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Preemptive Surrenders and Self-Fulfilling Life Insurance Crises

Koichi Takeda*

Abstract

This paper examines the mechanisms behind the preemptive surrender of life insurance policies which resulted in the series of bankruptcies of life insurance companies during the "life insurance crisis" between 1997 and 2001 in Japan. In this paper, we construct coordination games between policyholders in a situation where a life insurance company is likely to go bankrupt if many policyholders surrender their policies. We derive a unique equilibrium in a situation where policyholders are only able to obtain inaccurate information on the financial status of the life insurance company.

We conduct a comparative statics study and results show that, with regard to the equilibrium, there is an increased possibility that a life insurance company may be forced into bankruptcy when policyholders surrender their policies, even when the company's financial status is good, in a situation where, one, policyholders become more risk averse, two, there is a decline in the level of accuracy of information on the financial status of the company, three, there is an increase in the refund that can be recovered with the surrender of a policy, and, four, there is a decrease in the liquidity of assets owned by the company.

Furthermore, when there exist different types of policyholders in cases such as the coexistence of policyholders who are uncertainty-averse and policyholders who are highly risk-averse, there is a greater possibility that the life insurance company will go bankrupt as a result of the preemptive surrender of policies by policyholders. In addition, with regard to the injection of capital such as public funds into a life insurance company, the increase of capital injection up to a specific level will have the desirable effect of preventing a bankruptcy caused by the surrender of policies. However, an increase in the amount of capital injected beyond a certain level will produce the undesirable effect of postponing the disposition of excessive debts held by the life insurance company.

1. Introduction

1.1 Problem

Nissan Mutual Life Insurance Company, a medium-sized life insurance company, went bankrupt in April 1997. It was the first time since the post-war period that a life insurer had gone bankrupt in Japan. Parties related to the bankruptcy had no choice but to try and learn how to handle such a bankruptcy. The bankruptcy of Nissan Life was but the first episode in an historic series of life insurance bankruptcies known as the "life insurance crisis". Because

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of the non-existence of a protection system for policyholders and of a life insurance bankruptcy contingency scheme, the Japanese insurance market faced a severe and unprecedented challenge with the successive bankruptcies of four life insurance companies within a short span of four years, starting from the bankruptcy of Toho Life in June 1999, Daihyaku Life in May 2000, Taisho Life in August 2000, Kyoei Life and Chiyoda Life in October 2000, and finally Tokyo Life in March 2001.[1]

After the enactment of the new insurance business act in 1996 which was amended in 1995, the regulatory framework shifted from the so-called ‘convoy system’ which assumed that life insurance companies as a whole could not be allowed to fail, to one that is centered on post-inspection of a company’s management, based on solvency margins and the protection of the policyholders of bankrupt life insurance companies which together act as a safety net. However, at the time of the bankruptcy of Nissan Life in 1997, the limits of compensation for policyholders were not clear and the disclosure of information by the life insurance companies was still insufficient, therefore it was not possible for outsiders to get a good grasp of their company’s actual financial status. This meant that the new regulatory system was still inadequate. As a result, many policyholders became aware that they needed to know about the risks attendant on the reduction in the value of their insurance as a result of the bankruptcy of their life insurance company.

This situation was gradually resolved through the development of a series of protection systems for policyholders, starting with the disclosure of the life insurance companies’ solvency margin rate in 1998 and the establishment of the Life Insurance Policyholders Protection Corporation of Japan in the same year, along with the concurrent handling of the bankruptcies of companies such as Nissan Life. However, in the process of developing the new system after the bankruptcy of Nissan Life, many life insurance policyholders were unable to ignore the risks of bankruptcy on the part of several companies who had been identified by the mass media. They therefore began to study measures such as the surrender of their policies in earnest.

The main reasons for the bankruptcies of the life insurance companies have been studied by many researchers.[2] In these studies, the main factors behind the bankruptcies were the high financial burden of the so-called gyakuzaya or ‘reverse spread’ problem, which stems from factors such as the dilemma in a worsening operating environment between low market interest rates and the high target yield rates that were promised during the sale of high-savings insurance products in mass quantities during the bubble economy period, the rapid decrease in the actual value of assets in the portfolio due to the burst of the asset price bubble, the increase in bad loans in a deteriorating economy, the failures and inadequacies in the risk management of assets and liabilities, as well as the outflow of capital due to sharp increases in the surrender of existing policies alongside the inability to raise the number of new policies. These factors were identified on the basis of evidence from the financial data of the bankrupt-

[1] After Kyoei Life in March 2001, bankruptcies in Japan came to a stop for a while until 7 years later in October 2008 when Daiwa Life went bankrupt. Daiwa Life took over the policyholders of Taisho Life, which went bankrupt in 2000. Hence policyholders from Taisho Life suffered a second reduction in the value of their insurance with the second bankruptcy.

[2] The reasons behind the bankruptcies during the life insurance crisis in Japan were studied in detail by Fukao and JCER (2000), Kofuji (2001), etc.

ed life insurance companies which were disclosed after their bankruptcies.[3]

1.2 Analysis perspective

Life insurance companies at risk of bankruptcy faced a chain of serious problems, starting with the above-mentioned problem of negative spread. They were pushed financially into a situation where they had to sell their assets at huge losses in order to generate necessary liquidity. Under these dire circumstances, if the policyholders, worried about a bankruptcy and seeing only uncertainty in the future, surrender their policies en masse, life insurance companies cannot generate enough funds from what limited liquidity they have, and this increases the likelihood of bankruptcy.

In a situation where voluntary coordination among policyholders is difficult, policyholders' actions to recover the surrender value ahead of other policyholders before the life insurance company goes bankrupt as a result of the surrender of policies by other policyholders will henceforth be known as preemptive surrender. This kind of rush to surrender policies by policyholders can lead to the fire sale of assets by companies so that they can continue to survive, which is probably not socially optimal. In the view of the life insurance policyholders, their individual decisions are perfectly rational, therefore it is likely that the vicious cycle of preemptive surrenders will not stop of its own accord until the life insurance company goes bankrupt.

To the best of our knowledge, however, no paper in the literature has conducted a theoretical analysis that extends to the economic mechanisms behind the rapid escalation of life policy surrenders that were fueled by concerns about bankruptcies. Despite its importance in the life insurance crisis, the relationship between the preemptive surrenders, on the one hand, and the character of the life insurance companies and their policyholders, on the other, has not been examined theoretically.

Hence, by focusing on the failure in coordination between the policyholders in this study, we shall analyze theoretically the economic mechanism behind the preemptive surrenders. The study provides a perspective that helps us understand the life insurance companies' bankruptcies, but this in no way denies the factors behind the bankruptcies as pointed out in the literature. Rather, by revealing the mechanism that led to the bankruptcies of the life insurance companies as caused by the rapid spike in policy surrenders due to coordination failure among the policyholders, as opposed to the factors pointed out thus far, this study aims to highlight new and overlooked economic motives that led to the bankruptcies, and to understand the interaction between the fundamental factors found in earlier studies and the preemptive surrenders that were carried out on the basis of beliefs that became self-fulfilling. Therefore, this study is an attempt to show that the above factors are important in furthering our understanding of the truth behind the life insurance companies' bankruptcies.

[3] According to 1999 information on insurance earnings disclosed by various Japanese life insurance companies, income from insurance premiums, etc. fell below the increasing expenses of the insurance premiums and the lump sum payouts upon the maturity of premiums, etc. for 9 out of the 14 main life insurers, more than half. The insurers with the worst three insurance income versus expenditure ratios were Chiyoda Life Insurance, Kyoei Life Insurance, and Tokyo Life Insurance, and they went bankrupt over a period of one year (Fukao and JCER (2000)).

1.3 Approach

In this study, to examine the problems of preemptive surrenders caused by the failure among policyholders to coordinate amongst themselves, we used the theoretical framework of global games, a useful way to analyze coordination problems. Global games, pioneered by Carlsson and van Damme (1993), and further extended by Morris and Shin (1998) and others, are games with incomplete information, whose type space is determined by the players, each of whom observes a private noisy signal of the underlying state of affairs. Applications of a global games framework to the analyses of financial markets include Morris and Shin's (2004) model of debt pricing and Goldstein and Pauzner's (2005) model of bank runs. To the best of our knowledge, this paper is the first attempt to apply the framework of global games to analyze coordination problems among policyholders.

The rest of the paper is organized as follows. Section 2 explains the model used in this paper. Section 3 derives the equilibrium and explains its characteristics. Section 4 extends the basic model in the previous section to address the policy issues facing life insurance companies and presents some policy implications to mitigate problems of coordination among policyholders. Section 5 concludes the paper.

2. Model

2.1 Setting

A life insurance company that starts its activities in period 0 continues to period 2. For simplicity, the life insurance company does not have its own funds but procures all its funds from outside policyholders. The consolidation of policyholders is represented in the continuum of measure 1. The individual policyholders are assumed to provide only a small and negligible share of the total funds procured by the company. For simplicity, all the insurance policies belong to the same type of policy that pays a fixed annuity, and the maturity value of the insurance policy is $A > 0$. If the project return v of the life insurance company realized in period 2 exceeds the payout stipulated in the insurance policy, then the policyholders receive a payout according to their initial policy.[4]

Policyholders have the right to surrender their insurance policies in period 1. We assume that different policyholders make independent decisions at the same time on whether they should surrender their policies or wait until the policies mature and that they are not able to communicate with other policyholders beforehand to form an agreement. The insurance policies are maintained through the actuarial liability in period 1 and policyholders who surrender their policies are assumed to receive the surrender value of $K^* \in (0, A)$ in period 1. In cases where policyholders do not surrender their policies and wait until their maturity, the policyholders receive the maturity value that corresponds to the return from the life insurance company's project. The project's return realized in period 2 is determined by the fundamental $\theta \in \mathrm{R}$, which is associated with the financial status of the life insurance company, and determined

[4] This paper serves to narrow the focus to the problem of the uncertainty that the insurance policy holder faces before the maturity of their policy, with the simplified assumption that the policy holder receives a lump sum benefit at maturity. In practice, a payout upon the maturity of a policy has a variety of patterns, but any problems that follow from this are out of our scope and will not be discussed in this paper.

randomly, and the size of the damage of the early liquidation of the life insurance company's project's assets when the policyholders surrender their policies.[5]

For simplicity, we assume that θ has a uniform prior distribution. The proportion of policyholders who surrender their policy is represented as λ, and the realization of the return from the life insurance company's project is given by

$$v(\theta,\lambda)=\begin{cases}V & \text{if} \quad \delta\lambda<\theta \\ K_* & \text{if} \quad \delta\lambda\geq\theta\end{cases} \tag{1}$$

where $V > A$ is a constant that represents the return when the project is successful while $K_* \in [0, K^*)$ is the constant that represents the return when the project is a failure. $\delta > 0$ is a parameter that represents the low liquidity of the project's operating assets. As δ becomes larger, so does the damage caused by the early liquidation of assets to deal with the problems of the surrender of policies for the project's likelihood of success. The payoff function of the equation Equation (1) shows that if the fundamentals associated with the life insurance company's financial status are good enough to cover the damage from the surrender of policies, the project succeeds, and the life insurance company is able to achieve a return that is enough to pay the maturity value as specified in the initial policies of the company. However, if the damage from the surrender of policies is unable to be covered, then the project fails, and the life insurance company succumbs to bankruptcy. This means that the life insurance company can only pay a maturity value that is less than the amount specified in the initial policy.

By normalizing the return of the policyholders so that $A = 1$ and $K_* = 0$, the return for policyholders during the surrender of policies is given by $0 < \kappa < 1$, where $\kappa \equiv (K^* - K_*)/(A - K_*)$. The return ϖ for policyholders is 1 if the project succeeds and the policyholders wait until their policies mature. However, it is 0 if the project fails. In the case where policyholders surrender their policies, regardless of the project's success or failure, it is κ.
Policyholders are risk-averse and maximize the expected value of the utility as determined according to the constant absolute risk aversion (CARA) type utility function

$$\mathrm{U}(\varpi)=-\frac{\exp(-\rho\varpi)}{\rho}, \tag{2}$$

where the degree of risk aversion is given by $\rho > 0$, $\rho \neq 1$.

For simplicity, in a case where the expected utility when policyholders wait until their policies mature is equal to the expected utility when they surrender their policies, then the policyholders are assumed to surrender their policies.

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[5] The damage from early liquidation here refers specifically to the case where life insurance companies who receive requests to surrender policies from many policyholders and sell off their liquid or highly liquid assets will still be unable to obtain the funds necessary to pay the surrender values. Hence they have no choice but to increase liquidity while incurring losses through a fire sale of low liquidity assets. Between 1997 and 2001 at the time of the life insurance crisis in Japan, it became impossible to maintain the investment yield of the assets on a par with the policies' high, expected interest rates (guaranteed yield). As the negative net worth between the actual investment yield and the annual guaranteed yields widened, and as the financial structure continued to worsen with no solution in sight to resolve the problem of policy surrenders, the actual prices of assets, such as owned shares and real estate, during the burst of the assets bubble took a sharp dip. Meanwhile, there was a problem with increasing the number of new policies; thus there was an increased need to ensure liquidity in order to pay the surrender value and insurance benefits. For example, in the case of the bankrupt Kyoei Life, it sold off some saleable good assets and was stuck with low liquidity assets that were difficult to sell without suffering huge losses. These details came to light following a careful examination of their assets after their bankruptcy (with regard to individual cases of bankrupted life insurance companies, including Kyoei Life, refer to Fukao and JCER (2000)).

2.2 Information

Although policyholders do not observe the realization of the financial status θ of the life insurance company until period 2, they receive private signals regarding it during period 1. A typical policyholder i receives the private signal

$$x_i = \theta + \sigma\varepsilon_i , \tag{3}$$

where $\sigma > 0$ is a noise scale factor and ε_i is an idiosyncratic random variable in accordance with a standard normal distribution with mean 0. ε_i is i.i.d. across policyholders. We assume that the distribution of ε_i is common knowledge. For simplicity, we assume $\kappa < \exp\left[-\frac{1}{2}\rho\sigma^2\right]$.[6]

Until observed at period 2, θ is not common knowledge among policyholders. Upon receiving a respective signal at period 1, a policyholder infers the value of θ and the distribution of signals received by the other policyholders, as well as their estimates of θ. Likewise, all other policyholders form their beliefs by relying on their own private information. The assumption of incomplete information is the key to deriving the unique equilibrium.

3. Equilibrium and comparative statics

Before we figure out the equilibrium of the model configured in the earlier section, we shall discuss cases of complete information, which are special cases that act as a benchmark on the evaluation of cases of incomplete information. After our discussion on these special cases, we shall derive the equilibria of cases of incomplete information and detail their character.

3.1 Complete information cases

Suppose that policyholders perfectly know the value of θ before deciding whether to surrender their policies in period 1. Let us examine the types of optimal strategies that they would adopt. In the case of $\theta > \delta$, it would be best for policyholders to wait until the maturity of their policies. This is because even if all other policyholders surrender their policies, the life insurance company's project will be successful. In the case of $\theta \leq 0$, it would be best for policyholders to surrender their policies. This is because even if all other policyholders wait until their policies mature, the life insurance company's project will fail.

In the case where θ is at $(0, \delta]$, policyholders face strategic uncertainties, and coordination problems arise among policyholders. If the proportion of policyholders who wait until their policies mature is sufficiently large, then the life insurance company's project will be successful, so that waiting until their policies mature is the best step for policyholders. Conversely, if the proportion of policyholders who wait until their policies mature is sufficiently small, the life insurance company's project will fail, so surrendering their policies is the best step for policyholders. In other words, the best choice of each policyholder depends on the actions of other policyholders and cannot be determined in advance. Such coordination

[6] The assumption of $\kappa < \exp\left[-\frac{1}{2}\rho\sigma^2\right]$ enables us to focus on an analysis of cases where it is not obvious beforehand that the life insurance company will go bankrupt, because of policyholders' extreme risk aversion or the extreme inaccuracy of the private signals observed by policyholders, regardless of the life insurance company's financial status, by eliminating cases where the model has only a degenerated bankruptcy equilibrium because all policyholders always surrender their policies.

problems among policyholders are similar to problems that arise among depositors in Diamond and Dybvig's (1983) bank runs model. As Diamond and Dybvig (1983) articulated, in a complete information game where the fundamentals are common knowledge, multiple equilibria exist. In our model, the Pareto-superior strategy to wait until the maturity of the policies and the Pareto-inferior strategy to surrender the policies are both pure strategy Nash equilibria.

3.2 Incomplete information cases

Here, we shall derive the unique equilibrium under incomplete information and characterize the comparative statics properties of the equilibrium.

3.2.1 Derivation of the equilibrium

In our model, the incomplete information case refers to the global game framework among a continuum of small players as studied by Morris and Shin (2004) and others. Here, we shall derive the Bayesian equilibrium when policyholders adopt a switching strategy in which they surrender their policies if the signals they observe fall below the critical value x^*, and they wait until their policies mature if the signals go beyond the critical value x^*. In the global game in this class, there is no other equilibrium except for the switching strategy. As Morris and Shin (2004) have shown, by the iterative elimination of strictly dominated strategies, the switching strategy can be shown to be the only equilibrium strategy.

The unique equilibrium is characterized by the critical value θ^*, below which the life insurance company's project fails and the threshold signal x^*, below which policyholders surrender their policies. We shall derive two equilibrium conditions to get the two threshold values below.

Given the fundamental is θ, according to Equation (3), the probability that policyholder i observes the signal below x^* is

$$\Pr(x_i \leq x^* \mid \theta) = \Phi\left(\frac{x^* - \theta}{\sigma}\right), \tag{4}$$

where $\Phi(\cdot)$ is the cumulative distribution function for the standard normal.

Policyholders surrender their policies when they observe a signal below x^*. Since the noise $\{\varepsilon_i\}$ is i.i.d., the proportion of policyholders who surrender their policies λ is equivalent to the probability in Equation (4). We know from Equation (1) that the condition under which the project fails is $\theta \leq \delta\lambda$, which stands in equality when $\theta = \theta^*$. Therefore, the first equilibrium condition, i.e. the critical mass condition, which needs to be satisfied by the critical state θ^*, below which the project fails, is given by

$$\theta^* = \delta\Phi\left(\frac{x^* - \theta^*}{\sigma}\right) \tag{5}$$

Next, we shall discuss the optimal switching strategy of policyholder i who receives signal x_i given θ^*. The conditional probability that the fundamental θ exceeds the critical level θ^*, and is sufficient for the project to succeed, is given by

$$\Pr(\theta > \theta^* \mid x_i) = \Phi\left(\frac{x_i - \theta^*}{\sigma}\right)$$

Likewise, the conditional probability that the fundamental does not exceed the critical level, and is sufficiently low for the project to succeed, is given by Pr $(\theta \leq \theta^* \mid x_i) = \Phi((\theta^* - x_i)/\sigma)$. Provided that the expected utility based on returns if policyholders wait until their policies mature does not exceed the expected utility based on returns if policyholders surrender their policies, policyholders do not wait until their policies mature and surrender their policies.

Since the expected utility for the policyholder who receives the critical signal x^* must be equal to the expected utility for the policyholder who surrenders, the second equilibrium condition, i.e. the optimal cut-off condition, which needs to be satisfied by the critical level x^*, below which the policyholder surrenders, is given by

$$\mathrm{E}\left[U\left(\Phi\left(\frac{x^* - \theta^*}{\sigma}\right)\right)\right] = \mathrm{E}[U(\kappa)],$$

which implies

$$\Phi\left(\frac{x^* - \theta^*}{\sigma}\right)\exp\left[-\frac{1}{2}\rho\sigma^2\right] = \kappa. \tag{6}$$

The equilibrium value of θ^* can be obtained from the pair of Equations (5) and (6). From Equation (6), we obtain

$$x^* = \theta^* + \sigma\Phi^{-1}\left(\kappa\exp\left[\frac{1}{2}\rho\sigma^2\right]\right).$$

By substituting this in Equation (5), we obtain $\theta^* = \delta\kappa\exp\left[\rho\sigma^2/2\right]$.

To sum up, under incomplete information, there is a unique equilibrium in which policyholders use the switching strategy with a critical signal x^*, below which they surrender their policies. The two Equations (7) jointly determine the switching point x^* and the critical state θ^*, below which the project fails.

$$\begin{cases} x^* = \delta\kappa\exp\left[\frac{1}{2}\rho\sigma^2\right] + \sigma\Phi^{-1}\left(\kappa\exp\left[\frac{1}{2}\rho\sigma^2\right]\right) \\ \theta^* = \delta\kappa\exp\left[\frac{1}{2}\rho\sigma^2\right] \end{cases} \tag{7}$$

3.2.2 Characteristics of the equilibrium

From Equation (7) we know that the critical signal x^* and the critical fundamental θ^* move closer to $\delta\kappa$ when the size of the private signal's noise gets closer to 0, in other words, when the information associated with the life insurance company's financial situation as observed by policyholders becomes more accurate. In addition, θ^* moves closer to $\delta\kappa$ when policyholders' degree of aversion to risk ρ moves closer to 0, in other words, when policyholders get closer to risk neutral. The socially optimal critical fundamental is $\theta^* = 0$.

When the fundamental θ is at $\left(0, \delta\kappa\exp\left[\rho\sigma^2/2\right]\right]$, the bankruptcy of the life insurance company is a socially inefficient equilibrium resulting from coordination failure among policyholders. The actions of policyholders who surrender their policies before the life insurance company actually becomes bankrupt are the result of individually rational decisions. However, the equilibrium of the bankruptcies of the life insurance company causes losses to society. This equilibrium is Pareto-inferior compared to the equilibrium when the project succeeds. Since $\delta > 0$ and $\kappa > 0$, we know from Equation (7) that θ^* does not tend to 0 even if σ moves close to 0. This means that even if the noise associated with fundamentals becomes smaller, the strategic uncertainties associated with the actions of other policyholders are not eradicated and the possibility of inefficient bankruptcies still exists. On the other hand, the continued survival of the life insurance company when the fundamental θ is at $\left(\delta\kappa\exp\left[\rho\sigma^{2}/2\right],\delta\right]$ is a socially efficient equilibrium resulting from successful coordination among policyholders. This equilibrium is Pareto-superior compared to the equilibrium when the project fails.

Next, we conduct a comparative statics study on θ^{*}. According to Equation (7), we obtain the following comparative statics results.

Firstly, regarding the relationship between the surrender value and the critical fundamental θ^{*}, the following inequality holds.

$$\frac{\partial\theta^{*}}{\partial\kappa}>0$$

In other words, an increase in the amount of the surrender value which policyholders can recover increases the range of the fundamentals where policyholders preemptively surrender and raises the probability of the project’s failing.

Secondly, with regard to the relationship between the degree of risk aversion of policyholders ρ and the critical fundamental θ^{*}, the following inequality holds.

$$\frac{\partial\theta^{*}}{\partial\rho}>0$$

In other words, an increase in the degree of the policyholders’ aversion to risk increases the range of the fundamentals where policyholders preemptively surrender and raises the probability of project failure.

Thirdly, with regard to the size of the noise of the private signal observed by policyholders σ, and the critical fundamental θ^{*}, the following inequality holds.

$$\frac{\partial\theta^{*}}{\partial\sigma}>0$$

In other words, an increase in the inaccuracy of the information about the insurance company’s financial situation as observed by policyholders increases the range of the fundamentals where policyholders preemptively surrender and raises the probability of the project’s failure.

Fourthly, with regard to the relationship between the illiquidity of the life insurance company’s assets δ and the critical fundamental θ^{*}, the following inequality holds.

$$\frac{\partial\theta^{*}}{\partial\delta}>0$$

In other words, a decrease in the liquidity of the life insurance company's assets increases the range of the fundamentals where policyholders preemptively surrender and raises the probability of project failure.

Finally, we examine the probability of the occurrence of preemptive surrenders. We shall use Π to denote the conditional probability of the occurrence of preemptive surrenders where the fundamental θ is in the boundary $(0, \delta]$. Thus, $\Pi = \Pr(\theta \le \theta^* \mid \theta \in (0, \delta])$ holds. Therefore, Π is given by

$$\Pi = \kappa \exp\left[\frac{1}{2}\rho\sigma^2\right]. \tag{8}$$

Π can be interpreted as the conditional probability of the occurrence of preemptive surrenders, or the conditional probability of the bankruptcy of the life insurance company, under the condition that the bankruptcy of the life insurance company is determined by the policyholders' actions. In Equation (8), once the size of the signal noise σ and the extent of the policyholders' aversion to risk ρ moves closer to 0, Π moves toward κ, but does not tend toward 0. In addition, $\partial\Pi/\partial\kappa > 0$, $\partial\Pi/\partial\rho > 0$, $\partial\Pi/\partial\sigma > 0$ are established from Equation (8). In other words, the increase in the surrender value κ that policyholders can recover when they surrender their policies, the increase in the degree of policyholders' aversion to risk ρ, and the increase in the size of the signal noise σ have the effect of increasing the conditional probability of the bankruptcy of the life insurance company as a result of preemptive surrenders.

4. Discussion

In this section, we extend the model in section 2, and examine cases in which policyholders are averse to uncertainties, cases where there exist policyholders with different risk aversion levels, and cases where the life insurance company receives an injection of capital.

4.1 Policyholders averse to uncertainties

As discussed in section 3.1, in our model, there exist strategic uncertainties among policyholders within the boundary $\theta \in (0, \delta)$.[7] Here, in order to examine the actions of policyholders who are averse to uncertainties, we shall discuss how the equilibrium turns out when we assume that policyholders maximize not the expected utility but the Choquet expected utility. We assume that policyholders maximize the Choquet expected utility $\mathrm{CE}\left(U(\varpi)\right)$ as defined below:[8]

$$\begin{aligned} \mathrm{CE}\left(U(\varpi)\right) &= \int U(\varpi)\,d\psi(\varpi) \\ &= \int_{-\infty}^{0} \left(\psi\left(\{\varpi \mid U(\varpi) \ge z\}\right) - 1\right)dz + \int_{0}^{+\infty} \psi\left(\{\varpi \mid U(\varpi) \ge z\}\right)dz \end{aligned}, \tag{9}$$

where the integral on the right-hand side is the Riemann integral, and ψ is the convex non-additive probability measure (i.e. capacity).

According to Bauer (2005), in an incomplete information game in a class of our model, when the player maximizes the Choquet expected utility, the equilibrium is uniquely determined, and this equilibrium is the lowest end point of all possible equilibria.[9] In our model, the

[7] The uncertainty here refers to uncertainties in a case where the realization as well as its probability distribution are unknown. To differentiate such an uncertainty from risks with known probability distribution, we shall call this Knightian uncertainty, named after Knight (1921).

[8] With regard to Choquet expected utility, refer to Gilboa (1987) and Schmeidler (1989).

[9] With regard to the derivation of a unique equilibrium in a case when players maximize the Choquet expected utility, refer to Bauer (2005).

critical signal x^*_{CE} that characterizes the equilibrium in a case where policyholders maximize the Choquet expected utility is given by

$$x^*_{CE} = \delta .$$

This shows that at the equilibrium where policyholders maximize the Choquet expected utility, in a case where there is the possibility that the life insurance company will go bankrupt due to policyholders' surrenders, the optimal action for all policyholders is to surrender their policies, and this indicates that preemptive surrenders will cause the bankruptcy of the life insurance company.

4.2 Policyholders with varying levels of aversion to risk

In the model in section 2, it is assumed that all policyholders have the same levels of aversion to risk. However, in reality, it is likely that the level of aversion to risk varies among policyholders. Therefore, we assume that there are two types of policyholders with different levels of aversion to risk. We assume that the proportion of type H policyholders with a high risk-aversion level ρ_H is $\mu \in (0, 1)$, and the proportion of type L policyholders with a low risk-aversion level $\rho_L < \rho_H$ is $1-\mu$.

The critical signals where high risk-averse policyholders surrender their policies are denoted by x^*_H, while the critical signals where low risk-averse policyholders surrender their policies are denoted by x^*_L. Given $\{x^*_H, x^*_L\}$, the critical mass condition, which needs to be satisfied by the critical fundamental θ^*_R, below which the project fails, is given by

$$\theta^*_R = \delta\left(\mu\Phi\left(\frac{x^*_H - \theta^*_R}{\sigma}\right) + (1-\mu)\Phi\left(\frac{x^*_L - \theta^*_R}{\sigma}\right)\right) \tag{10}$$

Given θ^*_R, the optimal cutoff conditions, which need to be satisfied by the critical levels $\{x^*_H, x^*_L\}$, below which the respective policyholders surrender their policies, is given by

$$\begin{cases} \Phi\left(\frac{x^*_H - \theta^*_R}{\sigma}\right)\exp\left[-\frac{1}{2}\rho_H\sigma^2\right] = \kappa \\ \Phi\left(\frac{x^*_L - \theta^*_R}{\sigma}\right)\exp\left[-\frac{1}{2}\rho_L\sigma^2\right] = \kappa \end{cases} . \tag{11}$$

Based on Equations (10) and (11), the set of critical values $\{x^*_H, x^*_L, \theta^*_R\}$ that characterizes the equilibrium with multiple levels of policyholders' aversion to risk is given as follows.

$$\begin{cases} x^*_H = \delta\kappa\left(\mu\exp\left[\frac{1}{2}\rho_H\sigma^2\right] + (1-\mu)\exp\left[\frac{1}{2}\rho_L\sigma^2\right]\right) + \sigma\Phi^{-1}\left(\kappa\exp\left[\frac{1}{2}\rho_H\sigma^2\right]\right) \\ x^*_L = \delta\kappa\left(\mu\exp\left[\frac{1}{2}\rho_H\sigma^2\right] + (1-\mu)\exp\left[\frac{1}{2}\rho_L\sigma^2\right]\right) + \sigma\Phi^{-1}\left(\kappa\exp\left[\frac{1}{2}\rho_L\sigma^2\right]\right) \\ \theta^*_R = \delta\kappa\left(\mu\exp\left[\frac{1}{2}\rho_H\sigma^2\right] + (1-\mu)\exp\left[\frac{1}{2}\rho_L\sigma^2\right]\right) \end{cases}$$

Thus, from $\rho_L < \rho_H$, we obtain

$$\frac{\partial\theta^*_R}{\partial\mu} > 0 .$$

In other words, the higher the proportion of high risk-averse policyholders, the higher the likelihood that the life insurance company will go bankrupt as a result of preemptive surrenders.

4.3 Capital injection into the life insurance company

The U.S. Treasury Department injected capital totaling 40 billion dollars into the leading U.S. insurance company AIG by acquiring preferred shares in November 2008. AIG had been unable to raise funds as a result of the impact of the recession and the unstable financial market, which were caused by the subprime mortgage crisis. On top of that, in May 2009, the U.S. Treasury Department, using the Financial Stability Act, approved the injection of public funds into six leading U.S. insurance companies. The direct injection of funds into the life insurance companies by the government has been legitimized as an urgent policy measure, when the large-scale bankruptcies of life insurance companies would have led to the collapse of the entire financial system.

To examine the effect of such an injection of capital into a life insurance company, it is assumed that the life insurance company receives public funds $\gamma > 0$ in period 0, and the life insurance company's project return, determined according to equation (eq:v) in the model in section 2, is revised to $v_C(\theta, \lambda, \gamma)$.

$$v_C(\theta, \lambda, \gamma) = \begin{cases} V & \text{if} \quad \delta\lambda < \theta + \gamma \\ K_* & \text{if} \quad \delta\lambda \geq \theta + \gamma \end{cases}$$

Let us denote by $\{x_C^*, \theta_C^*\}$ the set of critical values that characterize the equilibrium when there is capital injection.

Given x_C^*, the critical mass condition, which needs to be satisfied by the critical fundamental θ_C^*, below which the project fails, is given by

$$\theta_C^* + \gamma = \delta\Phi\left(\frac{x_C^* - \theta_C^* + \gamma}{\sigma}\right). \tag{12}$$

Given θ_C^*, the optimal cut-off conditions, which need to be satisfied by the critical level x_C^*, below which the policyholders surrender their policies, is given by

$$\Phi\left(\frac{x_C^* - \theta_C^* + \gamma}{\sigma}\right)\exp\left[-\frac{1}{2}\rho\sigma^2\right] = \kappa \tag{13}$$

By figuring out the set of equilibrium conditions (12) and (13), the equilibrium $\{x_C^*, \theta_C^*\}$ is given by

$$\begin{cases} x_C^* = \delta\kappa\exp\left[\frac{1}{2}\rho\sigma^2\right] - \gamma + \sigma\Phi^{-1}\left(\kappa\exp\left[\frac{1}{2}\rho\sigma^2\right]\right) \\ \theta_C^* = \delta\kappa\exp\left[\frac{1}{2}\rho\sigma^2\right] - \gamma \end{cases}$$

The conditional probability of the bankruptcy of a life insurance company with an injection of capital in conditions where the bankruptcy of a life insurance company is determined by the actions of policyholders by Π_C is given by

$$\Pi_C = \begin{cases} \kappa\exp\left[\frac{1}{2}\rho\sigma^2\right] - \frac{\gamma}{\delta} & \text{if} \quad 0 < \gamma < \delta\kappa\exp\left[\frac{1}{2}\rho\sigma^2\right] \\ 0 & \text{if} \quad \delta\kappa\exp\left[\frac{1}{2}\rho\sigma^2\right] \leq \gamma \end{cases} \tag{14}$$

From Equations (12) and (14), if $0 < \gamma < \delta\kappa \exp\left[\rho\sigma^2/2\right]$, we get

$$\frac{\partial\Pi_C}{\partial\gamma} < 0, \frac{\partial\theta_C^*}{\partial\gamma} < 0.$$

After the amount of capital injected is increased, the critical fundamental θ_C^* moves closer to 0, which is the optimum value for society. Π_C also moves closer to 0. In other words, an increase in the amount of capital injected has the desirable marginal effect of decreasing the likelihood of an inefficient bankruptcy. If $\gamma = \delta\kappa \exp\left[\rho\sigma^2/2\right]$, we get θ_C^* = and $\Pi_C = 0$. An inefficient bankruptcy is prevented and a socially optimal situation is attained. This suggests that a sufficient amount of capital injection before preemptive surrenders begin is effective in preventing the bankruptcy of a life insurance company.

However, in the case of $\gamma = \delta\kappa \exp\left[\rho\sigma^2/2\right]$, we obtain $\theta_C^* < 0$, $\partial\theta_C^*/\partial\gamma < 0$ and $\partial\Pi_C/\partial\gamma = 0$. The increase in the amount of capital injected depresses θ_C^* to a negative value. Meanwhile Π_C does not move at 0. In other words, an increase in the amount of capital injected that exceeds $\delta\kappa \exp\left[\rho\sigma^2/2\right]$ has the undesirable marginal effect of increasing the possibility of the continued survival of companies with excessive debts. On the other hand, the desirable marginal effect that lowers the possibility of the occurrence of the inefficient bankruptcy of a life insurance company is attenuated.

The results of the above analysis show that the use of public funds for an injection of capital into a life insurance company has the desirable marginal effect of decreasing the likelihood of the inefficient bankruptcy of a life insurance company, up to a certain level. However, increasing the amount of capital injected beyond a necessary level brings about an undesirable marginal effect, such as extending the life, through the use of public funds, of an inefficient life insurance company with excessive debts. Thus, in the case of the use of policies to rescue life insurance companies during a crisis through the injection by government of capital, the marginal effects can either be positive or negative, depending on the amount of capital involved. Therefore, when carefully considering whether or not to apply such a rescue policy, special attention needs to be paid to all of the intricacies involved.

5. Conclusion

In this paper, we have studied the mechanisms behind the preemptive surrenders of life insurance policies that resulted in the series of bankruptcies of life insurance companies during the “life insurance crisis” between 1997 and 2001 in Japan. We constructed coordination games between policyholders in a situation where a life insurance company is likely to go bankrupt if many policyholders surrender their policies. We derived a unique equilibrium in a situation where policyholders are only able to obtain inaccurate information on the financial status of the life insurance company.

We conducted a comparative statics study and the results showed that, with regard to the equilibrium, there is an increased possibility that the life insurance company may be forced into bankruptcy when policyholders surrender their policies even when their company’s financial status is good if, one, policyholders become more risk averse, two, there is a decline in the level of accuracy of information on the financial status of the life insurance company, three, there is an increase in the value of the refunds that can be recovered through the surrender of policies, and, four, there is a decrease in the liquidity of the assets owned by the life insurance company.

Furthermore, when there exist different types of policyholders, in cases such as the coexistence of policyholders who are uncertainty-averse and of policyholders who are highly risk-averse, there is a greater possibility that the life insurance company will go bankrupt as a result of the preemptive surrender of policies by policyholders. In addition, with regard to the injection of capital, such as public funds, into the life insurance company, an increase in the amount of capital injected up to a specific level will have the desirable effect of preventing a bankruptcy caused by the surrender of policies. However, an increase in the amount of capital injected beyond a certain level will produce the undesirable effect of postponing the disposition of excessive debts by the life insurance company.

To prevent the socially inefficient bankruptcy of economically viable life insurance companies as a result of the fatal loss of assets due to coordination failure amongst policyholders, life insurance companies have to disclose important information on their financial status to policyholders in a timely fashion and prevent unnecessary apprehension among their policyholders. At the same time, in order that the outflow of insurance policies from preventive surrenders does not easily lead to the fire sale of illiquid assets, it is important to manage the liquidity of assets and liabilities and, whilst keeping in mind the setting of the actuarial liability and the surrender values, design and implement a policy during normal times so that the company can withstand difficult situations, such as a life insurance crisis.

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