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The Phillips-Mundell Relation in Theory and Data

Koichi MIYAZAKI

Introduction

In this paper the relation between the rate of inflation and the levels of production and employment is called as the Phillips relation. Usually "the Phillips *curve*" means the trade-off between the rate of inflation and the rate of *unemployment,* and it is to distinguish the above relation from this that we use the term "relation" instead of "curve." Moreover, the inflation connected with the term "the Phillips relation" here means the rise in the prices as against that in wage-rates.

The basic assumptions of the model of this paper are made in order to make it easier to describe and analyze theoretical relations between inflation and employment. Let us briefly summarize them as follows. (1) The wage-rate function with employment as its variable is assumed to be upward-sloping *in the short-run,* where the wage-rate is the nominal wage-rate. (See Fig. 1, below.) (2) A two-sector model is adopted in which the labor market and the commodity markets are assumed to be perfectly competitive. (3) A tatonnement price adjustment process is assumed to prevail in the system of flow markets as treated separately from the system of stock markets. And (4) it is assumed to the effect that the ratio between the prices of nondurable consumption good and durable good (which are the two kinds of commodity in the two-sector model) is all the time constant and that so is the ratio between the quantities produced of the two kinds of commodity. This last assumption is made in order for us to be able to avoid theoretical complications likely to arise from the possibility of changes in the relative price and in the proportions of the constituents of the aggregate production.

Two kinds of the Phillips relation are distinguished : that is, the positive correlation between employment and the *expected* rate of inflation, and the positive correlation between employment and the *actual* rate of inflation. Let us call them as " the first kind of the Phillips relation " and " the *short-run* Phillips relation," respectively. It is an aim of this paper to formulate a model which accommodates both the two kinds of the Phillips relation. Such a Keynes model as formulated by Chakrabarti in [2] satisfies the purpose, which is a two-sector model. Concerning the two-sector model to be applied to the analysis of inflation, two fundamental considerations are required. The first of them is about the problem of what assumptions must be made in order for us to be emancipated from the anxiety about the aggregation in the two-sector model, that is, about the index number and the real aggregate unit. The second of them is concerned with stability of the model, especially with the stability of the system of flow markets, which is basic both to the simple aggregation and to the analysis of inflation.

Chakrabarti [2] gives us valuable suggestions for the modelling in the present paper. Also we have to refer to Prof. Uzawa's model [13] as a formulation of the profit-maximizing firms on the basis of the perfectly competitive market assumption.

The above defined first kind of the Phillips relation, or the *Phillips-Mundell relation*, has been pointed out by Mundell [8] in the classical framework of the model by Metzler [6]. The presence of monetary claims such as bonds in the macroeconomy together with money and real capital is the most fundamental factor to the existence of the first kind of the Phillips relation.

Chapter 2 (which is in preparation to be separately published in this *Review*) is devoted to the statistical analysis of the Phillips relation and some related correlations. Data relevant to the Phillips relation are examined both conceptually and econometrically, based on the model formulated in Chapter 1. The factor of increases in labor productivity will be introduced in relation to the rate of increase of the nominal wage index, motivated by the econometric test of the correlation between the factor and employment. An explanation of the empirical correlation between the rate of increase of the wage index and employment, or the counterpart to the original Phillips curves in [10], will be stated as an alternative to the explanation by Phillips [10].

The Phillips-Mundell relation defined above will be positioned in Chapter 2 as one of the factors which explain the high correlation of the Phillips curve.

Chapter 1. Inflation Theory

Introduction

1. Summary of the Model Before the Aggregation and the Inflation Scheme

Commodity Production: There are two kinds of good, non-durable consumption good, denoted by subscript C, and durable good including consumers' durables, denoted by subscript D. The production function for j-good ($j=C, D$) is $X_j=F_j(N_j, K_j)$, where N_j and K_j are quantities of effective labor employed in the j-sector and capital assets held in the j-sector. This is the long-run production function. The short-run production function is denoted by $f_j(N_j)$ which is given by regarding K_j as fixed in $F_j(N_j, K_j)$. As for convexity, it is assumed only that for a fixed K_j the short-run function is convex above, reflecting diminishing returns to labor. The short-run function is assumed to shift up when K_j increases.

Demand for Commodities: Demand for durable good is a decreasing function of the expected real rate of interest $i^e-\pi^e$, where π^e denotes the expected rate of inflation. This is what is assumed in the short-run when the state of long-term expectation is given. Capital accumulation may shift the curve. But a fictitious situation which would prevail if there is a lower rate of inflation expected now to endure steadily in the future, lower than the actual expected rate of inflation, will be worth comparing with the actual situation. In this sense the short-run durable-good demand function matters in relation to the first kind of the Phillips relation.

Demand for nondurable consumption good equals $\bar{c}Y/P_C$, where $\bar{c}$ is a constant such that $1>\bar{c}>0$, $Y=P_CX_C+P_DX_D$, and P_C is the price level of C-good. X_C, X_D, and P_D are quantities produced of C-and D-goods, resp., and the price level of D-good.

Demand for Durable Good: Saving $Y - P_C X_C$ always equals the nominal rate of the purchase of D-good $P_D X_D$. Hence, $P_D X_D$ $(i^e - \pi^e)$, where $X_D(\cdot)$ denotes the durable-good demand function.

Demand for Money: Demand for money equals $L(i)Y$.

Supply of Money: Money supply equals to $\bar{M}W$, where $\bar{M}$ is a policy variable and W is the nominal wage-rate.

The Supply Curve of Labor: Demand for labor is a demand derived from demand for commodities. The supply curve of labor is represented by the nominal wage-rate function $W = W(N, t)$. The function measures the level of nominal wage-rate at which workers are willing to supply the quantity N of *effective* labor at time t.

It is assumed that, for each fixed t, $W(N, t)$ is an increasing function of N. (See Fig. 1). This curve in Fig. 1 is denoted by $W = W^*(N)$.

The function $W(N, t)$ contains the variable t because the short-run labor supply curve is assumed to be always shifting for the following two reasons: first, shifting upward at a pace of the inflation of the general price level, since workers get informed of the

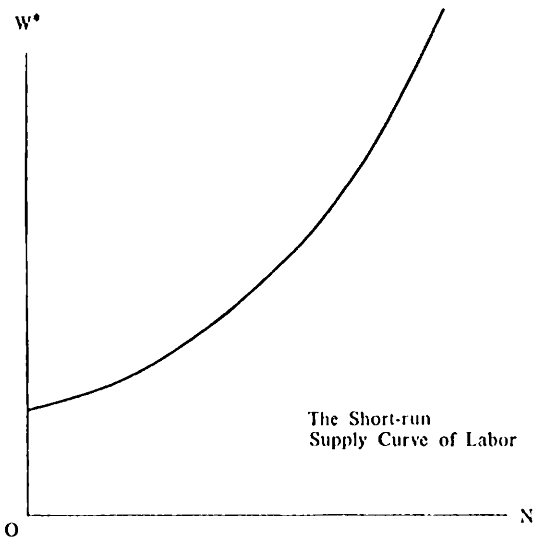

Fig. 1

reduction in real incomes by the inflation, sooner or later, despite their short-run money illusion. Second, it is shifting to the right, reflecting the increase of the quantity of effective work that is contained in one unit of work hour, caused by a rise in labor productivity. Behind the formulation $W=W(N, t)$ there is supposed the short-run money illusion on the part of workers as Friedman [4] states. Namely, workers tend to lag in being informed of a change in the general price level, because they are not able to obtain information about the price level so easily as producers are. Producers are much more accessible to such information since the general price level reflects a change in P_D, that is, the price of durable good which contains durable goods for industrial use.

Perfect Competition: Perfect competition prevails in the commodity markets and the labor market. It is assumed that no influential power on the market prices of goods and labor is held by the demanders and suppliers there. Instead, auctioneers are assumed to play the equilibrating roles in the markets.

The LM Curve: In the short-run the equilibrium in the money market requires $L(i)Y=\bar{M}W^*(N)$. In the short-run the argument N in $W^*(\cdot)$ reflects the movements of X_C and X_D: such movements of X_C and X_D may happen on account of a shift of durable-good demand. This is how the nominal wage-rate W changes even in the short-run. The change in W enters the money supply function $\bar{M}W$. P_j 's are *ceteris paribus* proportional to W, so that Y is *c. p.* proportional to W. Hence, in view of $L(i)Y/W^*(N)=\bar{M}$, the function $W^*(N)$ may be immaterial on the whole left-hand side of the equation: the production functions and the function $L(i)$ may be the only factors which determine the relation between real production and the nominal rate of interest. The real LM curve will thus be independent of the wage-rate function, a simple result justifying the formulation of money supply.

2. *The Inflation Scheme*

Suppose the economy is in the short-run equilibrium in the sense of the following equations:

$$\bar{s}Y=D(i^e-\pi^e),$$
$$\bar{M}W(N)=L(i)Y,$$
$$P=P_W(y)W(N),$$
$$Y=Py,$$

$$\pi^e=0,$$
$$i^e=i,$$
$$y=f(N),$$

where D denotes the nominal value of the purchase of durable good, $D=P_D(i^e-\pi^e)X_D(i^e-\pi^e)$, and P the nominal price (index). The function $P_W(y)$ is the inverse of the function $f'(f^{-1}(y))$, where $f^{-1}(y)$ is the inverse function of the function $f(N)$. Capital stock (K) is asssumed constant. This system of the short-run equilibrium describes a static state of the economy in the sense that prices and nominal values are constant as well as real variables. Indeed the equilibrium quantity of employed labor (N) is constant and $W(N)$ is constant, so that the nominal money supply $M^s=\bar{M}W(N)$ is constant. Similarly for P. Hence Py is constant, and so is $D=\bar{s}Y$. The expected rate of inflation equals zero. Let us call this state as 'the short-run static equilibrium.'

Let us proceed to consider how inflation can consistently creep into the equilibrium system. The following is a simple logical mechanism to expain this: first, suppose the real stock of money supply is increased (by increased production of precious metals, by printing more bank notes, etc.) The real LM curve shifts to the right, and the price level P rises as real production and employment rise. The wage-rate rises as employment rises.

Thus, a once and for all increase of the real stock of money causes once and for all rises in price level, wage-rate, production and employment. But the rises will stop as soon as the increase in $\bar{M}$ does. The economy shifts to the new short-run static equilibrium.

The rate of interest falls, and there arises a discrepancy between the rate and the long-term expectation about the future rate of interest. Let us call such a process *'the short-run rise-in-money process.'*

Now, instead of supposing that $\bar{M}$ initially rises, assume that the durable-good-demand function shifts autonomously upward (by discoveries of new opportunities of investment, etc.) Subsequently, the price level rises as production and employment rise. The wage-rate rises as employment rises. The rate of interest rises. Hence the discrepancy between the actual and expected rates of interest (of the opposite direction to that in the above case of $\bar{M}$'s increase.) Let us call such a process the short-run 'shift-of-mec' process. ('Mec' represents 'the marginal-efficiency schedule of capital assets.')

Thus, we defined the two short-run processes of the changes in the supposedly given economic conditions. Let us then imagine that these processes alternately happen in the short-run, without reaching the ceiling of full employment. Then, the equilibrium rate of interest alternately falls and rises, whereas price and wage-rate keep rising. Suppose, for simplicity, that the shift-of-mec process is everytime just as effective as cancels out the preceding fall of the interest rate caused by the rise-in-money process. Let us call such an alternate process as 'the interest-neutral process.'

The price level and wage-rate continue to rise in the interest-neutral process. Suppose the rate of change in real production (y) is constant through the process. Suppose further that marginal product of labor happens to be kept from falling much notwithstanding the steady rise in the rate of production. This implies that the supply functions of the two goods are assumed to shift downward. Such shifts of the supply curves are supposed to be caused by capital accumulation and technical progress: marginal productivity is assumed to rise owing to the productivity effect of capital accumulation and technical progress, hence the downward shifts of the marginal cost curves. The process is no longer a short-run process but a long-run process, in the sense that it reflects effects of accumulation of physical capital assets and technical progress. Distinguishing such a process with constant marginal product of labor from the short-run interest-neutral process, let us call this process 'the long-run interest-neutral process.' In this process, the price level and the wage-rate rise in a constant proportion, and the nominal money supply and the nominal rate of production rise in a constant proportion. The nominal rate of production rises in a greater growth rate than the actual inflation rate by the growth rate of the real rate of production.

If such a long-run process has been experienced for some sufficient length of time, the long-term future inflation will come to be psychologically built in people's economic behavior. This is how π^e becomes a positive magnitude.

After a long-term expectation of positive inflation has been formed, the short-run equilibrium investment per unit of time rises relative to the level before the formation of the expectation. As will be explained in Section 2-4, in the full equilibrium with the new expectation of inflation the time paths of the price level and the wage rate are uniformly higher than in the full equilibrium with zero expected inflation. The wage/price ratio will be lower, however, provided there arises no offsetting rise in labor productivity, relative to the zero expected inflation. Once the new full equilibrium is reached, the short-run equilibrium conditions are of course fulfilled. To this is added the effects of capital accumulation and technical progress, which shifts the supply curves of the goods in a harmonized proportion and at the same time shifts the marginal product function of labor, so as to keep the equilibrium marginal product of labor from falling. We are now in a stylized picture of the long-run full equilibrium growth path of the economy.

Section 1. Statics

1. The Demand Schedule of Durable Good

In this papar only one kind of durable good is assumed to exist, and the production period of the durable good is assumed to be so short that capital accumulation during one period of production of durable good may be regarded as negligible. Thus, production of durable good is completed in the short-run.

For simplicity let us assume that (1) there is not any depreciation of real capital, and that (2) each unit of durable good is expected (by the investing producers and the consumers of consumers' durables) to earn a constant and everlasting flow of extra net profit or imputed benefit which can be measured in terms of nondurable consumption goods. The extra net profit or imputed benefit which is expected to be earned by each marginal real purchase of the durable good reflects its productive service or utility benefit at each point of time in the future.

Let $x(X_D)dX_D$ denote the constant flow of extra net profit or imputed real benefit pertinent to the marginal purchase of D-good dX_D. Let π^e be the expected rate of inflation. Then, $P_C x(X_D)dX_D \exp(\pi^e t)$ equals the nominal value of the flow pertinent to dX_D at time t ($t \geqq 0$.) Hence, $P_C x(X_D)dX_D \exp(\pi^e t)dt$ equals the accumulated value of it from t to $t+dt$. Its dimension is stock of money at time t. Therefore,

$$\int_0^\infty P_C x(X_D)dX_D \exp((\pi^e - i^e)t)dt = (P_C x(X_D)/(i^e - \pi^e))dX_D$$

equals the present value of the flow of extra net profit or imputed benefit expected to be earned by the dX_D from $t=0$ to indefinite

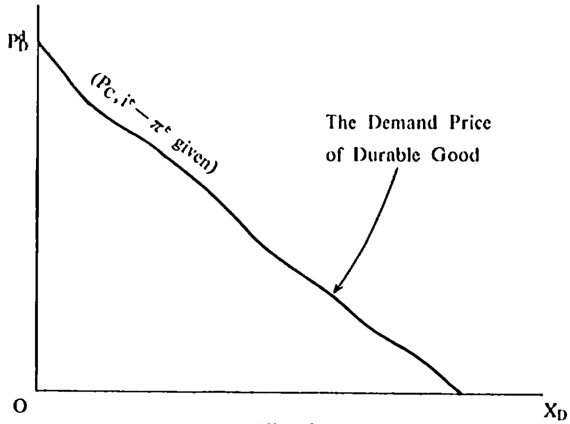

Fig. 2

future. Let us denote this by $P_D^d(X_D, P_C, i^e-\pi^e)dX_D$. The inverse function of the function P_D^d with respect to X_D is denoted by X_D^d $(P_D, P_C, i^e-\pi^e)$. Suppose $P_D^d(X_D, P_C, i^e-\pi^e)dX_D > P_D dX_D$, where P_D is the market price of D-good. This means that the present value of that dX_D exceeds the cost of purchasing it. They will therefore purchase all of such dX_D. Thus, the function $X_D^d(P_D, P_C, i^e-\pi^e)$ represents the demand curve of D-good, measuring how much D-good the firms and the consumers demand, given P_C and their expectation $i^e-\pi^e$. (Fig. 2) It is clear that we may write $X_d^D(P_D, P_C, i^e-\pi^e)=\varphi((P_D/P_C)(i^e-\pi^e))$.

Finance of Investment: An Implicit Assumption

It must be remarked that even if the producers of durable good are ready to supply the good at a supply price, the investing firms and the demanders of consumers' durables cannot purchase the good to the extent sufficient for them *unless* opportunities to finance the purchase are available : if they do not have enough money they must be able to borrow from other economic agents. We assume, therefore, that such opportunities are always secured.

This assumption amounts to the same thing as saying that the durable-good purchasers are supposed to be free to supply monetary claim. Because to supply monetary claim means to give to other agents claim to the funds by which they finance the purchases of durable good. This assumption will be institutionally satisfied if they are able to issue bonds and stocks, or to borrow from banks, etc.

Investment Creates Savings?

The Kahn-Keynes income multiplier process explains how a change in the nominal rate of the purchase of durable good gives rise to the same change in the nominal rate of saving in the wider sense, that is, the remainder of nominal income after buying nondurable consumption good. In our Keynes model a change in the purchase of durable good logically precedes that in the saving. This means that the amount of finance of the purchase of durable good, such as that of new lending by the banks to the firms or to the consumers, is considered to be able to be changed more or less independent of how much the past flow of the saving in the wider sense amounts to. Such flexibility of supply of loan is assumed to be satisfied.

2. *The Supply Price*

$f_j(N_j)$ is assumed to fulfil the convexity denoted by $f'_j>0$ and $f''_j<0$ for all $N_j>0$. The producers of j-good ($j=C, D$) try to maximize profit, and the price of the good and the wage-rate are assumed to be given by the auctioneers of the market of the good and of the labor market. Thus, they maximize profit $P_jf_j(N_j)-WN_j$, to determine N_j such that $f'_j(N_j)=W/P_j$, that is, marginal physical product of labor equals real wage-rate in terms of the

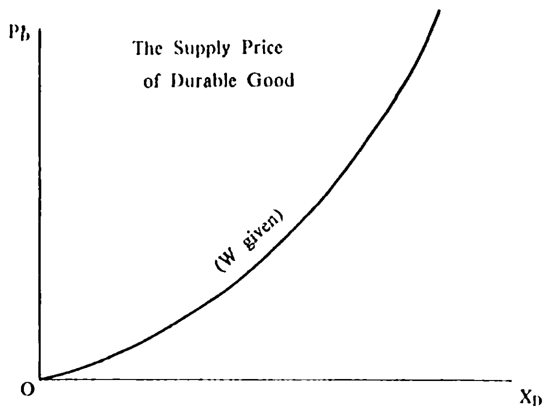

Fig. 3

good. Thus, $P_j = W/f'_j(N_j)$ as a function of N_j represents the supply price of the j-good at which producers are willing to produce by employing the quantity N_j of labor. By $X_j = f_j(N_j)$, the supply curve of the good can be written as $X^s_j = X^s_j(P_j, W) = f_j(\lambda_j(W/P_j))$, $j = C, D$, where $\lambda_j(\cdot)$ is the inverse function of $f'_j(N_j)$. (Fig. 3)

3. *Stability of the Price Adjustments in the Flow Markets*

Walrasian tatonnement price adjustment rules (see [1]) are assumed in the flow markets, that is, in the markets of C- and D-goods and the labor market. In the good markets, it is assumed that the auctioneers' price adjustment functions are written as

$$dP_D/d\tau = \alpha_1(X^d_D(P_D, P_C, \bar{i}^e - \bar{\pi}^e) - X^s_D(P_D, W)) \qquad \alpha_1 > 0$$
$$dP_C/d\tau = \alpha_2(X^d_C(P_D, P_C, W) - X^s_C(P_C, W)) \qquad \alpha_2 > 0$$

where[1]

$$X^d_D(P_D, P_C, \bar{i}^e - \bar{\pi}^e) = \varphi((P_D/P_C)(\bar{i}^e - \bar{\pi}^e)),$$
$$X^d_C(P_D, P_C, W) = (\bar{c}(P_D X^s_D(P_D, W) + P_C X^s_C(P_C, W))/P_C,$$

and $X^s_j(P_j, W) = f_j(\lambda_j(W/P_j))$, $j = C, D$. X^d_C is formulated in such a way that the auctioneer correctly anticipates the full result of the consumption generating process, with the multiplicand of the propensity $\bar{c}$ considered to be the real factor incomes (in terms of C-good), that is, real wage and profit, earned in both C and D-sectors, at each vector of the prices P_j and wage-rate W.

In the labor market the auctioneer is assumed to adjust the nominal wage-rate by the following rule before actual transaction is made.

$$dW/d\tau = \alpha_3(N^d(P_D, P_C, W) - N^s(W)), \qquad \alpha_3 > 0$$

where $N^d = N^d_D + N^d_C = \lambda_D(W/P_D) + \lambda_C(W/P_C)$ and N^s is the inverse function of $W = W(N, t)$. The shift factor t can be adequately ignored in the tatonnement process above.

The auctioneers in the three markets are assumed to complete the adjustment process before actual transactions start. The variable τ should be interpreted as distinct from the ordinary time variable t. τ is a fictitious adjustment time variable of the Hicksian temporary equilibrium, which is considered to have been attained at every point of time t. (Fig. 4)

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1) I wish to thank Prof. S. Akashi, Seijo University, for a suggestion on X^d_C at the meeting of the T. C. E. R., which motivated me to improve the formulation.

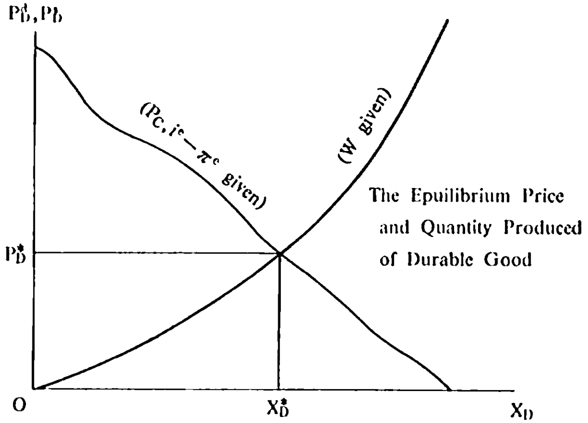

Fig. 4

Theorem 1. Suppose $\bar{i}^e - \bar{\pi}^e > 0$ and $\varphi' < 0$. Suppose, further, that the price elasticity of short-run supply of nondurable consumption good is greater than, or equal to, that of durable good. Then, the price adjustment system of the flow markets is locally stable if and only if

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} > 0, \quad |J| \equiv \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} < 0,$$

where

$$J = \partial(E_D, E_C, E_N)/\partial(P_D, P_C, W),$$

namely the Jacobian matrix of the excess demand functions $E_D = X^d_D - X^s_D$, etc., with respect to the prices, evaluated at an equilibrium price vector.

Lemma. If all commodities in a market system are weak gross substitutes, and if the Metzler matrix pertinent to the market system is indecomposable and acyclic, then the conditions for local stability independent of the speeds of adjustment are identical with the Hicks conditions for perfect stability.

Proof. By the Frobenius Theorems (see Takayama, [12], p. 372 and 375) a non-negative square matrix has its Frobenius root to be positive and unique in the sense that there is no other characteristic root with its modulus equal to that of the Frobenius root (Takayama, [12], p. 378) whenever the matrix is indecomposable and acyclic. It is known that the argument in the proof of Metzler's theorem

(Metzler [7], p. 286–290, especially p. 286–288) holds good exactly as it is if the corresponding Frobenius root (Metzler denotes it as ρ in his Eq. (9) in p. 286 of Metzler [7]) is positive and unique even when some of non-diagonal coefficients (Metzler's a_{ij}) equal zero. Thus, the Lemma is proved as a corollary of the Metzler theorem. Q. E. D.

Proof of Theorem 1. By the assumed inequalities it can easily be seen, in view of $f_j'>0$ etc., that the Jacobian matrix J is an indecomposable and acyclic Metzler matrix. Hence, by Lemma, the system is locally stable irrespective of the speeds of adjustment α_1, α_2, and α_3 (as long as they are all positive), if and only if the Hicksian condition for perfect stability are satisfied. Since $a_{11}=\alpha_1 (\varphi'(\bar{i}^e-\bar{\pi}^e)/P_C+f_D'\lambda_D' W/P_D^2)$ is negative, the second and the third inequalities of the Hicksian conditions are necessary and sufficient for the stability. (*Appendix* treats a detailed acount.) Q. E. D.

Section 2. The Model with the Stock Concepts

1. Simplifying Assumptions For the Aggregation

As the following argument will show, there exists a set of assumptions concerned with elasticities of supply with respect to price and capital and so on under which the two commodities and their prices behave as if they were aggregated into one commodity and one price, without any ambiguities about making indeces.

Let K_j denote real capital stock held by the j-good producers, $j=C, D$.

In the following, the flow-market adjustment is assumed to have been done at every moment.

Assumption 1. The supply function of j-good for each given $\overline{W}$ can be written as

$$X_j=\gamma_j(K_j)\chi_j(P_j/\overline{W}) \qquad j=C, D.$$

Assumption 2. It is assumed that

$$(d\ln\gamma_j(K_j))/(d\ln K_j)=\bar{b} \quad \text{for all } K_j>0,\ j=C, D,$$

where $\bar{b}$ is a constant independent of j. Assumptions 1 and 2 imply that the same rate of capital accumulation is assumed to expand the short-run supply curve of either good proportionally along the horizontal axis, the porportion being constant for all $P_j>0$ and the same for both the goods.

Assumption 3. (*The Elasiticity Condition*)

$$(\partial \ln X_j)/(\partial \ln P_j)=\bar{g}, \qquad j=C, D.$$

Namely the elasticities of the short-run supply curves of the goods are constant and equal.

Assumption 4. The part of the total quantity demanded of durable good for every level of its price which is shared to the firms as against the consumers is always divided in the same proportion between the two sectors as that in which the capital stock (for industrial use) is divided between the two. This assumption implies

$$d \ln K_D = d \ln K_C,$$

where, as repeatedly will appear below, d denotes derivative with respect to time. (Fig. 5)

Theorem 2. Under Assumptions 1 through 4, the rates of change in P_C and P_D are equal, and the rates of change in X_C and X_D are equal.

Proof. By Assumption 1,

$$d \ln X_j = d \ln \gamma_j(K_j) + d \ln \chi_j(P_j/\bar{W}) \qquad j=C, D. \qquad (1)$$

By Assumptions 1 and 3,

$$d \ln \chi_j = \bar{g}(d \ln P_j), \qquad j=C, D. \qquad (2)$$

By Assumption 2 and Eqs. (1) and (2),

$$d \ln X_j = \bar{g}(d \ln P_j) + \bar{b}(d \ln K_j), \qquad j=C, D. \qquad (3)$$

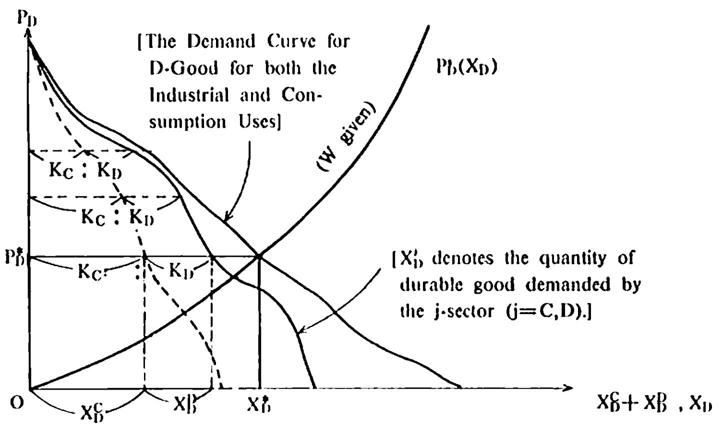

Fig. 5

By $S=\bar{s}Y$, we have $P_D X_D = h P_C X_C$, so that $d\ln P_D + d\ln X_D = d\ln P_C + d\ln X_C$. Hence,

$$d\ln X_D - d\ln X_C = -(d\ln P_D - d\ln P_C).$$

But by Assumption 4 and Eq. (3), we have

$$d\ln X_D - d\ln X_C = \bar{g}(d\ln P_D - d\ln P_C).$$

Since $\bar{g}$ is positive (the positive-slopedness of the short-run supply curves), we have

$$d\ln P_D = d\ln P_C$$
$$d\ln X_D = d\ln X_C. \qquad \text{Q. E. D.}$$

Clearly the same proposition holds good if Assumption 1 is replaced by

$$P_j/\bar{W} = \beta_j(K_j)\eta_j(X_j) \quad j=C, D.$$

and Assumption 3 by $(\partial \ln P_j)/(\partial \ln X_j)=\bar{k}$, $j=C, D$. In this case, Assumption 2 implies that the same rate of capital accumulation is assumed to expand the short-run supply curve of either good proportionally along the *vertical* axis.

An important implication of Theorem 2 can be summarized in the following

Corollary. As long as Assumptions 1 through 4 are fulfilled, the two goods can be treated as if they constituted one commodity, which holds good independently of what curvature the demand schedule for durable good takes.

The equilibrium price of durable good may change either by a shift of the demand curve or by a shift of the supply curve. In either case, the local and global curvature of the demand curve appears to be one of the possible causes of disturbing a proportional balance between P_D and P_C (the equilibrium prices). The above corollary emancipates us from such an anxiety : though the rate of change in price level may change over time, the rates of change of both the prices are always the same, and, moreover, the rates of change of quantities of the goods produced are always the same, which holds regardless of the shape of the demand curve for durable good.

Assumption 5.

$$F_j(0, K_j)=0, \quad j=C, D.$$

Theorem 3. Under Assumptions 3 and 5, the short-run production

function are of the Cobb-Douglas type with respect to labor.
Proof. Let us omit subscript j for brevity in this proof. The two basic relations are $f'(N)=\bar{W}/P$ and $X=f(N)$. The situation we are concerned here is that in the short-run. Let us denote the inverse function of $f(\cdot)$ by $N=N(X)$, and $f'(N(X))$ by $h(X)$, which denotes the marginal product of labor when the short-run production level equals X. In view of $f'(N)=\bar{W}/P$, we have, by Assumption 3, $(d\ln h(X))/(d\ln X)=\bar{z}$, where $\bar{z}=-(1/\bar{g})$. This implies $h'(X)/h(X)=\bar{z}/X$, so that we have $\ln h(X)=\bar{z}(\ln X)+A_0$, where A_0 is a constant of integration. We have $h(X)=AX^{\bar{z}}$. Since $h(X)=f'(N(X))$, we have $1=dX/dX=d(f(N(X))/dX=f'(N(X))N'(X)=h(X)N'(X)$. Hence $N'(X)=(h(X))^{-1}=A^{-1}X^{-\bar{z}}$. Assumption 5 implies that $N(X)=BX^{1-\bar{z}}$, where B is a positive constant. Hence, we have $X=A_1N^{(1/(1-\bar{z}))}$, where A_1 is a positive constant. Q. E. D.
Corollary. Under Assumptions 3 and 5, the elasticities of demand for labor with respect to the levels of production of C- and D-goods are the same in the short-run. Under Assumptions 1 through 5, therefore, the rates of change in demand for labor of both the sectors are equal.
Theorem 4. Under Assumptions 3 and 5, the relative shares of labor in both the sectors equal $\bar{g}/(1+\bar{g})$.
Proof. By the proof of Theorem 3 we have $X_j=A_{1j}N_j^{(\bar{g}/(1+\bar{g}))}$, $j=C, D$, so that the relative share of labor in the j-sector equals $\bar{W}N_j/(P_jX_j)=\bar{g}/(1+\bar{g})$. Q. E. D.

2. *Compound Interest and the Exponential Function: A Digression*

'Monetary claim' may be an uncountable word just like money or liquidity. This word is introduced here to replace the word 'bonds.' Monetary claim is the claim to a certain amount of money. The value of a quantity of monetary claim equals the value of the money to which its holders have claim. The quantity of monetary claim is measured by the quantity of money to which its holders have claim. Thus, the quantity of monetary claim and its nominal value coincide with each other.

In the following, a fundamental consideration about multiplicative increase of monetary claim will be examined. As formulated above, the demand price of durable good is determined as the value of monetary claim which is equivalent to the marginal unit of durable good with respect to the present value of expected flow of returns. The following is an analysis of multiplicative increases of monetary claim. For the sake of analytical simplicity, the continuous (as against discrete) model of monetary claim and interest is adopted.

Let V_0 denote the nominal value of a quantity of monetary claim at time 0, and i the nominal rate of interest. The value of the total of the principal and interest at compound interest at time t is denoted by $V(t)$. Then, in this continuous scheme, it holds that $dV(t)=(idt)V(t)$, which expresses that the value of claim is added by $V(t)$, or the value of claim at time t, *times* idt or the rate of interest for the infinitesimal period of time dt starting from time t. This holds for all time t. Hence, $V'(t)=iV(t)$ for all $t>0$. The solution of this differential equation is $V(t)=V_0 \exp(it)$. This proves that: *if the rate of interest is constant for all* $t \geqq 0$, then the total of principal and interest at compound interest equals the exponential function with initial value to be the value of the claim at $t=0$.

It might be interesting to point out as follows that there exists an argument in terms of calculation at *simple* interest by which the total of principal and interest at *compound* interest is shown to equal $1+\sum_{k=1}^{\infty}(it)^k/(k!)$ *times* V_0, dispensing with the above argument in terms of the differential equation and without using the expansion formula for the function e^x. The argument is as follows.

For simplicity suppose $V_0=1$. This value of claim produces interest *on this initial value* at the rate of i per unit of time: the interest *on this initial value only* is a linear function of time t, *i.e.*, equals ti. (Example: if $t=0.32$ year and $i=6.0\%$/year, then the simple interest on the principal 1 yen equals $(0.32)(0.060)(1)=0.0192$ (yen).) Put $J_1(t)=ti$, defined for all t, $t \geqq 0$ with i constant.

Fig. 6 depicts the function $J_1(t)$ for all $t \geqq 0$. Now, fix arbitrarily time $t_1(>0)$. Take any t such that $0 \leqq t \leqq t_1$. Then, there is some small piece of interest which was produced by the principal 1 during some infinitesimal period of time starting at each time t. This small piece of interest can be written as $(dJ_1(t)/dt)dt=idt$. This piece of interest at time t has been producing interest at simple interest since time t until time t_1 when we now are. The accumulated simple interest on this piece of $J_1(t_1)$ equals $[(t_1-t)i](idt)$. This formula holds generally for all t, $0 \leqq t \leqq t_1$. The integral of this differential expression from $t=0$ to $t=t_1$ will denote the

Fig. 6

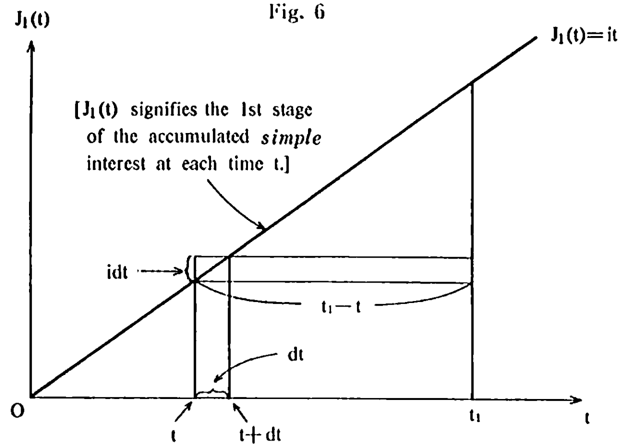

Fig. 6: The infinitesimal piece idt at time t which is a part of $J_1(t_1)$ has been producing simple interest $(idt)i$ per unit of time since t until t_1. The piece has accumulated the simple interest by $(t_1-t)((idt)i)$. We integrate this from $t=0$ to $t=t_1$, to obtain $J_2(t_1)$.

total of simple interest on every infinitesimal piece of $J_1(t_1)$ accumulated since the birth of that piece until time t. This integral is denoted by $J_2(t_1)$. Clearly $J_2(t)$ is defined for all t, $t \geqq 0$, and is seen to be an increasing function of t. Just as the function $J_2(t_1)$ is derived from the function $J_1(t)$, we can derive the function $J_3(t_1)$, say, which signifies the total of simple interest on every infinitesimal piece of $J_2(t_1)$ accumulated since the birth of that piece until time t_1. It is seen to hold that

$$J_3(t_1)=\int_0^{t_1}[(t_1-t)i](dJ_2(t)/dt)dt. \qquad (4)$$

Similarly the functions $J_4(t)$, $J_5(t)$, etc. will be derived recursively by the formula

$$J_{k+1}(t_1)=\int_0^{t_1}[(t_1-t)i](dJ_k(t)/dt)dt, \qquad (5)$$

for $k=1,2,3,\ldots$ etc. For the economic meaning of (5), see the legend for Fig. 7.

Since

$$J_2(t_1)=\int_0^{t_1}[(t_1-t)i](idt)=i^2(t_1^2/2!),$$

we have the general formula

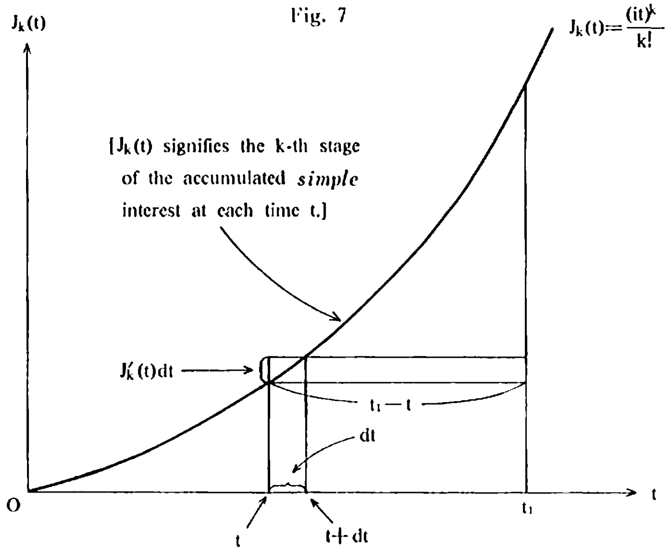

Fig. 7: The infinitesimal piece $J_k'(t)dt$ which is a part of $J_k(t_1)$ has been producing simple interest $(J_k'(t)dt)i$ per unit of time since t until t_1. The piece has accumulated the simple interest by (t_1-t) $((J_k'(t)dt)i)$. We integrate this from $t=0$ to $t=t_1$, to obtain $J_{k+1}(t_1)$.

$$J_k(t)=(it)^k/k!, \quad k=1, 2, 3, \dots, \text{ etc.} \tag{6}$$

via the equation

$$\int_0^{t_1}(t_1-t)(t^j/j!)dt=t_1^{j+2}/(j+2)! \tag{7}$$

The total of 1, $J_1(t_1)$, $J_2(t_1)$,..., etc. represents the total of principal and interest which comprises all repercussions of simple interest on any interest produced before time t_1, each of which has been entirely added to the stock of monetary claim (Fig. 8). This total equals

$$1+it+(it)^2/2!+(it)^3/3!+\dots\text{etc.}$$

and the k-th term of this series has that meaning, represented by (5).

It can be easily found that the above line of reasoning applies also to the case where the future rate of interest is not constant but may change over time, or can be written as a function $i(t)$ of time t. It is well known that the total compound interest on unit of the principal in this case may be expressed by $\sum_{k=1}^{\infty}I_k(t)$, where

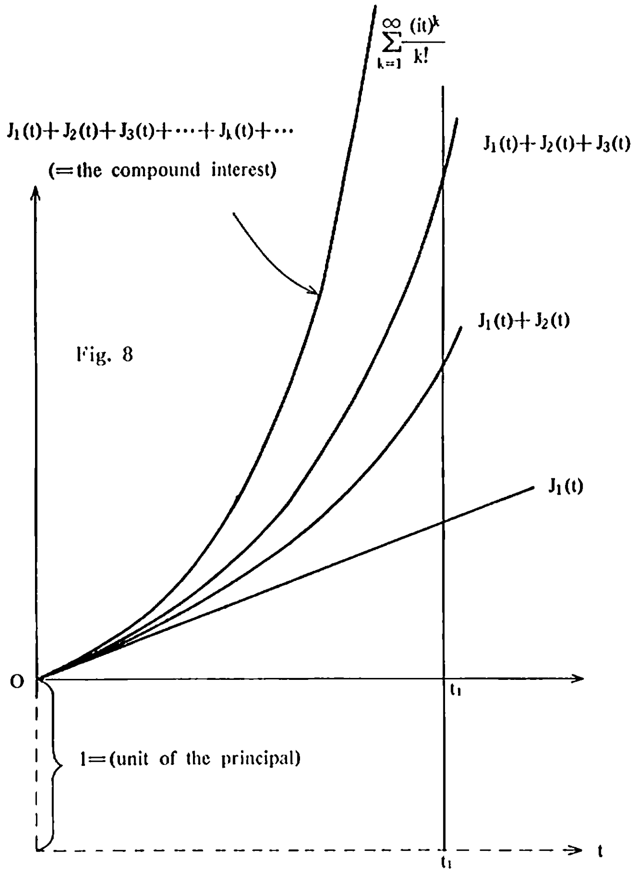

Fig. 8: The total of all the stages of the accumulated simple interest equals $(\exp(it))-1$ at time t_1, because the k-th stage of it is calculated to equal $(it_1)^k/(k!)$ (see the text.) Thus, the k-th term of the expanded series of $\exp(it_1)$ (which can be shown to measure the total of principal and interest at ***compound*** interest via the alternative route) may be interpreted as the k-th stage of the accumulated ***simple*** interest.

$$I_k(t)=\frac{1}{k!}\left(\int_0^t i(\tau)d\tau\right)^k. \tag{8}$$

What might be interesting is that it follows from the argument that each term of the above series allows a direct interpretation similar to that of the term $(it)^k/(k!)$ in the previous case. *Proof:* The term for $k=1$ signifies the total of the (first-stage) simple interest on the unit principal, that is, $I_1=\int_0^t i(\tau)d\tau$. Suppose, for some $k=1,2,\ldots$, that the equation (8) holds. Suppose, further, that $I_{k+1}(t)$ is in the following relation to $I_k(t)$, the relation which permits the economic interpretation similar to that of Eq. (5):-

$$I_{k+1}(t_1)=\int_0^{t_1}\left(\int_t^{t_1}i(\tau)d\tau\right)I'_k(t)dt. \qquad (9)$$

Integrating by parts, since $I_k(0)=0$, we have

$$I_{k+1}(t_1)=\left[\left(\int_t^{t_1}i(\tau)d\tau\right)I_k(t)\right]_0^{t_1}-\int_0^{t_1}-i(t)I_k(t)dt$$

$$=\int_0^{t_1}\left[\frac{\left(\int_0^t i(\tau)d\tau\right)^{k+1}}{(k+1)!}\right]'dt$$

$$=\frac{\left(\int_0^{t_1}i(\tau)d\tau\right)^{k+1}}{(k+1)!}.$$

Hence, Eq. (8) holds good for all $k=1,2,\ldots$ under the assumptions. This means that the expression on the right-hand side of Eq. (8) allows, for any $k=1,2,\ldots$, the recursive economic interpretation indicated by Eq. (9).

The economic interpretation of the terms of the series might be interesting as analogous to that of the terms of the Kahn-Keynes income multiplier series, to that of the terms of the classical series of multiplicative credit creation, etc.

3. *The Effects of Inflation on the Nominal and Real Rates of Interest*

Why Incremental Capital Assets and Stock Money Supply in the General Theory?

Why does capital assets appear in the incremental variable, i. e., in investment in *the short-run* equilibrium conditions, whereas the other kind of accumulated assets, collectively called 'money,' appears there as a stock variable? The fixity of capital assets explains the reason for it—this is widely known and understood. Here, I should like to enlarge the explanation for a deeper understanding of such a way of Keynes' short-run formulation. The point is that it is in comparison with that of money that the fixity of capital assets enters the consideration behind the formulation.

For example, unlike materials or fuels, a machine used in a manufacturing factory will endure physically and functionally for such a long time as 20 years. This means that even after 5 years passed the machine will still work, and even then, the machine will not have completed the long process of taking back the principal of the money invested in itself. Materials and fuels are consumed and totally transformed into the products which are being made by use of them, but capital assets is not : it remains, so that it strongly tends to be accumulated. In other words, the period of the taking-back of the fund is much longer in the case of manufacturing industries' long-term investment in machines and capital assets, than in the case of trading and commercial industries' 'short-term investment' in (or temporary purchase of) general goods, including investment good or machines.

It is here that money is compared with capital assets with respect to the short-run modelling, i. e., stock versus flow. First, the stock of money, once invested in a purchase of commodities to resell, will soon be taken back on finding demanders of them and selling them. But the fund for the stock of capital assets, on the other hand, once invested in factories for instance, will take much time to complete the taking-back process or program of it. This means that money soon comes back in the total sense or in the stock sense, whereas the fund for capital assets does not soon come back in that sense. Second, money changes its owner from one economic agent to another in its entirety, whereas capital assets changes its owner not much in its entirety, but mainly only partly, in the additional or replacing sense. For those two interrelated reasons capital assets does not appear as a stock variable in the short-run equilibrium conditions or equations. Especially, money changes its owner in its entirety in the short-run, so that it appears as a stock variable in the short-run equilibrium conditions. It does so, because it comes back to the traders soon or in the short-run, who pay it out to purchase things and sell them with trading margins. Money appears in its totality thus in the short-run picture of the macroeconomic transactions.

The Effects of Inflation on the Nominal and Real Rates of Interest

Monetary claim increases at compound interest if its interest continues to be added to the principal. Money does not bear any interest and it is barren in that sense. Money has liquidity, whose benefit balances with the interest of monetary claims. A model of a schedule of the liquidity premium for successive marginal units of money is depicted in Fig. 9. This schedule is downward-sloping and it is assumed in this paper that the curve expands along the money-axis in the same proportion as the nominal rate of production (Y) changes. (See Fig. 10) Behind this there is made the following

Assumption 7-1: The demand for money is proportional to the nominal rate of monetary transaction: the quantity of such a portion of money that bears each level of the rate of liquidity service is assumed to be doubled (say) if the rate of monetary transaction doubles.

Hence, the formulation $M^d = L(i)Y$ (p. 4). However, more fundamentally, there is placed the following assumption behind the formulation of the function $L(i)$ itself.

Assumption 7-2: Any change in the expected rate of inflation (π^e) does not influence the people's asset-choice between money and monetary claims, so that the rate of interest which is relevant to the asset-choice between money and monetary claim is the *nominal* rate i, as against the real rate $i-\pi^e$.

This assumption stems from the viewpoint about the motive to liquidity as follows: people want to hold money mainly only for short time in the sense that the stock of money they hold will soon

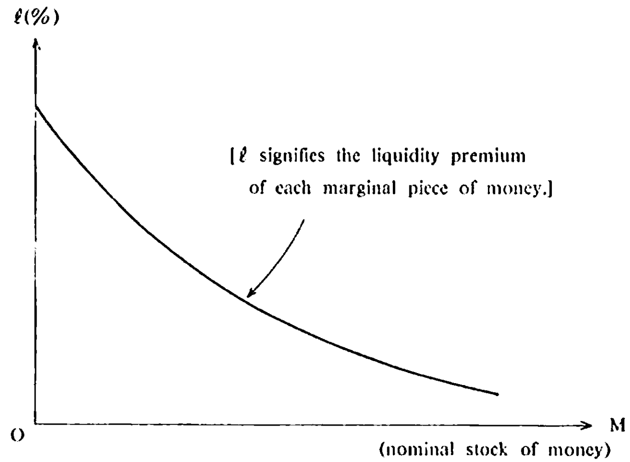

Fig. 9

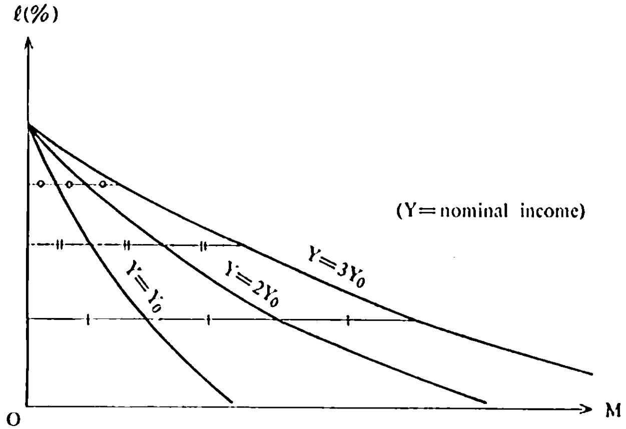

Fig. 10

be paid out in transaction and be replaced by the money that they receive in transaction. In other words the turnover ratio of the stock of money is high as compared with other kinds of asset. It follows that the price people compare with the liquidity premium of money is the short-term rate of interest rather than the long-term rate : people try to know how much an amount of monetary claim will earn as interest in one or two years rather than five or ten when determining the asset-balance between money and monetary claim. This implies that the interest they question in this case is interest in one or two years rather than the compound interest to be earned in five or ten years : The simple interest will do in this context.

This will be more easily understood in the case of commercial and business motives to liquidity : the stock of money the traders and firms hold for commercial and transaction purposes is a *short-term* revolving fund, in contrast to the fund for machines and equipments purchased by them for investment purposes which usually takes long time to take back the whole of itself. Liquidity premium includes the commercial margins which the traders earn by paying the stock of money to buy commodities and by taking it back with margins and profits *in the short-run*. They purchase commodities from the producers to sell them to the customers in the short-run, and earn the commercial margins or profits. The liquidity premium reflects the ratio of such a commercial margin to the revolving fund of money. They will compare, therefore, the liquidity premium with the *short-term* rate of interest when they consider how much money they hold in balance with monetary claim that bears interest. Here, the more relevant interest is the simple interest rather than the compound interest in the comparison.

Here enters the factor of inflation. Suppose inflation is expected to raise prices at 6% per year for one or two years ahead. What difference happens in the asset-choice between money and monetary claim, as compared with the case when the expected inflation is 4% instead of 6? For simplicity suppose further that the nominal rate of interest is constant at 8% in both the cases. The effect of the change in the expected rate of inflation on the choice between money and monetary claim is this: Disregarding the liquidity benefit and the interest of them, it *raises* the expected *real* values *after one or two years,* of unit of the *present* money and of unit of the *present* monetary claim *by the same percentage,* that is, by 2%(=6−4) per year which is the change in the expected rate of inflation. Hence, apart from difference between the liquidity premium of a marginal unit of money and the nominal rate of interest of a marginal unit of monetary claim, the difference in stock value between the two kinds of asset will be unchanged.

Let us denote the liquidity premium by l (%) and the expected rate of inflation by π^e (%). Then if the nominal rate of interest is i (%), the real benefits of money and monetary claim after one year will equal $l-\pi^e$ and $i-\pi^e$, respectively. The difference equals $l-i$, which is independent of π^e. Hence the formulation of $L(i)$ independent of π^e.

Let us now return to the equation $M^d=L(i)Y$. Since $Y=Py$, inflation implies the expansion of the nominal money demand curve along the money-axis in proportion to the rate of inflation (Fig. 10). But this expansion will be cancelled out by the proportional shift of the vertical nominal money supply line, which, in our formulation, is naturally assumed to shift at the inflation rate when the inflation is a long-run steady one (since $M^s=\bar{M}W$ and W/P is constant in a steady state.) Thus, the expansion of the nominal money demand curve and the shift of the nominal supply line in this case cause no effect on the equilibrium *nominal* rate of interest. (The comparison of equilibria is between two ideally similar economic states which do not differ except the expected rate of inflation, a thought experiment.)

As long as $\bar{M}$ is constant and $L(i)$ does not shift, inflation does not shift the real LM curve (Fig. 12). However, inflation shifts the real IS curve upward, as explained in the next section (Fig. 12). Hence the equilibrium levels of the real variables of the system are affected by a change in π^e. The equilibrium rates of real and nominal purchase of durable good (X_D and D) are raised by inflation, as well as the equilibrium nominal rate of interest is. As long as the durable-good demand function does not shift autonomously in addition to its shift caused by the change in π^e, the equilibrium *real* rate of interest *decreases.*

4. *The Aggregate Commodity and Stability of the Interest-Rate Adjustments*

All the assumptions in Subsection 1 ensure that any movements or changes in X_C, P_C, N_C, and K_C are proportionally exactly the same as those in X_D, P_D, N_D, and K_D, resp. The initial proportions of X_C, P_C, N_C, and K_C to X_D, P_D, N_D, and K_D, resp. are preserved in the strict sense. This is the case both in the short and long-run.

An important corollary to this result is that we can separate in the very simple way of multiplication the changes in prices and those in real quantities. Once units of the two commodities are specified the aggregate commodity can be defined in one of the transparent ways: the aggregate good may be measured either in terms of C-good or D-good. Correspondingly, the general price level becomes to be measured in terms of money, with the initial level equal to that of the price of C-good or D-good, respectively. In either case the nominal total production Y can be written as Py. Once which of these two ways of defining the common measure of goods is determined, the initial values of the aggregate production y and the general price level P are uniquely determined. The changes in those aggregate variables are proportional to those of X_D and P_D, respectively.

From Theorem 3 we know that N_C preserves the same proportion to N_D, since X_C does so to X_D and since, by Assumption 4, K_C preserves the same proportion to K_D.

Since y/X_D is constant, we have $y = A_1 X_D$ as an identity, and $X_D = X_D(i^e - \bar{\pi}^e)$. The IS curve is described by those relations. The LM curve in a simple form can be derived as follows. The equation of demand and supply of money is $L(i)(P_C X_C + P_D X_D) = \overline{M} W^*(N)$. In the short-run both the sectors have $W^*(N)/P_j = f_j'(N_j)$ fulfilled, so that $f_C'(N_C) = (P_D/P_C) f_D'(N_D)$. If the unit of goods is determined

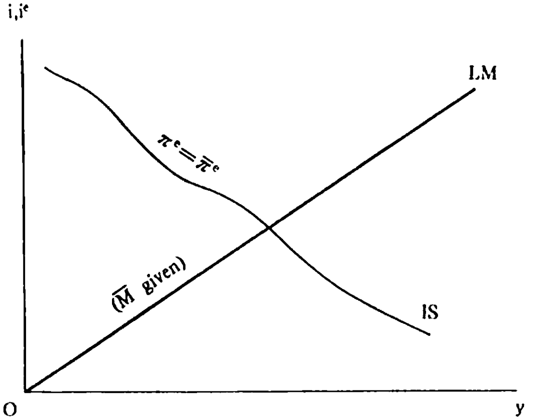

Fig. 11

in terms of consumption good, then $y = X_C + (P_D/P_C)X_D$ and $P = P_C$. We have, therefore, $L(i)(X_C + (P_D/P_C)X_D) = \bar{M}(P_D/P_C)f_D'(N_D)$. Since P_D/P_C and X_D/X_C are constant, we may write $L(i)A_2X_D = \bar{M}f_D'$ (N_D), where A_2 is a positive constant. By putting $h_D(X_D) = f_D'(f_D^{-1}$ $(X_D))$, the LM equation is derived by $L(i)A_2X_D = \bar{M}h_D(X_D)$ together with $y = A_1X_D$. The IS and LM equations are depicted in Fig. 11.

We have to distinguish i and i^e, that is, the actual and expected rates of interest. It is the expected rate that matters in the demand for durable good, and the expected rate does not necessarily move always keeping equality to the actual rate. The expected rate may be rather sticky in some cases in face of a substantial movement of the actual rate. But it seems that the adaptive expectation hypothesis as follows is an impeccable assumption as the first approximation. Namely

$$\dot{i}^e = \xi \cdot (i - i^e), \qquad \xi > 0, \qquad (4)$$

where $\dot{i}^e$ denotes the derivative of i^e by time t, as distinct from τ which is pertinent only to the temporary equilibrium flow-price adjustments. As remarked in Page 11, the temporary price adjustments is considered to have been finished at every point of time t. Correspondingly total real production y is assumed to keep its relation to i^e of the form $y = A_1X_D(i^e - \bar{\pi}^e)$ at every t in the short-run.

The actual rate of interest, on the other hand, is of course governed by the market powers in the money market. The actual rate is assumed to be adjusted by the transaction-out-of-equilibrium rule (as for this term, see Arrow and Hahn [1]) as follows.

$$\dot{i}=\mu\cdot(L(i)A_3y(i^e-\bar{\pi}^e)-\bar{M}h_D(y(i^e-\bar{\pi}^e)/A_1)),\qquad \mu>0, \qquad (5)$$

where A_1 and $A_3=A_2/A_1$ are positive constant and $y(\cdot)=A_1X_D(\cdot)$. It is assumed to be the demand, that is, $L(i)A_3y(i^e-\bar{\pi}^e)$, which is actually held by the public. The supply $\bar{M}h_D(y/A_1)$ is regarded here as the indicative supply of money, which, by influencing the actual rate of interest, pulls the quantity demanded of money towards itself.

Such a system of (4) and (5) ignores a change in the stock of capital. It is assumed that the adjustments of (4) and (5) are almost completed in the short-run.

Theorem 5. The short-run transaction-out-of-equilibrium adjustment mechanism which consists of Eqs. (4) and (5) is locally stable irrespective of the magnitudes of the speeds of adjustment ξ and μ.

Proof. Let the Jacobian matrix of the functions on the right-hand sides of Eqs. (4) and (5) with respect to i^e and i, evaluated at an equilibrium, be denoted by T. Then, by the sign conditions of the elements of T, it is easily seen that the characteristic polinomial of T has all its coefficients positive. Since this polinomial is of order two, it follows, by the well-known theorem (see e. g. [11]), that the differential system is locally stable. Q. E. D.

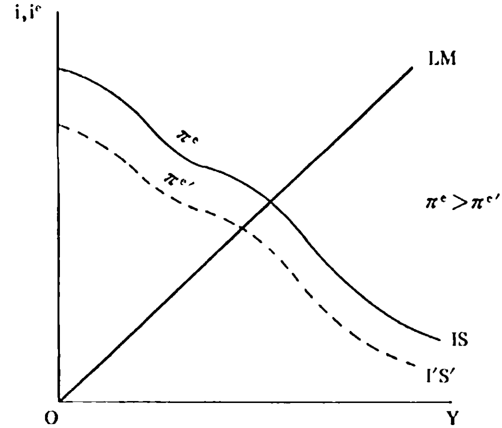

Fig. 12

Conclusion of Chapter I

The Two Kinds of the Phillips Relation

The short-run Phillips relation should be clearly distinguished from the first kind of the Phillips relation. The short-run Phillips relation is the positive relation between the actual rate of inflation and employment. Suppose there happens an upward shift of demand curve for durable good. Then the production of D-good, and hence of C-good, increase. Since the real wage-rate changes to the opposite direction to that of the change in production, the rate of change in the price level exceeds that in the nominal wage-rate. Hence an upward shift of the demand curve causes an inflationary pressure to the price level. Similarly for a downward shift. But the shift of the demand curve for durable good may be caused by a change in the expectation of the future rate of inflation, or may be caused by other reasons.

On the other hand, the first kind of the Phillips relation is related to the *expectation* of the rate of inflation. Two experimental situations may be imagined : one in which the rate of inflation has been kept low so that the expected rate of future inflation is low, and the other in which the rate of inflation has been high so that the expected rate of future inflation is high. One situated in the former will find, *ceteris paribus*, a lower equilibrium level of producion than when situated in the latter, because the IS curve is positioned lower in the former case than in the latter, so that the intersection point of the IS and LM curves shows a lower level of equilibrium production. (Fig. 12)

The Friedman argument of the "vertical long-run Phillips curve" in [4] can be reexamined in view of the Keynes-type model developed in this paper. Friedman's argument does not make the distinction between the nominal rate of interest entering the liquidity-preference function and the real rate of interest entering the durable-good demand function (Mundell, [8]). Money is a substitute for monetary claim (i.e., bonds, etc.) as well as for capital. Inflation has the tendency to make real capital regarded as more valuable relative to money and bonds (and other kinds of monetary claim) than in the case without it. This tendency of inflation seems to be underestimated by Friedman.

Appendix to Section 1.

The Sign Conditions of the Jacobian Matrix J

The differential system in Section 1 is written in full as follows.

$$dP_D/d\tau = \alpha_1(\varphi((P_D/P_C)(\bar{i}^e - \bar{\pi}^e)) - f_D(\lambda_D(W/P_D)))$$
$$dP_C/d\tau = \alpha_2(\bar{c}(P_D f_D(\lambda_D(W/P_D)) + P_C f_C(\lambda_C(W/P_C)))/P_C) - f_C(\lambda_C(W/P_C)))$$
$$dW/d\tau = \alpha_3(\lambda_D(W/P_D) + \lambda_C(W/P_C) - N^S(W)).$$

Its Jacobian $J = (a_{ij})$ evaluated at an equilibrium can be calculated as follows. First,

$$a_{11} = \alpha_1((\bar{i}^e - \bar{\pi}^e)\varphi'/P_C + f'_D\lambda'_D W/P_D^2), \quad a_{12} = -\alpha_1(\bar{i}^e - \bar{\pi}^e)\varphi'(P_D/P_C^2),$$
$$a_{21} = \alpha_2\bar{c}(f_D - Wf'_D\lambda'_D/P_D),$$
$$a_{31} = -\alpha_3\lambda'_D/P_D^2,\ a_{32} = -\alpha_3\lambda'_C/P_C^2,\ a_{33} = \alpha_3(\lambda'_D/P_D + \lambda'_C/P_C - (N^S)')$$
and $a_{13} = -\alpha_1 f'_D\lambda'_D/P_D$.

By assumption, $\bar{i}^e - \bar{\pi}^e > 0$, $\varphi' < 0$, $f'_j > 0$, $\lambda'_j < 0$, $j = C, D$, and all other symbols are positive. Hence all the above coefficients a_{ij} fulfil the sign conditions for a Metzler matrix. Second, $a_{22} = \alpha_2\partial(\bar{c}(P_D f_D + P_C f_C)/P_C - f_C)/\partial P_C = \alpha_2\partial(\bar{s}((\bar{c}/\bar{s})P_D f_D/P_C - f_C)/\partial P_C$ which is obviously negative. Finally, as for the remaining sign condition for a_{23}, it is to be seen that the assumption of the theorem that the price elasticity of short-run supply of C-good is greater than or equal to that of D-good implies that the wage-rate elasticity of short-run supply of C-good is greater than or equal to that of D-good in absolute value. Mathematically this is an easy consequence of homogeneity of the short-run supply functions with respect to P_j and W. On the other hand, since the Jacobian matrix is evaluated at an equilibrium, we have $(\bar{c}/\bar{s})P_D f_D/P_C - f_C = 0$. Therefore by the above wage-rate elasticity condition we have

$$a_{23} = \alpha_3\partial(\bar{s}((\bar{c}/\bar{s})P_D f_D/P_C - f_C))/\partial W \gtrless 0.$$

Thus the sign pattern of the matrix J can be indicated by

$$\begin{bmatrix} - & + & + \\ + & - & 0 \\ + & + & - \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} - & + & + \\ + & - & + \\ + & + & - \end{bmatrix}.$$

The matrix with the former sign pattern is indecomposable and acyclic as well as that with the latter sign pattern. In either case, the Metzler theorem is applicable. Hence the conclusion of

Theorem 1.

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