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NEGATIVE INCOME TAX, THE LABOR SUPPLY EFFECT, AND A DEPARTURE FROM MARGINAL COST PRICING

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We analyze social optimality of marginal cost pricing equilibria under simplifying assumptions of two goods and two classes, workers and capitalists. The possible loss of the public sector is financed by taxation on profits. The negative income tax scheme is superimposed on this. Leisure, as well as the two goods, are assumed to enter in the workers' utility function. In such a model, if a change in the level of production of the public sector has a non-negligible effect on the total supply of labor, a departure from the Bonnisseau-Cornet marginal cost pricing equilibrium can be socially optimal.

Recently, Bonnisseau and Cornet (1990) proved the existence of marginal cost pricing equilibria in a general framework. The marginal cost pricing equilibria in their paper are those which are optimal only in the sense of Pareto.

In this paper, we will construct a simplified two-good, two-class model of the Bonnisseau-Cornet many-good, many-consumer model, and consider social optimality of the marginal cost

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pricing equilibrium. Our model in this paper assumes redistribution of income between the two classes, the capitalists and the workers, as well as the taxation on the profits for the subsidy for the possible loss of the public sector.

It is assumed that the product of the public sector is of different nature from that of the so-called pure and impure public goods. This means that any indivisual consumer can be excluded from the consumption of the product of the public sector when he (or she) does not pay the cost of the quantity of the product he consumes. We will call the product of the public sector simply "Good 1."

In such a model, we will show that the marginal cost pricing equilibrium can be non-optimal, and that social optimality is attained by a different "quasi-" marginal cost pricing equilibrium. The crucial assumptions for the social non-optimality of the Bonnisseau-Cornet equilibrium and the optimality of the other quasi-marginal cost pricing equilibrium are (i) that there is the income-redistribution scheme, and (ii) that the public sector occupies a non-negligible share in the total demand for labor so that a change in the level of production of the public good has a non-negligible effect on the total labor supply determined at the labor market.

We will show that social optimality in our model with the income-redistribution requires Good 1 to be priced a little *lower* than its marginal cost when a change in the level of production of the Good 1 has a non-negligible *positive* effect on the total supply of labor through its effect on the market real wage-rate.

The quasi-marginal cost in our model will be defined as the sum of the Bonnisseau-Cornet marginal cost[1)] *and* a term which depends on, and is of the *opposite* sign to that of, the change in the total supply of labor caused by a change in the level of production of Good 1 in consideration through its effect on the market wage-rate.

Bonnisseau-Cornet (1990)[2] are concerned with the existence of marginal cost pricing equilibria which follow the marginal cost pricing 'rule' a rule which requires that the prices of commodities are to be set equal to the marginal costs.

A case of the model considered in this paper will provide an explicit account of social optimality of the marginal cost pricing which Wiseman (1957)[3] pointed out to have been missed.[4]

In Section 1, we will make basic assumptions. In Section 2, we will analyze subjective optimizations (or constrained utility-maximization) of the workers and the capitalists, and the 'representative consumer' will be defined for the analysis of social optimality of marginal cost pricing. In Section 3, we will clarify the general market-equilibrium conditions of the economy as a whole, and formulate the market adjustment process in which the general market equilibrium is attained.

The 'quasi-' marginal cost pricing will be defined in Section 5. There will be defined the 'consumption possibility frontier,' which illustrates the trade-off relationship between the technologically feasible quantities consumed of Good 1 and Good 2.

In Section 6, the market equilibrium demand function for Good 1 will be defined, and its monotonely decreasing property will be proved.

Finally in Section 7, the necessary condition for the socially optimal level of the price of Good 1 will be clarified, and the social optimality of the quasi-marginal cost pricing will be proved.

1. Assumptions

In our model, uncertainty is neglected. Commodities and factors of production are assumed to be perfectly divisible. We will assume that aggregation of all consumers is possible. Our analysis will use the Gorman condition concerning marginal propensities to consume. (See Gorman (1953).)

We assume that there are two commodities, Good 1 and Good 2 which are produced respectively by the public and the private sectors.

Along with Goods 1 and 2, leisure is also taken in consideration of the utility functions.

Two primary factors are assumed to exist, capital and labor, the capital being held constant in both the sectors since we consider only statical equilibria.

Good 1 is produced by the public sector which is supervised by the government, its production function being described as $x_1 = F_1(L_1, m)$, where L_1 denotes labor employed in the sector and m the input of Good 2 used in the public sector.[5] The production function for Good 2 is described as $x_2 = F_2(L_2)$, and Good 2 is produced by the private sector which operates perfectly competitively (in the sense that it is a price-taker at each market, as assumed in the neoclassical two-sector growth model in Uzawa (1961)).[6]

Good 1 is assumed to be a consumption good, but it does not of course comprise all consumption goods. The remaining consumption goods are included in Good 2 which is regarded as an aggregative concept which comprises all products produced in the private sector under perfect competition.

A part of Good 2 is assumed to be used as an input in the production of Good 1.[7]

For simplicity, Good 1 is assumed to be a pure consumption good. Correspondingly, Good 1 is assumed not to be demanded as an input by the private sector.

We assume that decreasing returns (or increasing cost) prevail in the private sector, so that we assume $F_2'(L_2) > 0$ and $F_2''(L_2) < 0$ for all $L_2 > 0$. Meanwhile, the public sector may be under increasing returns all over, or over some parts of, the domain of its production function.

We assume that the capitalists absorb all profits or losses, and that the workers earn the total wage.[8] The capitalists absorb a loss of the public sector in the sense that they transfer a part of their profit income which is earned at the private sector to the public sector when it incurred a loss. The transfer is assumed to be implemented by the taxation by the government on the capitalists' profits, with which the government subsidizes the public sector.[9]

The workers' utility function is denoted by $U_w(x_{1w}, x^c_{2w}, l_w)$, where x_{1w}, x^c_{2w}, and l_w denote the workers' consumption of Good 1, Good 2, and their demand for leisure.

Similarly, the capitalists' utility function is denoted by $U_c(x_{1c}, x^c_{2c}, l_c)$ where x^c_1, x^c_{2c}, l_c denote the consumption of Good 1 and Good 2, and their leisure.

The utility functions are assumed to be quasi-concave, twice-differentiable, and to fulfil other ordinary assumptions.

We assume that the workers and the capitalists maximize their respective utility subject to their income and the market prices p_1 and p_2 (p_1 is written simply as p and p_2 is fixed to equal unity as the price of the numeraire) and the market wage-rate w.

The markets of labor and Good 2 are assumed to be perfectly competitive in the sense that both the demanders and suppliers at these markets are price takers. Walrasian auctioneers are assumed to function at the markets.

Finally, the production function of the first sector $F_1(L_1, m)$ is assumed to be quasi-concave, and the production set of the first sector A ($\subset R^3$) is assumed to be closed and to have the property of free disposal with respect to the input vector (L_1, m). The production functions, $F_1(L_1, m)$ and $F_2(L_2)$ are assumed twice continuously differentiable.

Given each level of output x_1, the first sector is assumed to minimize cost of producing it, subject to the input price vector $(w, p_2)=(w, 1)$, where w denotes the market wage-rate in terms of Good 2 and $p_2= 1$ the fixed unitary price of Good 2 as another input for the production of Good 1.

We denote by $L_1(x_1; w)$ and $m(x_1; w)$ the elements of the cost-minimizing input vectors (L_1, m) at the given $(x_1; w)$. It is assumed that these are uniquely determined for any $(x_1; w) > 0$, that the functions L_1 and m are differentiable with respect both to x_1 and w and that

(1-1) $\partial L_1(x_1; w)/\partial x_1 \geqq 0$ and $\partial m(x_1; w)/\partial x_1 \geqq 0$

for all $(x_1; w) > 0$.[10]

We assume that the adjustment for the equilibrium wage-rate is instantaneously completed. The auctioneer at the labor market equilibrates by the tatonnement wage-rate adjustment process at infinite speed, so that we assume that the labor market is always in equilibrium for any given level of production of Good 1. The market equilibrium level of production of Good 2 is determined consequently by the market wage-rate.

Provided that the labor market is always in equilibrium, the Walras law implies that the market of Good 1 is equilibrated simultaneously with that of Good 2.

The demand and supply of Good 1 is assumed to be equilibrated in one of the following two alternative market adjustment processes. (1) The Walrasian price adjustment: At each level of production and each level of the price of Good 1, if the demand exceeds (falls short of) the supply of Good 1, the price is raised (lowered) by the auctioneer at the market of Good 1;(2) The Keynesian quantity adjustment: At each level of production and at each level of the price of Good 1, if the demand exceeds (falls short of) the supply of Good 1, the quantity produced of Good 1 is increased (decreased).

We assume that both of these alternative market adjustment processes are tatonnement processes, or market adjustment processes before actual transactions are made (Arrow and Hahn (1971)).[11]

The workers as a whole and the capitalists as a whole are assumed to own fixed initial endowments of available time, H_j, $j = w, c$. It is assumed that the workers may sell a part, say L, of H_w as labor at the labor market. For simplicity, the capitalists are assumed not to sell any part of H_c.

We also assume that the capitalists do not supply any part of their available time at the labor market, and that they do not directly demand leisure for their own consumption (services such as barbers are classified into the private good or Good 2).[12)]

We denote by Π_1 and Π_2 the profits from the two sectors. It is assumed that the profits from the two sectors, or Π_1 and Π_2, entirely accrue to the capitalists.

Π_1 may be negative, and if it is so, the absolute value of Π_1 equals the tax on the profit from the second sector levied by the government which subsidizes the first sector with the same amount (in short, the transfer from the capitalists to the first sector). If Π_1 is zero or positive, it signifies the ordinary profit distributed from the first sector to the capitalists.

It is convenient for our analysis below to define the income before the payment of Π_1 between the first sector and the capitalists, which we will call the capitalists' basic income, denoted by I_c.

$$(1\text{-}2) \qquad I_c = \Pi_2$$

It is assumed that I_c is always as large as sufficient to cover the loss of the first sector.

Now, we assume that the utility functions of the two classes are of the forms such that aggregation is possible in the Gorman sense: or such that there can be a representative consumer *of the goods* (excluding leisure) which is consistent with the utility functions of the two classes. This means that we assume

$$(1\text{-}3) \qquad U_j = x_{1j}^{b_{j1}} (x_{2j}^c)^{b_{j2}} l_j^{b_{j3}}$$

for $j = w, c$, where $b_{jk} > 0$ are constants, $j = w, c$, $k = 1, 2, 3$, $b_{j1} +$

$b_{j2} + b_{j3} = 1$, and

$$(1\text{-}4) \qquad b_{w1}/b_{w2} = b_{c1}/b_{c2}$$

The income-redistribution we formulate is based on the idea of the positive and negative income taxes. We consider income-redistribution between the aggregate capitalists and the aggregate workers, so that we do not consider any income-redistribution among *intra*-class groups of different income levels.

The income-redistribution is based on the idea of social welfare concerning the utility level of the aggregate workers, U_w, so that the positive and negative income tax scheme is formulated on the welfare consideration that the tax on the wage-income has to be such that it stabilizes the level of the workers' real income Y_w which is a cardinal indicator of their level of utility.

If there were not for the income-redistribution, the workers' utility-maximization problem can be written

$$(1\text{-}5) \qquad \text{Max.}\ U_w = x_{1w}^{b_{w1}} (x_{2w}^{c})^{b_{w2}} l_w^{b_{w3}}$$

$$\text{sub. to} \qquad px_{1w} + x_{2w}^{c} + wl_w \leqq Y_w = wH_w$$

so that the maximum attainable level of utility of the workers corresponds to the level of their income Y_w.

As w changes, the workers' utility U_w also changes in the same direction. For the welfare purpose, the government tries to stabilize the change in U_w by the negative income tax.

Though we do not consider a spectrum of various groups of workers with various levels of income, we here adopt the idea of negative income tax as follows. If the workers' total wage Y_w is greater (smaller) than a certain aggregate 'break-even' level, B (a constant), then the government taxes the workers (the capitalists) and transfers thus levied tax revenue entirely to the capitalists (the workers).

In the case where the capitalists transfer income to the workers (or $Y_w < B$), the workers are regarded to be poor enough to be provided welfare benefit of various forms from the capitalists through the government's institutions of income transfer.

The aggregate break-even wage-bill B is assumed to be determined exogenously from welfare consideration, and so to be a constant.

We assume that the aggregate break-even wage-income is so high that the income transfer is ordinarily made from the capitalists to the workers. The transfer is made in the opposite direction only in the case of an exceptionally high real wage-rate.[13]

We formulate the positive and negative wage-income tax scheme as follows.

$$(1-6) \qquad T = g(B - Y_w) = T(w),$$

$$(1-7) \qquad n = Y_w + T(w) = n(w) > 0 \text{ and } n'(w) \geqq 0 \text{ for all } w > 0,$$

where g is a constant, $g > 0$, T denotes the transfer from the capitalists to the workers, and n the workers' income after the transfer.

By (1−6) and (1−7), it follows that $0 < g \leqq 1$, and $n(w)$ is of the form

$$(1-8) \qquad n(w) = n^0 + vY_w$$

where $n^0 = gB$, $v = 1 - g$ and $0 \leqq v < 1$.[14]

Correspondingly, the capitalists' income after this income transfer equals

$$(1-9) \qquad y_c = Y_c - T(w)$$

The workers' utility-maximization problem in the presence of the income-redistribution scheme is written

$$(1-10) \qquad \text{[Problem 1]} \qquad \text{Max. } U_w$$

$$\text{sub. to} \qquad px_{1w} + x_{2w}^c + wl_w \leqq n(w)$$

2. Subjective Optimizations and the Representative Consumer

The necessary conditions for the solution of Problem 1 are written

$$(2\text{-}1) \qquad b_{u1} x_{1u}^{b_{u1}-1} (x_{2u}^c)^{b_{u2}} l_u^{b_{u3}} - qp = 0$$

$$(2\text{-}2) \qquad b_{u2} x_{1u}^{b_{u1}} (x_{2u}^c)^{b_{u2}-1} l_u^{b_{u3}} - q = 0$$

$$(2\text{-}3) \qquad b_{u3} x_{1u}^{b_{u1}} (x_{2u}^c)^{b_{u2}} l_u^{b_{u3}-1} - qw = 0$$

where q denotes the Lagrange multiplier for the budget constraint. Multiplying (2–1), (2–2), and (2–3) respectively by x_{1u}, x_{2u}^c, and l_u, we have

$$(2\text{-}4) \qquad b_{u1} x_{1u}^{b_{u1}} (x_{2u}^c)^{b_{u2}} l_u^{b_{u3}} - qpx_{1u} = 0$$

$$(2\text{-}5) \qquad b_{u2} x_{1u}^{b_{u1}} (x_{2u}^c)^{b_{u2}} l_u^{b_{u3}} - qx_{2u}^c = 0$$

$$(2\text{-}6) \qquad b_{u3} x_{1u}^{b_{u1}} (x_{2u}^c)^{b_{u2}} l_u^{b_{u3}} - qwl_u = 0$$

Summing (2–4), (2–5), and (2–6) through, we have

$$(2\text{-}7) \qquad x_{1u}^{b_{u1}} (x_{2u}^c)^{b_{u2}} l_u^{b_{u3}} - qn(w) = 0$$

owing to the linear homogeniety of U_u.

Hence,

$$(2\text{-}8) \qquad q = x_{1u}^{b_{u1}} (x_{2u}^c)^{b_{u2}} l_u^{b_{u3}} / n(w)$$

By (2–4), (2–5), (2–6), and (2–8), we have

$$(2\text{-}9) \qquad x_{1u}^* = b_{u1} n(w)/p$$

$$(2\text{-}10) \qquad x_{2u}^{c*} = b_{u2} n(w)$$

$$(2\text{-}11) \qquad l_u^* = b_{u3} n(w)/w$$

where x_{1u}^*, x_{2u}^{c*}, and l_u^* denote the workers' demand for Goods 1 and 2 and that for leisure. The sufficient condition for maximization is ensured if we assume that p and w are positive.

By (1–7) and (2–11) we have

(2-12) $$l_u^* = b_{u3}n(w)/w = b_{u3}(n^0+vY_u)/w = b_{u3}((n^0/w)+vH_u)$$

It follows that the workers' optimal total supply of labor, $L = H_u - l_u^* = (1 - vb_{u3})\ H_u - (b_{u3}n^0/w)$, so that we have

(2-13) $$L'(w) > 0$$

Now we define the following 'sub' problem for the workers.

(2-14) [Problem 2] $\text{Max.}\ x_{1u}^{d_1}(x_{2u}^c)^{d_2}$

sub. to $px_{1u} + x_{2u}^c \leqq y_u$

where $d_i = b_{ui}/(b_{u1}+b_{u2})$, $i = 1, 2$, and

(2-15) $$y_u = wL(w) + T(w)$$

where y_u denotes the workers' real disposable income after buying back leisure in terms of Good 1.

The necessary conditions for the solution of Problem 2 are

(2-16) $$d_1 x_{1u}^{d_1-1}(x_{2u}^c)^{d_2} - rp = 0$$

(2-17) $$d_1 x_{1u}^{d_1}(x_{2u}^c)^{d_2-1} - r = 0$$

where r is the Lagrange multiplier for the budget constraint for the goods.

By the similar calculation used in deriving the three demand functions, (2-9), (2-10), and (2-11) from the three conditions (2-4), (2-5), and (2-6), we have

(2-18) $$x_{1u}^{**} = d_1 y_u/p$$

(2-19) $$x_{2u}^{c**} = d_2 y_u$$

where x_{1u}^{**} and x_{2u}^{c**} denote the quantities demanded by the workers of Good 1 and Good 2 when the workers are confronted with Problem 2 in (2-14).

By the definitions of $n(w)$ and y_u in (1-7) and (2-15), we have

$$(2\text{-}20) \qquad y_w = n(w) - wl_w^*$$

By the workers' demand function of leisure in (2–12), (2–10) implies

$$(2\text{-}21) \qquad y_w = (n^0 + vwH_w) - b_{w3}(n^0 + vwH_w)$$
$$= (b_{w1} + b_{w2})(n^0 + vwH_w)$$

so that, by $d_i = b_{wi}/(b_{w1} + b_{w2})$, $i = 1, 2$, we have

$$(2\text{-}22) \qquad d_i y_w = b_{wi} n(w), \; i = 1, 2$$

It follows that the workers' demands for Goods 1 and 2 in (2–18) and (2–19) when confronted with Problem 2 are verified to equal the workers' demand for the goods in (2–9) and (2–10) when they are confronted with the initial Problem 1 in (1–10). That is,

$$(2\text{-}23) \qquad x_{1w}^{**} = x_{1w}^{*} = b_{w1} n(w)/p$$
$$(2\text{-}24) \qquad x_{2w}^{c**} = x_{2w}^{c*} = b_{w2} n(w)$$

The capitalists' utility-maximization problem is written[15]

$$(2\text{-}25) \qquad [\text{Problem 3}] \qquad \text{Max. } U_c = (x_{1c})^{b_{c1}} (x_{2c}^c)^{b_{c2}} H_c^{b_{c3}}$$
$$\text{sub. to} \qquad px_{1c} + x_{2c} \leq y_c$$

where

$$(2\text{-}26) \qquad y_c = (H_1 + H_2) - T(w)$$

By (1–4) and (2–14), we have

$$(2\text{-}27) \qquad d_1/d_2 = b_{c1}/b_{c2} \,,$$

it is easy to verify that the marginal propensities to consume Good 1 and Good 2 *from* y_w and y_c, are constants and the same between the workers (as confronted with Problem 2) and the capitalists (as confronted with Problem 3). By Gorman (1953), the utility functions of the workers for the subproblem (or Problem 2) and the capitalists for Problem 3 can be aggregated into one 'community' utility function for the goods, and the total of the workers' and the capitalists' demand for Goods 1 and 2 (which are respectively derived from the classwise utility maximizations in Problems 2 and 3) can be regarded as the representative consumer's demand functions for the goods which are generated from the maximization of the aggregated utility function subject to the price vector $(p, 1)$ and the total income which equals the sum of the classwise incomes, y_w and y_c, which is written by (2–15) and (2–26),

$$(2\text{–}28) \qquad y_w + y_c = wL(w) + (\Pi_1 + \Pi_2)$$

We will denote the utility function of the representative consumer for the goods by $U(x_1, x_2^c)$, where $x_1 = x_{1w} + x_{1c}$ and $x_2^c = x_{2w}^c + x_{2c}^c$.

Then, U has all the regular properties for the well-behaved utility function.

We denote by y the total income of the representative consumer. Then, by $y = y_w + y_c$ and (2–28), we have

$$(2\text{–}29) \qquad y = wL(w) + (\Pi_1 + \Pi_2)$$

Suppose the workers and the capitalists respectively are confronted with Problem 1 in (1–10) and Problem 3 in (2–25). Then, by (2–23) and (2–24), the resulting demand functions of the workers for the goods always coincide with those generated when confronted with Problem 2 in (2–14).

Of course, it follows that the demand functions for the goods of the workers and the capitalists confronted with Problems 1 and 3 equal those generated when confronted with Problems 2 and 3.

It is convenient for our analysis below to define the income of the representative consumer for the goods *before* the payment of Π_1 between the first sector and the representative consumer, which we will call the basic income, denoted by I.

$$(2\text{–}30) \qquad I = wL(w) + I_c = wL_1 + (wL_2 + \Pi_2) = y - \Pi_1$$

In the case where the first sector incurs a loss, I denotes the income before tax of the representative consumer, since we can then write $I = y + |H_1|$, where $|H_1|$ represents the amount of the tax which the consumer pays to the government. In this deficit case, therefore, y denotes the disposable income all of which the representataive consumer can spend for consumption. This is also the case when the first sector earns non-negative profit.

3. The General Market Equilibrium Conditions

We denote by C^* the total cost of production at the first sector. (See footnote 5 for further detail.)

$$(3\text{-}1) \qquad C^* = wL_1 + m$$

where m is the sum of the material cost, G, the fixed and variable deprecation, $D_f + D_v$ (footnote 5).

Hence, we have

$$(3\text{-}2) \qquad H_1 = px_1 - C^* = px_1 - (wL_1 + m)$$

The total cost of production of the second sector is defined to equal wL_2, since x_2 is defined to represent the net output of the second sector after subtracting the depreciation from the gross total production of Good 2. (See footnote 6.)

Hence, we have

$$(3\text{-}3) \qquad H_2 = x_2 - wL_2.$$

By (2–29), (3–2), and (3–3), we have

$$(3\text{-}4) \qquad \begin{aligned} y &= (wL_1 + H_1) + (wL_2 + H_2) \\ &= (px_1 - m) + x_2 \\ &= px_1 + (x_2 - m) = px_1 + x_2^c \end{aligned}$$

where x_2^c denotes the quantity consumed of Good 2.

By (2–30) and (3–2), we also have

(3–5) $y = I + (px_1 - C^*)$

The equilibrium condition for the labor market is written

(3–6) $L_1(x_1; w) + F_2'^{-1}(w) = L(w)$

where $L_1(x_1; w)$ is the demand function for labor of the first sector, and $F_2'^{-1}(\cdot)$ denotes the inverse function of the marginal product function of labor at the second sector, $F'_2(L_2)$. So $F_2'^{-1}(w)$ is the demand function for labor of the second sector. L denotes the total supply of labor.[16]

Given each level of x_1, the market equilibrium wage-rate is uniquely determined as a function of the parameter x_1, which we denote by $w(x_1)$.

Since $\partial L_1(x_1; w)/\partial x_1 > 0$ by assumption, the decreasing property of both the functions L_1 and $F_2'^{-1}$ with respect to w implies

(3–7) $w = w(x_1)$

(3–8) $w'(x_1) > 0$

The total cost of the first sector can be written

(3–9) $C^* = wL_1 + m$
$= wL_1(x_1; w) + m(x_1; w)$

Substituting (3–7) into (3–9), C^* is seen to be a function of the level of production of Good 1, or x_1. We denote this function by $C^*(x_1)$.

By (2–30) and (3–3), the basic income can be written

(3–10) $I = wL_1 + x_2$
$= wL_1(x_1; w) + F_2(L_2)$

Noting that $L_2 = F_2'^{-1}(w)$, we can further write

(3–11) $I = wL_1(x_1; w) + F_2(F_2'^{-1}(w))$

Substituting $w = w(x_1)$, we can regard I as a function of x_1, which we denote by $I(x_1)$.

By (3–5), we have

$$(3\text{–}12) \qquad y = I(x_1) + (px_1 - C^*(x_1)) = y(x_1, p)$$

Given the level of production of Good 1 (x_1) and the price of Good 1 (p), the consumer's demand for Good 1 is equal to $h_1(p, y) = h_1(p, y(x_1, p))$. The supply of Good 1 equals x_1. Hence, the market equilibrium condition for Good 1 is written

$$(3\text{–}13) \qquad h_1(p, y(x_1, p)) = x_1.$$

4. The Market Adjustment Process

There can be two alternative formulations for the market adjustment process for Good 1. One is the adjustment by the price and the other by the level of production. In this section, the former will be considered.

The latter market adjustment process will be considered in the Appendix.

In the former type of market adjustment, we have to assume that the government specifies an appropriate level of x_1 within the range of all levels x_1 such that there is a market equilibrium solution for p corresponding to each level of x_1. For various levels of x_1, the market equilibrium price is determined as the function of x_1, which we denote by $p(x_1)$.

In this case we consider a stability in the sense that any displacement of the price from the market equilibrium level is reduced to zero by the market forces of the excess demand and supply.

Indeed, under the assumption that x_1 is fixed at any level such that there is some p fulfilling (3–13), it may not be unnatural for us to consider the following Walrasian price adjustment process.

(4-1) $\dot{p} = b(h_1(p, y(x_1, p)) - x_1)$

where b is a positive constant. The stability of this adjustment process depends on the sign of the total derivative of h_1 with respect to p, which is written

(4-2) $(\partial h_1/\partial p)_{x_1:\text{const.}} = (\partial h_1/\partial p)_{y:\text{const.}} + (\partial h_1/\partial y)_{p:\text{const.}} x_1$

By the Slutsky equation

(4-3) $(\partial h_1/\partial p)_{y:\text{const}} = (\partial h_1/\partial p)_{U:\text{const.}} + (\partial h_1/\partial y)_{p:\text{const.}} x_1$

we have

(4-4) $(\partial h_1/\partial p)_{x_1:\text{const.}} = (\partial h_1/\partial p)_{U:\text{const}}$

where the right-hand side represents the slope of the compensated demand function for Good 1.

Eq. (4-4) can be explained as follows: As p changes by dp, the disposable income of the consumer changes by $(dp)x_1$ because, in the case of deficit, if p rises, the deficit of the first sector is alleviated by $(dp)x_1$, so that the tax which the consumer is levied by the government to subsidize the sector is reduced by $(dp)x_1$. This increases the disposable income y by this amount. In the case of non-negative profit, the profit increases by $(dp)x_1$, by which the disposable income is also increased.

As p changes, therefore, the consumer is always just compensated so as to keep the real purchasing power of his disposable income after the price change, in the sense that he could at least continue to purchase the same consumption vector which he demanded at the price before the change.

Since the compensated own price effect is always negative, it follows that the above price adjustment process is always globally stable.

Furthermore, since we took such x_1 that there exists some $p \geqq 0$ which satisfies the market equilibrium condition (3-13), we can rewrite the same adjustment process as

$$(4\text{-}5) \qquad \dot{p} = b(h_1^*(p,\ U^*(x_1)) - x_1),$$

where $h_1^*(p,\ U^*(x_1))$ represents the compensated demand function for the utility level $U^*(x_1) = U(x_1,\ x_2^c)$ which is attained at the particular market equilibrium consumption vector $(x_1,\ x_2^c)$ corresponding to the given x_1.

$h_1^*(p,\ U^*(x_1))$ can be also illustrated by the x_1-coordinate of such a point on the indifference curve corresponding to the particular level of utility $U^*(x_1)$, that the absolute value of the tangency of the indifference curve equals to the argument p of the function $h_1^*(p,\ U^*(x_1))$ (Fig. 1).

The above defined function $p(x_1)$, which represents the market equilibrium price for each fixed x_1, is depicted as the tangency of the particular indifference curve which intersects (or possibly touches) the consumption possibility frontier at the consumption vector $(x_1,\ x_2^c)$ whose x_1-coordinate equals the given level of x_1

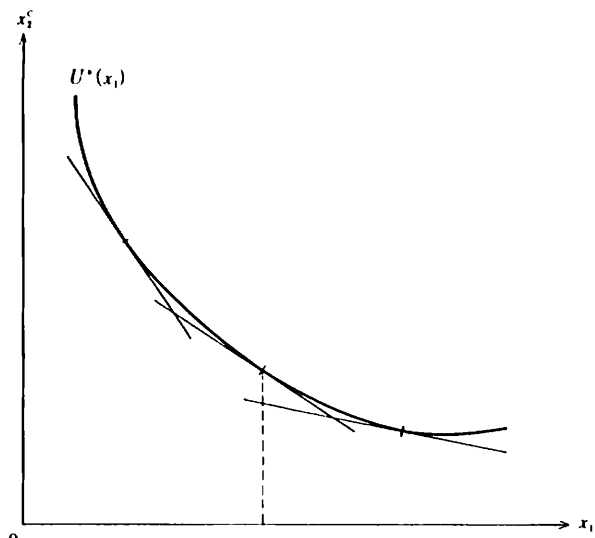

Fig.1

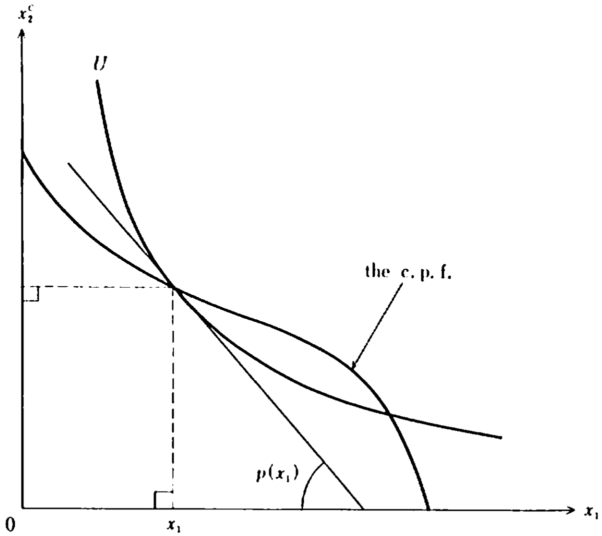

Fig.2

(Fig. 2).

5. The Market Equilibrium Demand Function for Good 1

In this section, we will prove that the function $p(x_1)$ has its inverse function, which we denote by $x_1(p)$, and that the inverse function $x_1(p)$ is monotonely decreasing.

We call this function $x_1(p)$ the "market equilibrium demand function for Good 1."

We will now prove the following

PROPOSITION 1: There exists the inverse function $x_1(p)$ which is monotonely decreasing over its domain.

Proof. By defining $f(x_1)$ by $f(x_1) = C^*(x_1) - I(x_1)$, (3-12) implies $y(x_1, p) = px_1 - f(x_1)$. Then the market equilibrium condition (3-13) can be rewritten

(5–1) $\qquad h_1(p, px_1 - f(x_1)) = x_1.$

Substituting $p = p(x_1)$, we have the identity

(5–2) $\qquad h_1(p(x_1), p(x_1)x_1 - f(x_1)) = x_1,$

over the domain of $p(x_1)$.

We take some x_1^0 in the domain and denote $p^0 = p(x_1^0)$. Then, by (5–2), we have

(5–3) $\qquad h_1(p^0, p^0x_1^0 - f(x_1^0)) = x_1^0$

Take some x_1^1 such that $p(x_1^1) > p^0$ and $p(x_1^1)$ belongs to the range of $p(x_1)$. We denote $p(x_1^1)$ by p^1. By Fig. 3, we will show that

(5–4) $\qquad h_1(p_1, p^1x_1^0 - f(x_1^0)) < x_1^0$

In Fig. 3, the point P_0 depicts the market equilibrium points for $x_1 = x_1^0$, and P'_0 the point of the consumer's optimal choice for

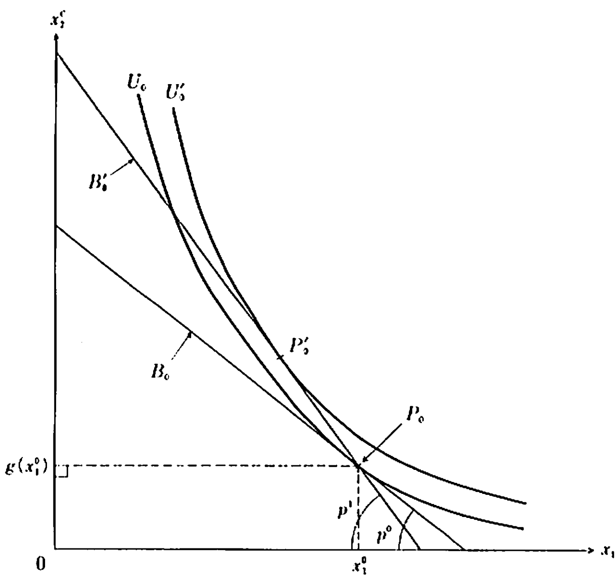

Fig.3

the price-income combination $(p^1, p^1x_1^0 - f(x_1^0))$, at which the budget lines B_0 and B_0' touch the indifference curves U_0 and U_0', respectively.

If we denote $P_0' = (x_1', x_2')$, the definition of P_0' implies that its horizontal coordinate x_1' equals $h_1(p^1, p^1x_1^0 - f(x_1^0))$, so that

$$(5\text{–}5) \qquad P_0' = (x_1', x_2') = (h_1(p^1, p^1x_1^0 - f(x_1^0)), x_2')$$

The slopes of the budget lines B_0 and B_0' equal $-p^0$ and $-p^1$ and their vertical intersepts equal $p^0x_1^0 - f(x_1^0)$ and $p^1x_1^0 - f(x_1^0)$, respectively.

Since the horizontal coordinate of the point P_0 equals x_1^0 and since the budget line B_0 goes through it, the vertical coordinate of P_0 is verified to equal $-f(x_1^0) = g(x_1^0)$, which implies that P_0 is on the c.p.f.[17] Hence,

$$(5\text{–}6) \qquad P_0 = (x_1^0, g(x_1^0))$$

Since the slope and the vertical intercept of the budget line B_0' equal $-p^1$ and $p^1x_1^0 - f(x_1^0)$, respectively, (5–6) implies that this budget line also goes through the point P_0.

It follows that the positions of the two budget lines are such that the budget line B_0' lies above (below) B_0 for all x_1 such that $x_1 < (>, \text{resp.})\ x_1^0$. This implies that the point P_0' lies to the northwest of P_0, so that $x_1' < x_1^0$. Hence, by (5–5), we have (5–4).

For the fixed p^1, the excess demand

$$(5\text{–}7) \qquad h_1(p^1, p^1x_1 - f(x_1)) - x_1$$

is a function of x_1, which we denote by $E(x_1)$. Then, by Proposition 2 in the Appendix which ensures the fulfillment of the stability condition for the alternative Keynesian market adjustment process (A–1), we have

$$(5\text{–}8) \qquad E'(x_1) < 0$$

for all $x_1 > 0$ belonging to a relevant range for x_1.

It follows that, as x_1 continuously *falls* from x_1^0 to some level, say x_1'', of x_1, $E(x_1)$ increases from negative up to zero, when $E(x_1'') = 0$, or

$$(5\text{-}9) \qquad h_1(p^1,\ p^1x_1'' - f(x_1'')) - x_1'' = 0$$

Since this means that p^1 equals the market equilibrium level of p corresponding to x_1'' we must have $p^1 = p(x_1'') = p(x_1^1)$. This implies that $x_1^1 = x_1'' < x_1^0$, or $x_1^1 < x_1^0$. This proves that $x_1(p)$ exists and is a decreasing function of p. Q.E.D.

By Fig. 3 in the above proof of Proposition 1, it will be seen that, for any p belonging to the domain of $x_1(p)$, p represents the (sign-inversed) tangency of the particular indifference curve which passes the market equilibrium point (i.e., the point on the c.p.f.) corresponding to the p, at that point.

For every p belonging to the domain of $x_1(p)$, the market adjustment process determines x_1 to be equal to $x_1(p)$, so that the market-equilibrium disposable income y of the representative consumer for the goods equals $y(x_1(p),\ p)$ for every such p.

6. The Consumption Possibility Frontier and the Quasi-Marginal Cost

By (3–12), we have $y(x_1,\ p) = px_1 - (C^*(x_1) - I(x_1))$. We defined that

$$(6\text{-}1) \qquad f(x_1) = C^*(x_1) - I(x_1)$$

Then, we had

$$(6\text{-}2) \qquad y(x_1,\ p) = px_1 - f(x_1)$$

By (3–9) and (3–10), we have

$$(6\text{-}3) \qquad f(x_1) = m(x_1;\ w(x_1)) - F_2(F_2'^{-1}(w(x_1)))$$

We now define

(6–4) $$z(x_1) = F_2(F_2'^{-1}(w(x_1)))$$

$z(x_1)$ represents the market equilibrium quantity of Good 2 when the quantity x_1 of Good 1 is produced and when the wage-rate is adjusted to clear the labor market for the given x_1.

Then we also define

(6–5) $$g(x_1) = z(x_1) - m(x_1; w(x_1)))$$

$g(x_1)$ represents the market equilibrium level of consumption of Good 2 corresponding to the level x_1 of that of Good 1.

By (6–4) and (6–5), we have

(6–6) $$f(x_1) =-g(x_1).$$

Therefore, diagrammatically, the function $f(x_1)$ represents (the sign-inversed) x_2^c-coordinate of the point (x_1, x_2^c) on the consumption possibility frontier.

Since the first sector may be under increasing returns by assumption, the c.p.f. may be partly concave-shaped outwards, an example of which is depicted in Fig. 4.

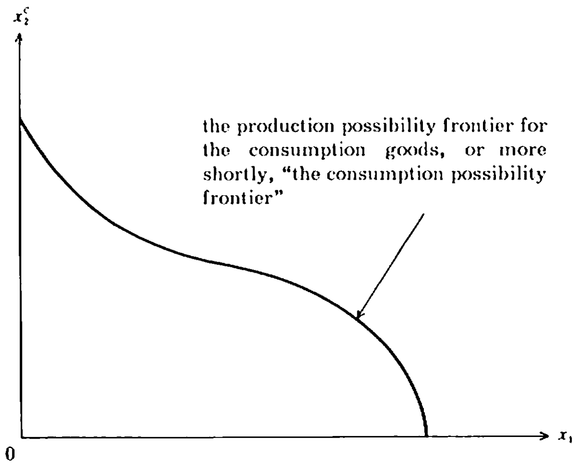

Fig.4

The data concerning the taste or the preference relation of the representative consumer for the goods is expressed by a set of indifference curves depicted in the same plane as that for the c.p.f. For simplicity we neglect any cases of corner solutions. Then there will be easily classified four qualitatively different positional combinations between the c.p.f. and the representative consumer's indifference curves for the goods, as depicted in Figs. 5–1 through 5–4. (Similar figures have been already presented in Brown and Heal (1979) and Dierker (1991).)

Since $F_2^{c\,-1}(w) = L_2 = L(w) - L_1(x_1; w)$, (6–5) can be rewritten

(6–7) $$g(x_1) = F_2(L(w) - L_1(x_1; w(x_1))) - m(x_1; w(x_1))$$

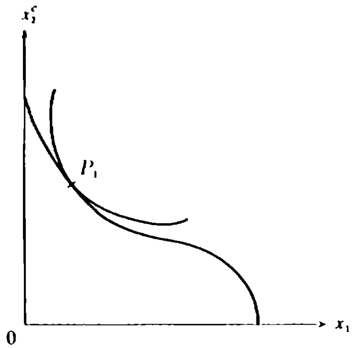

Fig.5–1

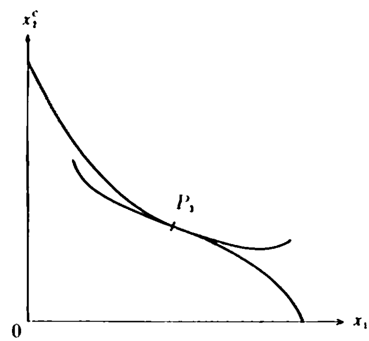

Fig.5–3

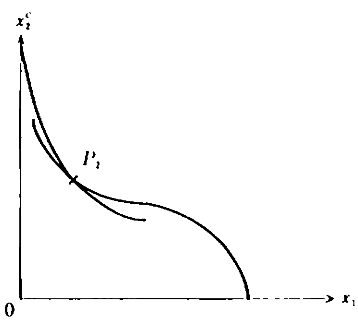

Fig.5–2

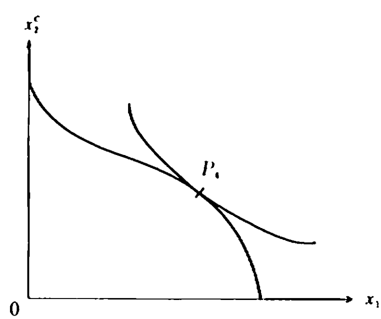

Fig.5–4

Hence, noting $F_2'(L_2) = w$, the slope of the c.p.f. equals

$$(6\text{–}8) \qquad g'(x_1) = w(x_1)(L'(w)w'(x_1) - (\partial L_1/\partial x_1) - (\partial L_1/\partial w)w'(x_1)) - ((\partial m/\partial x_1) + (\partial m/\partial w)w'(x_1))$$

By the well-known theorem (e.g., Chambers (1988)), we have

$$(6\text{–}9) \qquad w.(\partial L_1/\partial w) + 1.(\partial m/\partial w) = 0,$$

so that (6–8) becomes

$$(6\text{–}10) \qquad g'(x_1) = (w(x_1)(L'(w)w'(x_1) - (\partial L_1/\partial x_1)) - (\partial m/\partial x_1)$$

If $g'(x_1) > 0$, the c.p.f. is sloped upward at the x_1. The partly upward-sloping c.p.f. seems to be an economically excludable case, since it implies a violation of the a priori assumption of free disposal concerning the c.p.f. So, we may here assume that

$$(6\text{–}11) \qquad g'(x_1) \leqq 0$$

for all x_1 in the domain of the c.p.f.

By (6–6) and (6–10), we have

$$(6\text{–}12) \qquad f'(x_1) = [w(x_1).(\partial L_1/\partial x_1)) + (\partial m/\partial x_1)] - w(x_1)L'(w)w'(x_1)$$

We denote by $MC(x_1)$ the Bonnisseau-Cornet marginal cost for x_1, which is defined as

$$(6\text{–}13) \qquad MC(x_1) = w(x_1).(\partial L_1/\partial x_1)) + (\partial m/\partial x_1)$$

$f'(x_1)$ then equals the $MC(x_1)$ *plus* the residual term, $-wL'w'$.

We call this function $f'(x_1)$ as the *quasi-marginal cost* for x_1, which is denoted by $QMC(x_1)$.

$$(6\text{–}14) \qquad QMC(x_1) = MC(x_1) - w(x_1)L'(w)w'(x_1)$$

7. The Social Optimization

Since the representative consumer maximizes utility even in market equilibrium as well as in market disequilibrium, he maximizes utility given the price-income combination $(p, y(x_1(p), p))$ in market equilibrium.

The maximum level of utility of the representative consumer for the goods for any given price-income combination (p, y) is expressed by the indirect utility function

$$(7\text{–}1) \qquad V(p, y) = U(h_1(p, y), h_2(p, y))$$

where $h_i(p, y)$ denotes the demand function for Good i for the given (p, y), $i = 1, 2$. By substituting

$$(7\text{–}2) \qquad y = y(x_1(p), p)$$

into (7–1), the maximum level of utility in market equilibrium is written

$$(7\text{–}3) \qquad V(p, y(x_1(p), p))$$

for every p belonging to the domain of $x_1(p)$, which we denote by $V^*(p)$.

By the differentiability assumptions concerning the utility and production functions, $V^*(p)$ is a differentiable function.

If there exists some social optimum level of p, then it must satisfy that

$$(7\text{–}4) \qquad V^{*\prime}(p^*) = V_p + V_y(dy/dp) = 0$$

where p^* denotes the social optimum level of p, V_p and V_y the partial derivatives of the indirect utility function with respect to p and y, respectively.

We will now consider $V^{*\prime}(p)$ for all p more explicitly.

By Roy's identity, we have

(7–5) $V_p(p, y)/V_y(p, y) = -h_1(p, y)$

Hence, by $V_y > 0$ (the assumption that Good 1 is a normal good), we have

(7–6) $V^{*\prime}(p) = (-h_1(p, y(x_1(p), p)) + (dy/dp))V_y(p, y(x_1(p), p))$

By the market equilibrium identity (5–2), this can be re-written

(7–7) $V^{*\prime}(p) = (-x_1(p) + (dy/dp))\ V_y(p, y(x_1(p), p))$

Since $y(x_1(p), p) = px_1(p) - f(x_1(p))$, we have

(7–8) $dy/dp = x_1(p) + px_1'(p) - f'x_1'(p)$

so that (7–7) is further rewritten

(7–9) $V^{*\prime}(p) = (px_1'(p) - f'x_1'(p))\ V_y(p, y(x_1(p), p))$
$= (p - f'(x_1(p)))\ x_1'(p)\ V_y(p, y(x_1(p), p))$

By the monotonely decreasing property of $x_1(p)$, we have $x_1'(p) < 0$, so that (7–9) implies that the sign of $V^{*\prime}(p)$ is always opposite to that of $p - f'(x_1(p))$.

Hence, the necessary condition for p^* to be the socially optimal level is written $p^* = f'(x_1(p^*)) = QMC(x_1(p^*))$, or

(7–10) $p^* = QMC(x_1(p^*))$

or the equality between the price of Good 1 and its QMC.

The sufficient condition for p^* to be the socially optimal level of the price is that

(7–11) $V^{*\prime}(p) > (<)0$ if $p < (>$, respectively) p^*.

By (7–9), this condition is equivalent to

(7–12) $p < (>)\ QMC(x_1(p))$ if $p < (>$, resp.) p^*.

The sufficient condition for p^* to be the socially optimal level of p is interpreted that the level of the price of Good 1 is lower (higher) than its QMC corresponding to that level of the price, whenever the price is lower (higher, resp.) than p^*.

Now, we compare the cases with and without the income-redistribution scheme. We call these cases simply Case 1 and Case 2.

In Case 2, by $T = 0$ and (1−7), we have $n(w) = wH_w$, so that

$$(7\text{–}13) \qquad L'(w) = 0$$

It follows that, in this case, the social optimality is attained at p^{**} such that

$$(7\text{–}14) \qquad MC(x_1(p^{**})) = p^{**}$$

or at p^{**} which is set by the MC pricing.

Since we have $L'(w) > 0$ in Case 1 and $L'(w) = 0$ in Case 2, the socially optimal level of the price of Good 1 in Case 1 is lower than that in Case 2, by the absolute value of the term $-w(x_1)L'(w)w'(x_1)$ in (6−12).

The absolute value of the discrepancy term $-w(x_1)\ L'(w)w'(x_1)$ is interpreted as the increment of the real market value (in terms of Good 2) of the total labor owing to the (algebraical) increase in the total supply of labor itself, caused by one unit of increase in the level of production of Good 1.

With the minus sign taken in consideration, the discrepancy term as a whole means that, since $L'(w) > 0$, one unit of increase in x_1 will raise the market wage-rate, thus encourage the total supply of labor. This total supply effect will lighten the social cost of additionally producing one unit of Good 1, and hence the true marginal *social* cost (or the quasi-marginal cost) becomes smaller than the marginal cost in the technological (or cost-minimizing) sense, by the amount just equal to the market value (evaluated at the market equilibrium wage-rate after the adjustment) of the increase in L induced and mobilized by the encouragement of the rise in the real wage-rate by the unit increase of x_1.

Thus, the quasi-marginal cost consists of two qualitatively different terms: One is the sacrifice, caused by the unit increase in x_1 in the absence of the total labor supply effect, of the quantity produced of the private good; and the other is the term which expresses the total labor supply effect, caused by the unit increase of x_1, in terms of the quantity produced of the private good.

The former term represents nothing but the Bonnisseau-Cornet marginal cost.

Since $L'(w) > 0$, the latter term is negative so that it is a negative sacrifice and hence so to speak a social marginal benefit as a whole. The quasi-marginal cost is smaller than the marginal cost by the amount of the social marginal benefit.

Conclusions

The critical reader may argue (1) that the above analysis says nothing new, and that it only rephrases the Dupuit-Hotelling classical thesis for marginal cost pricing (Dupuit (1844) and Hotelling (1938)), or (2) that the above analysis is not more than an example of the Bonnisseau-Cornet model.

On the first point, we have to first agree that, so far as we are concerned with the case in which the total labor supply effect can be neglected, it may be criticized that there is not found anything new in the conclusion of our argument for the optimality of the equality between the price and the marginal cost of production of the good in question. Even then, however, we may say that what is new is in that we present the general equilibrium analysis for the classical thesis of marginal cost pricing, as an alternative to the partial equilibrium theory by Dupuit and Hotelling. Our analytical tools consist of the sets of indifference curves of the consumers, the production functions of the goods, and the government's taxation and subsidization scheme. The concept of the consumer's surplus on which Dupuit and Hotelling depend, is not used in our analysis.

Our main argument lies in the analysis of the case where the income-redistribution is assumed and where the total labor supply effect is non-negligible. In this case, we have shown the possibility of the social non-optimality of the classical thesis of the marginal cost pricing.

On the second point, we have to note that our model cannot necessarily be regarded as a special case of the Bonnisseau-Cornet model, as long as the income-redistribution scheme is incorporated.

Our simple model will also possess some Bonnisseau-Cornet marginal cost pricing equilibrium.

We have also shown that, if there is the income-redistribution scheme and if the total labor supply effect is not negligible, then the Bonnisseau-Cornet MC pricing equilibrium may well be socially non-optimal and the QMC pricing equilibrium is socially optimal.

In summary, we have clarified that, if there is the income-redistribution and if the aggregation of the utility functions of the workers and the capitalists is possible so that the social welfare can be defined, the QMC pricing equilibrium is more desirable than the Bonnisseau-Cornet MC pricing equilibrium, provided the total labor supply effect is non-negligible.

APPENDIX

The Alternative Market Adjustment Process

In this Appendix, we consider the stability of the alternative Keynesian market adjustment process.

It is assumed that the government specifies a level of p among all levels of the price p such that there is a market equilibrium solution for x_1 corresponding to each level of p. The market equilibrium level of production of Good 1 is determined as the function of p, which we denote by $x_1(p)$. This function $x_1(p)$ is the inverse function of $p(x_1)$.

Under the assumption that p is fixed at any level such that there is some x_1 fulfilling the market equilibrium condition (3–13), we assume the following Keynesian quantity adjustment process.

$$\text{(A-1)} \qquad \dot{x}_1 = c(h_1(p, y(x_1, p)) - x_1)$$

where c is a positive constant.

This adjustment process is now interpreted as follows: Suppose, for a given fixed level of p, the first sector initially produces the quantity x_1^0 of Good 1. The profit or loss resulting from the production of x_1^0 is absorbed by the representative consumer, so that his disposable income then equals $y(x_1^0, p)$. His demand for Good 1 for the fixed p and the total income for the goods $y(x_1^0, p)$ is $h_1(p, y(x_1^0, p))$, whereas the supply of Good 1 is x_1^0.

If the demand (supply) exceeds the supply (demand, respectively), there is shortage (over-production, resp.) of Good 1, so that the (in this case, quantity-adjusting) auctioneer indicates the first sector to increase (decrease, resp.) its level of production from the initial level x_1^0.

The stability of this adjustment process depends on whether the total derivative of h_1 with respect to x_1 is algebraically less than unity or not.

Therefore, it will be in order for us here to consider this total derivative of h_1 by x_1 with p fixed at some level belonging to the domain of $x_1(p)$.

$$\text{(A-2)} \qquad (\partial h_1/\partial x_1)_{p:\text{const.}} = (\partial h_1/\partial y)(\partial y/\partial x_1)_{p:\text{const.}}$$

By the identity $y(x_1, p) = px_1 - f(x_1)$ in (6–2), we can write

(A−3) $\quad (\partial y/\partial x_1)_{p\text{ const.}} = p - f'(x_1)$

It follows that the Keynesian quantity adjustment process (A−1) is stable if and only if

(A−4) $\quad (\partial h_1/\partial y)(p - f'(x_1)) < 1.$

By the assumption (6−11) in the text, we have $f'(x_1) \geqq 0$.

The non-negativity of $f'(x_1)$ is used in the proof of the following proposition.

PROPOSITION 2: The Keynesian market quantity adjustment process (A−1) is always globally stable for all $p \geqq 0$.

Proof. In the case of $p = 0$, the stability condition (A−4) becomes $-(\partial h_1/\partial y)f'(x_1) < 1$, which always holds since $\partial h_1/\partial y > 0$ and $f'(x_1) \geqq 0$.

Suppose $p > 0$. By the assumption of $b_{jk} > 0$, $j = w, c$, $k = 1, 2$, it follows that the marginal propensity to consume Good 1 from the income for the goods is positive and less than unity. Hence, we have

(A−5) $\quad 1 > p(\partial h_1/\partial y) > 0.$

By $f'(x_1) \geqq 0$ and (A−5), we have $1 > p(\partial h_1/\partial y) - f'(x_1)(\partial h_1/\partial y)$, so that

(A−6) $\quad 1 > (\partial h_1/\partial y)(p - f'(x_1))$

which shows that the stability condition (A−4) for the adjustment process (A−1) is fulfilled. Hence (A−1) is globally stable.

FOOTNOTES

1) The marginal cost in the Bonnisseau-Cornet equilibrium to which the price is equalized, is defined as the sum of the derivatives, with respect of the level of the output, of the elements (including labor) of the cost-minimizing input vector, weighted and *multiplied by* the

equilibrium input prices (including the wage-rate. See Bonnisseau-Cornet (1990), pp.666–667.)

2) Bonnisseau and Cornet's (1990) is one of the most recent papers in the following (not necessarily exhaustive) series of related works: Guesnerie (1975), Dierker, Fourgeaud, and Neuefeind (1976), Calsamiglia (1977), Brown and Heal (1979, 1983), Beato (1982), Beato and Mas-Colell (1983), Brown, Heal, Khan, and Vohra (1986), Kahn and Vohra (1987), Cornet (1988a, b), Bonnisseau and Cornet (1988a, b), Kamiya (1988a, b), Vohra (1988), Bonnisseau (1988), Dehez and Dreze (1988a, b), Dierker and Neuefeind (1988) Jouini (1988), and Dierker (1991).

3) Many years ago, Wiseman (1957) pointed out that no general pricing rules had been proved unambiguously to bring about an optimum use of resources by public utilities. Wiseman called the marginal cost pricing rule an empty box. In his paper, he criticized a marginal cost pricing from the aspects of uncertainty, indivisibility, and the effects of taxation and subsidies on the distribution of income. Unfortunately, however, as Wiseman (1957) suggested, there had not been any explicitly formulated general equilibrium analysis for the social optimality of the marginal cost pricing which did not depend on the concept of consumers' surplus. Such a general equilibrium analysis could have been done in a simple aggregative framework which may not necessarily be immune from all of Wiseman's criticism. (See also Blaug (1990).)

4) Very recently, such a model has been presented in Samuelson (1990). In his paper, Samuelson assumed one commodity and one consumer, whose utility depends on both the commodity and leisure. In this paper, we will present an alternative framework to that of Samuelson's (1990). Instead of one commodity and leisure, we formulate two kinds of commodities and leisure in the utility functions.

5) The input, m, is assumed to include depreciation, denoted by D, of the capital of the first sector. Depreciation consists of the fixed part, denoted by D_f, which is independent of the level of production of the sector, and the variable part, D_v, which is a variable depending on the level of production, $D = D_f + D_v$. The input m consists of the ordinary input, denoted by G, used as meterials, etc., in the production, and the depreciation D, so that $m = G + D = G + D_f + D_v$. The prime cost of the first sector, denoted by C_p, is defined as the sum of the factor costs, the cost of ordinary (material and other) input, G, and the variable depreciation, D, (Keynes (1936), Chapter 6). The factor costs consist only of labor cost, wL_1.

The total cost of the first sector, C^*, equals $C_p + D_f$. Since D_f is fixed, marginal cost = marginal prime cost in this statical framework.

6) Rigorously, $F_2(L_2)$ is defined as the net output of the second sector. It equals the total production of Good 2 *less* the total input of Good 2 for the second sector itself. The total input equals the sum of the ordinary input (used by the second sector itself as materials, etc.) and the total depreciation of the second sector.

7) Behind this assumption, we suppose, before aggregation, that there holds perfect substitutability both in technology and taste between the privately produced consumption goods and the privately produced intermediate goods used as inputs in the first sector.

8) Beato and Mas-Colell (1985) assumed two consumers, the first consumer "neither has nor cares about the input good, absorbs all profits or losses, while the second consumer derives income only from selling inputs" (Beato and Mas-Colell (1985), p.358).

9) The government is assumed to play the following roles in our model. (1) It redistributes income between the capitalists and the workers, ordinarily transferring a part of income of the capitalists to the workers. It transfers from the workers to the capitalists when the real wage-income is exceptionally high. (2) It supervises the first sector to minimize cost at each and every level of output and at each and every market input-price vector. (3) If the first sector incurs a loss in spite of its best effort of minimizing cost, the government taxes the profit income of the capitalists which is earned at the second sector and subsidizes the first sector by the amount equal to the deficit of the first sector. (4) It specifies some initial levels of production and the price of Good 1 at appropriate levels. (5) The government supervises the auctioneers who work to equilibrate demand and supply at the markets of labor, Good 1, and Good 2. (6) The government supervises the first sector to adjust the level of the price (production, respectively) of Good 1 to that of the quasi-marginal cost pricing.

10) By McFadden (1978) and Uzawa (1964), the cost function is well-defined for all fixed level of output $x_1 > 0$ and all level of the market wage-rate $w > 0$ as

$$(\text{F}-1) \qquad C(x_1; w) = \min_{v \in A(x_1)} (wL_1 + m),$$

where $A(x_1)$ denotes the set of all input vectors (L_1, m) which can possibly produce the level of output x_1.

11) If we assume them to be non-tatonnement processes, we would have to introduce some third commodity, called "money," to play the role of the medium of exchange, which would unnecessarily complicate the analysis of the adjustment processes for this paper.

12) The capitalists' 'consumption' of leisure (which may not always be consumption of leisure for personal enjoyment but possibly be that for business purposes) is always equal to the initial endowment H_c, so that $U_c(x_{1c}, x^c_{2c}, H_c)$, where H_c is a constant.

13) In the case when the government taxes the workers, the tax revenue is assumed to be transferred to the capitalists, because the workers are regarded to be rich enough to give a transfer to the capitalists. However, such a case seems very unrealistic.

14) We allow the case where $g = 1$. In this special case, $n(w) = B$, a constant. The negative income tax scheme is such that the utility level of the workers is kept constant by the redistribution of income.

15) The following analysis is still valid if we assume the capitalists' utility function to be of the form

$$(\text{F}-2) \qquad U_c = (x_{1c} - x^0_{1c})^{b_{c1}} (x^c_{2c} - x^{c0}_{2c})^{b_{c2}} (l_c - l^0_c)^{b_{c3}}.$$

where x^0_{1c}, x^{c0}_{2c}, and l^0_c are constants and may be of any signs, and $l^0_c < H_c$. U_c is defined for all $x_{1c} \geqq x^0_{1c}$, $x^c_{2c} \geqq x^{c0}_{2c}$, and $l_c \geqq l^0_c$.

Since $b_{ci} = b_{ci}/(b_{c1} + b_{c2})$, $i = 1, 2$, and since x^0_{1c}, x^{c0}_{2c} are constants, the marginal propensities to consume Goods 1 and 2 of the workers confronted with Problem 2 and of the capitalists are constant and the same, so that the aggregation for the representative consumer for the goods is still possible in this more general case. See Gorman (1953).

16) The wage-rate w is assumed to be adjusted by the following tatonnement process.

$$(\text{F}-3) \qquad \dot{w} = a((L_1(x_1; w) + F'^{-1}_2(w)) - L(w))$$

where a is a positive constant. We note that $L'(w) > 0$. The quasi-concavity of the production set of the second sector implies that $\partial L_1(x_1; w)/\partial w < 0$ (the input substitution effect between L_1 and m with respect to a change in their relative price). Furthermore, $F'^{-1}_2(w)$ is a decreasing function by the assumption of $F''_2(L_2) < 0$ (the decreasing marginal product of labor). It follows that the market wage-rate adjustment process is globally stable.

17) The c.p.f. is not used in this proof and we do not depict it in Fig. 3. For simplicity, we neglect the case of a corner solution at which only one of the two goods is consumed.

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