negative effect on the steady state of capital. As long as the cash-in-advance constraint applies only to consumption, (7) says that the shadow price ratio of capital to money is unaltered, indicating that capital is costlessly obtained by barter (Stockman, 1981). But when $\Gamma > 0$, higher inflation makes capital more expensive than real balances, and induces decreases in capital stock. The constraint on part or all of investment operates as taxes on capital goods.

Whether ρ is increasing or decreasing in c, a Tobin effect, a positive relationship between inflation and capital stock, never emerges as long as the CP condition (11) hold. When $\rho_c > 0$, however, the effect of inflation on capital stock is mitigated. When $\Gamma > 0$, holding capital is relatively expensive, and inflation lowers the level of consumption as well as capital. Lower consumption makes an economic agent with $\rho_c > 0$ more patient, moderating a decrease in capital stock. On the other hand, since people with $\rho_c < 0$ tend to be more impatient and save less, higher inflation accelerates a decrease in capital stock.

As discussed in the introduction, Wang and Yip (1992) established a qualitative equivalence among the TC, CIA, and MIU approaches using a monetary model with an elastic labor supply and constant time preference. On the other hand, Chen et al. (2008) showed an equivalence between the TC and MIU approaches using a model with inelastic labor decisions and endogenous time preference. A reverse Tobin effect emerges when the discounting rate is decreasing in real balances in the MIU model and in consumption in the TC model. Such a decreasing discounting rate is supported by empirical results.[13] With the results in the previous section, we can establish a qualitative equivalence among the TC, CIA, and MIU models with inelastic labor decisions and endogenous time preference. Higher inflation lowers steady state capital, money, consumption, and welfare in the long run when the degree of impatience is decreasing in money in the MIU approach and in consumption in the TC approach and when money is required not only to for consumption purchases but also for part or all of investment in the CIA approach.

As shown in Chen et al. (2008), a Tobin effect emerges when the discounting rate is increasing in consumption and money in the TC and MIU models. In our model, the positive slope of the discounting rate function only appeases the severity of the negative relationship between inflation and capital, and cannot generate a Tobin effect. That is, the CIA constraint's effect reducing capital stocks dominates the impatience effect increasing capital stocks. Similarly, if a labor decision were introduced in the TC and MIU models, the endogenous labor effect might be dominant over the impatience

[13] See Chen et al. (2008) for empirical literature regarding endogenous time preference.

effect, independent of the slope of the discounting rate. Therefore, exploring a qualitative equivalence among the three monetary models with endogenous labor decisions and endogenous time preference would be, although complicated, an intriguing future task.

Appendix

This appendix provides the necessary and sufficient conditions for optimization in which the economic agent maximizes (3) subject to (1) and (2) and the initial and transversality conditions.

We denote investment by $x = \dot{k}$.[14] The present value Hamiltonian is

$$H = \exp(-\Delta)[u(c) + \xi(f(k) + v - c - \pi m - x) + \eta x + \lambda(m - c - \Gamma x) - \phi\rho(c)],$$

where λ is the Lagrange multiplier for the cash-in-advance constraints, and ξ, η and ϕ are the costate variables for $\dot{m}$, $\dot{k}$, and $\dot{\Delta}$, respectively. As shown in the next paragraph, ϕ is negative when $u < 0$.

The first order-conditions yield:

$$\begin{aligned} u_c - \phi\rho_c &= \xi + \lambda \\ \eta &= \xi + \lambda\Gamma \\ \dot{\xi} &= (\rho + \pi)\xi - \lambda \end{aligned}$$

[14] Capital depreciation is assumed to be zero in order to compare our results with those of Wang and Yip (1992) and Chen et al. (2008). Introducing capital depreciation still produces the same comparative statics results.

$$\dot{\eta} = \rho\eta - f_k\xi$$
$$\dot{\phi} = -u + \phi\rho, \qquad (13)$$

and the initial conditions and the transversality conditions: $\lim_{t\to\infty} \xi_t m_t \exp\{-\Delta_t\} = 0$, $\lim_{t\to\infty} \eta_t k_t \exp\{-\Delta_t\} = 0$ and $\lim_{t\to\infty} \Delta_t \phi_t \exp\{-\Delta_t\} = 0$. Notice that the last differential equation (13) with the last transversality conditon has (3) as a solution, leading to $\phi < 0$ when $u < 0$.

For a sufficient condition for the maximization problem, we should assume the Hamiltonian H is concave with respect to c, m, k, a, and Δ for any $\tilde{\lambda} \geq 0$, $\tilde{\xi} \geq 0$, $\tilde{\eta} \geq 0$, and $\tilde{\phi} \leq 0$, where $\tilde{\lambda} = \exp(-\Delta)\lambda$, $\tilde{\xi} = \exp(-\Delta)\xi$, $\tilde{\eta} = \exp(-\Delta)\eta$, and $\tilde{\phi} = \exp(-\Delta)\phi$. A sufficient condition for concavity is that $e^{-\Delta}u(c)$ is concave. Such concavity holds if $u < 0$, $u_{cc} < 0$, and $uu_{cc} - u_c^2 > 0$ for all c, excluding the possibility that $u > 0$. We assume the felicity function satisfies these conditions.[15]

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[15] An example satisfying the conditions is $u(c) = c^{1-\sigma}/(1-\sigma)$, $\sigma > 1$.

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