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The following symbols will be used below.

K=(the capital assets of the firms)
N=(the effective work employed)
Y=(the total production)
M_f=(the total money balance of the firms)
$F(N, K)$=(the production function)
I=(investment)
S=(savings)
$\bar{i}$=(the rate of interest)
$\bar{d}$=(the ratio of depreciation to the capital assets)
(Those with bars are constant.)

Other symbols will be introduced in the text.

Introduction

The *LM-IS* scheme formulated by Hicks [3], which is the simplest version of Keynes' static macro-economic model, contains two independent variables and consists of the two equilibrium conditions, i. e., the investment-saving equation (the *IS* equation) and the demand-supply equation of money (the *LM* equation.) In symbols, $I(i)=S(Y)$, and $L(Y, i)=M$, where Y and i are the total production and the rate of interest, I, S and L are investment, savings and money demand functions resp., and M is the given money supply.

In real life, however, the ***kotei-buai*** (the short-term rate of interest that the central bank can more or less arbitrarily change) is a policy variable rather than an endogenous which the system determines. It has been suspected that M is not really a directly controllable variable, that what is directly changed by the monetary authority is the ***kotei-buai*** rather than M and that M is changed mainly through the policy of changing the rate of interest. As long as this suspicion is realistic, the Hicksian view of regarding M as a given or exogenous policy variable will become unfounded. (Inapplicability of this Hicksian *IS-LM* model of Keynes' Theory to Japan's economy has also been pointed out e.g. by Y. Suzuki in [11], p. 75–78.)

In this paper, this gap between the Keynes model and the actual working of the monetary economy will be filled by an alternative model. In the model introduced in this paper, M is an adjustment variable, and *not* a given constant. Instead of regarding M as given, I regard the rate of interest i as a given constant, or a policy variable. The rate of interest will *not* be viewed here as an adjustment variable. (Suzuki's criticism of the inapplicability of Keynes' theory to Japan will thus come to be avoidable.)

If M comes to be viewed as an adjustment variable just like the level of production, it becomes an *indirectly* controlled variable just like Y, being much affected by the interest rate policy. The behavior of the point (Y, M) will be governed and turned to a direction by that policy. In fact the policy of changing the *kotei-buai* is executed mainly for influencing Y and M, (as well as coping with inflation which will be assumed away in this paper.) Thus, the causal ordering we hypothesize is $i \rightarrow$[the locus of (Y, M)].

Behind the Keynesian causal scheme $M \rightarrow i \rightarrow Y$, or especially the link $i \rightarrow Y$ which has been popularized by Samuelson [8], it is assumed that Y is determined by investment I, which is determined by i and the schedule of the marginal efficiency of capital. Is the Keynesian scheme $M \rightarrow i \rightarrow Y$ really pursuasive in the light of empirical observations? As noted above the view of regarding M as a policy variable is unrealistic at least for Japan's economy.

It is assumed in my previous paper that the main agent affecting the macro-economic conditions of a country is the firms, and the empirical investigations in this paper are concerned with the relations among the time series of the firms' turnover ratio (=the net sales/the total assets), their (money/the total assets) ratio and their (gross profit/the total assets) ratio and the *kotei-buai*. The starting point of the research is the basic model in my previous paper, and the new model with M as an adjustment variable is built as a theoretical outcome of those empirical observations.

In the new framework for the analysis of the interest rate policy, there will be assumed the ***horizontal*** money supply curve instead of the vertical.

The Firms' Liquidity-Maximization and the Marginal Efficiency of Capital Assets

The firms' final purpose is to keep themselves, feed themselves and grow. They try to maximize their own total value. They try to do so initially by borrowing liquidity (or money) to the maximum:—this is the central idea of the model of my previous paper. Behind this hypothesis it is assumed that they have sufficient opportunities to use their liquidity at hand to obtain capital gain or to earn returns on investment it finances. In order to borrow stock of liquidity from the banks and the individuals, however, they have to be capable to pay interest on it. Therefore they have to currently earn as much profit as possible to pay interest on liquidity. The profit maximization can thus be derived from the hypothesis of the firms' liquidity-maximization.

For what do the firms invest and grow except for animal spirits? —It is certain that they cannot much extend their current capacity of earning profit without investment or growth. Do they try to invest in order to add to the profit they earn? If so, for what do they try to increase profit?

Now the first thing needed when they try to invest is liquidity. They need liquidity before they invest. Therefore, if the firms try to invest in order to invest further, they need *the* liquidity to invest *further*. So, let us assume that they try to maximize liquidity *of* the next period (or at some future point of time), as well as *of* the present, instead of assuming that they try to maximize their present or future investment.

Even if they want to invest, they cannot do so without liquidity. But if only they have enough liquidity, they can invest whenever they want. Thus the liquidity-maximization comes before the investment-maximization. But if they invest little at present, they will not be able to increase profit in the next period and therefore to increase their liquidity stock. On the other hand if they try to maximize investment in the present period without any allowance for future liquidity, or to increase investment with little money left, they will be in double trouble in the next period: they would lack enough money at hand, so that they would not be able to speculate or invest in the next period as freely as they can in the first period, and the marginal efficiency of the maximized investment in the first period will be too low to earn enough additional profit on it to pay interest on the additional liquidity they want to borrow in the next period.

In order to avoid this dilemma, they had better obey at present the criterion of the maximization *of the liquidity of the next period* rather than that of the maximization of the investment at present. Let us assume, therefore, that the firms follow this former criterion, as well as that of the maximization *of the liquidity at present.*

Now in order to maximize liquidity of the next period, they only have to maximize the *additional* liquidity from the present to the next period, since the liquidity at present has already been maximized by the other criterion. Just as they add investment to their stock of capital assets (fixed and inventory assets), they try to add as much as possible to their stock of monetary assets from the present to the next period.

In order to add to the liquidity stock, they have to borrow the additional liquidity in the next period, and pay interest on it from their additional profit in the next period. However they also have to pay interest in the next period on the investment done in the present period, also from their additional profit in the next period. Therefore, by the hypothesis of the firms' maximization of the liquidity *of the next period*, they will determine investment so as to maximize [(the profit in the next period *less* the profit in the present period) *less* (the interest payment in the next period on the fund which finance the investment from the present to the next period)], which equals [(the increase in profit from the present to the next period which is expectedly caused by the investment from the present to the next period) *less* (the financial cost of the investment from the present to the next period)].

In terms of the concept of the schedule of the marginal efficiency of capital (the M. E. C.), this equals (the area of the trapezoid below the schedule of the M. E. C.) *less* (the area of the rectangle, or the level of investment multiplied by the M. E. C. at the level of investment), which equals (the Marshallian surplus area below the schedule of the M. E. C.) !

Thus, the liquidity in the next period is maximized if and only if the Marshallian triangular surplus area below the M. E. C. curve is maximized.

Summary and Conclusions

(1) *The True and Full Return on the Firms' Total Assets is the Gross Profit*

How should the *full* return on the firms' *total* assets (the sum of the monetary and capital assets) be defined? Are there any statistically calculated measures which appropriately represent it? Such a measure of the return must include the non-operating incomes along with the operating profit, since the former must be regarded as returns on items which constitute a part of the firms' total assets.

Such a measure of the full return on the total assets must be able to be considered equal to the full *cost of "capital", or of the sum of the liabilities and the net worth.* It should be substantially greater than the so called net profit, because it must also comprise the cost of capital on the liabilities, or the so-called financial charges, along with the net profit.

For both of these reasons the known concept *"the gross profit"* in the firms' profit and loss accounts comes to be closed up.

(2) *The Numerical Closeness of the Rate of interest and the Gross Profit Rate*

The gross profit rate, or (the gross profit)/(the total assets) ratio, deserves statistical comparison with the rates of interest. The gross profit rates of all industries and of the trading industry are remarkably close to the long- and the short-term rates of interest, resp., from 1953 to 1977. (See Fig. 1, P. 14 and Fig. 3, P. 18, resp.) This result is of much interest in that it seems to sustain statistically the following equation implied by the hypothesis of the firms' liquidity-maximization in my paper [6],

$$\frac{(F_K(N(Y),\ K)-d)K}{K+M_f}=i.$$

(3) *An Empirical Study Supporting the Liquidity-Maximization Hypothesis*

Important consequences of the formulation in my previous paper [6] are: that the interest-elasticity of the demand for money is *always* greater than one, and that the elasticity is the greater for *the higher* rate of interest. If the latter consequence is right, Keynes' so called "liquidity trap" hypothesis will be denied, because, according to that thesis, it is supposed that the demand for money is more elastic for *the lower* rate of interest.

However an empirical support has been given by Barth, Kraft and Kraft [1] for both of these two anticipations. Their research seems to prove the above two points at least for the data from 1951 to 1970, though concerning with American economy.

(4) *Rebuilding Investment Theory on the Liquidity-Maximization*

Hypothesis

Keynes' investment theory maintaining that the level of investment is determined by the marginal efficiency of capital and the rate of interest is one of the central theses of his general theory along with the theory of liquidity preference. I tried to theoretically reconstruct the latter theory with the hypothesis of the firms' liquidity-maximization in [6]. However is not it possible to found Keynes' investment theory on the basis of the hypothesis of the firms' maximization of their liquidity in some near future? A new formulation of investment theory along this line is given in the text. (See Secs. 1, 2 and 3 of Chap. 2.)

Thus both of these main theses come to be able to be derived from the hypothesis of the firms' maximization of their liquidity in the present and in the future. Keynes' whole system can be reformulated with the hypotheses of the consumers' stable propensity to consume and of the firms' liquidity-maximization in the present and the future.

(5) *The Clockwise Cycles in Theory and Fact of the Points of (e, m) and (r, m)*

From the Hicksian pair of equations $M=L(Y, i)$ and $I(i)=S(Y)$, with the interest rate exogenous and the money endogenous, there are derived the two functions $i_{LM}=i_{LM}(Y, M)$ and $i_{IS}=i_{IS}(Y)$, resp. Instead of the conventional IS and LM curves in the (Y, i) plane, the newly relevant Marshallian scissors will be the "i-LM" and "IS-LM" curves in the (Y, M) plane which represent the sets of the points (Y, M) satisfying $i_{LM}(Y, M)=i$ and $i_{IS}(Y)=i_{LM}(Y, M)$, resp. (See Fig. 6, P. 31 in the text.)

As the empirical variables which counterpart Y and M are adopted the turnover ratio (e) and (the money/the total assets) ratio (m). Based upon the empirical preview it is assumed that the $I(i)$ function shifts periodically and that the rate of interest (the *kotei-buai*) to be changed accordingly *with a lag* by the central bank so as to stabilize the potentially cumulative change in both Y and M. Correspondingly the i-LM and IS-LM curves periodically shift, and the resulting locus of the actual point (Y, M) becomes cyclical. This theoretical movement of the point is clockwise. (See Fig. 15, P. 43.)

The factual locus of the point combining the turnover ratio, e (or its close proxy variable r, or the gross profit rate) and (the money/the total assets) ratio, m, is diagrammatized from 1953 to 1977 (Figs. 17-20). These factual locuses pattern obviously clockwise shifting cycles. (The most striking is the case of (r, m) for the manufacturing industries. See Fig. 20.) This seems to prove the reality of our model in respect to that facet of business cycle.

Chapter 1 Empirical Findings

The statistical investigations in this paper have been based on the theoretical model presented in [6]. In the process that the statistical results are examined in view of the model, the new model will naturally emerge.

(1) A Related Research On the Money Demand Function

Summary of Our Model in the Previous Paper

In my previous paper [6], I introduced the hypothesis of the firms' liquidity-maximization, which can be recapitulated as follows: the firms try to maximize their money holding (M_f) by borrowing subject to the constraints of their capacity to pay interest on it and of the constancy of the capital assets (K). The capacity of interest payment $(F_K(N(Y), K)K-dK)$ is governed by the level of total production (Y), with K constant. Symbolically, they maximize M_f sub. to $i(K+M_f)\leqq(F_K(N(Y), K)-d)K$, where i is the rate of interest which is assumed given.

Thus the firms' demand curve for money has a shape of a right-angle hyperbola *shifted to the left by the magnitude of K.*

It follows from this shape of the money demand curve that the elasticity of the firms' money demand in respect of the rate of interest is the higher for the higher rate of interest, and that the level of the elasticity is greater than unity for all possible levels of the interest rate.

An empirical research was found in one of the major American economic periodicals which seems to give some facts which may be explained by our theory. The second of the above consequences is perfectly compatible with the result in their Table 1 shown below. The first of them is compatible with the figures in Table 1 for the period from 1951 to 1970. Their results for the other three periods seem less relevant since they contain the period of the Great Depression. In the authors' words, '[o]ne must still...interpret the ...results for the 1920–1940 period with some caution.' (p. 220, [1])

A Research Related to Our Hypothesis

The work of Barth, Kraft and Kraft [1] is interesting from the viewpoint of our theory summarized above. Using spline functions they calculate the elasticities of money in respect of the rate of interest for high and low interest levels. Specifically they measure "money" as 'coin and currency outside commercial banks plus demand deposits adjusted' (p. 220, [1]) and "the rate of interest" as 'the three-month Treasury bill rate' (p. 220, [1].) Their 'somewhat surprizing' (p. 221, [1]) finding is that 'the elasticities [of money to the interest rate] are highest for high interest rates and lowest for low interest rates' (my brackets). They continue that '[t]hese results indicate that the interest elasticity of the demand for money varies directly with the rate of interest, evidence inconsistent with the liquidity-trap hypothesis. Furthermore, none of the piecewise segments of the estimated spline functions indicates that the money demand function becomes horizontal at low interest rates'. (p. 221,

[1]) Though they do not indicate the numerical levels of r_1, r_2 and r_3, their calculative result of the interest elasticities of demand for money is reproduced as follows.

Table 1: Interest Elasticity coefficients

	1920–1970	1929–1970	1951–1970	1920–1940
r_1	−0.122	−0.127	−1.206	−0.115
r_2	−2.698	−2.881	−2.983	−1.815
r_3	−3.904	−5.126	−3.793	−2.906

Note that $r_1 < r_2 < r_3$. (The source of this table is p. 221, [1].)

(2) (The Gross Profit/the Total Assets) Ratio: A Measure

In selecting a measure of the firms' rate of return on the total assets, two measures come to be candidates, (1) the profit before interest payments and (2) the gross profit. These two measures are quantitatively close each other, but substantially different from the other measures, e. g., the operating profit and the net profit.

Table 2: Statement of Income (all industries, the first half of 1971)

Net sales	3087
less Cost of sales	2683
less Selling, administrative and general expenses (including depreciation)	252
Operating profit	152
plus Non-operating incomes	75
(=Interests, discounts, dividends and rents received, and capital gains on sales of securities, etc.)	
Gross profit	*227*
less Non-operating charges	143
(=Interests, discounts, and rents paid, amortization of bond premium, capital losses on sales of securities, inventory devaluations, business taxes etc.)	
Net profit	84
plus Financial charges	117
(=Interests, discounts paid and amortization of bond premium)	
Profit before interest payments	*201*

Source: [10]. Unit: 100 million yen.

(This should be distinguished from the 'net profit' in the terminology of my paper [6], which means a different concept.) Table 2 above shows the four definitions of profit.

Especially the net profit does not represent the full return on the total assets because though the total assets are equal to the total liabilities and stock holders' equities (Table 3), the net profit does *not* include the interest payments on the borrowed funds from the banks and individuals which are included in the liabilities, since, as shown in Table 2, the net profit equals the gross profit *less* the non-operating charges of which the interest payments are dominant components.

The operating profit falls short of the full return on the firms' total assets, because the interests, discounts, dividends and rents received must be regarded to accrue on some parts of the total assets. (See Table 3.) E. g., dividends received accrue on the equity shares of other firms which form a part of the assets of the firms.

Again, the net profit cannot be regarded to be the full return on the firms' total assets because it excludes the parts of the full return which the firms earn and pay out to the other agents who

Table 3: The Firms' Balance Sheet

Assets	*Liabilities and Equities*
Current assets	Liabilities
Quick assets	Current liabilities
Cash on hand and in hand	Accounts and notes payable
Accounts and notes receivable	Short-term loans
Marketable securities	Other current liabilities (including) provisions for taxes, etc.)
Inventories	
Other current assets	Fixed liabilities
Fixed assets	Debentures
Net property	Long-term loans
Goodwill, patents and trademarks	Other fixed liabilities (including provisions for staff superannuation, etc.)
Investments	
Deferred charges (=Prepaid costs, etc.)	Stock holders' equities
	Shares
	Surpluses

are claimants to a part of the firms' total assets. Interests and discounts paid are such parts of the earnings.

Thus we will arrive at the gross profit *or* the profit before interest payments when we seek for the full return on the total assets among the available profit measures. (See Table 2.)

Which of these should we choose as the full return on the total assets? This is a difficult conceptual problem. Though the two definitions of profit may both be regarded to approximate the full return on the total assets, for simpler data-processing let us adopt the gross profit in the investigation below.

Let us call (the gross profit/the total assets) ratio "the gross profit rate" hereafter. This rate is calculated for the three categories of industries (00), (01) and (02) (consisting of large corporations) as listed in Table A 5. (Also, see A 3.)

The results of the calculated rate for each of these industries are tabulated in Table A 7.

(3) The Rates of Interest and the Gross Profit Rate

It will naturally be anticipated from the model presented in my previous paper that (1) the true *average* rate of return on the firms' *total assets* will be at least roughly equal in magnitude to *some* rate of interest. This prediction may be considered to be verifiable by Fig. 1 which visualizes the time-series data of the representative *long*-term rate of interest (or the non-regulated interest rate of all banks) and the average rate of return on the total assets (or their gross profit rate) of large corporations in all industries.

Though the correlation of the two (with or without a lag) is not necessarily very high, it may be remarkable that the *absolute levels* of the two time-series data have been very close each other. Moreover it may be seen in Fig. 1 that the long-term rate of interest has behaved as if it were the center of fluctuation of the gross profit rate of all industries.

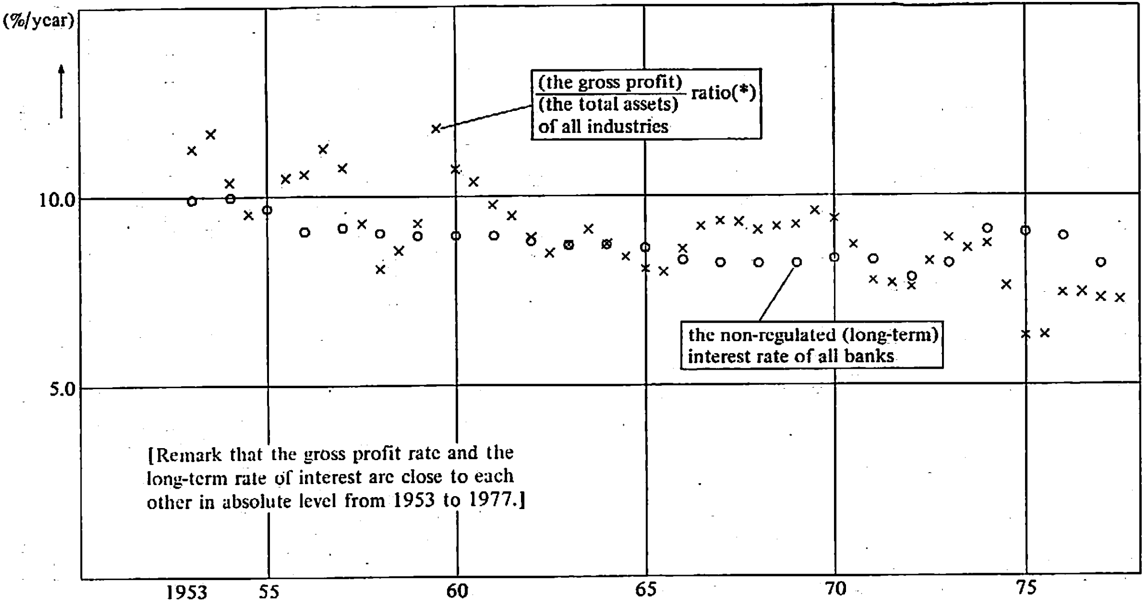


Fig. 1

(*) : (the gross profit *of the present period*) ×2 divided by (the total assets *of the present period*).
Source: [10] for the gross profit rate and [9] for the long-term interest rate.

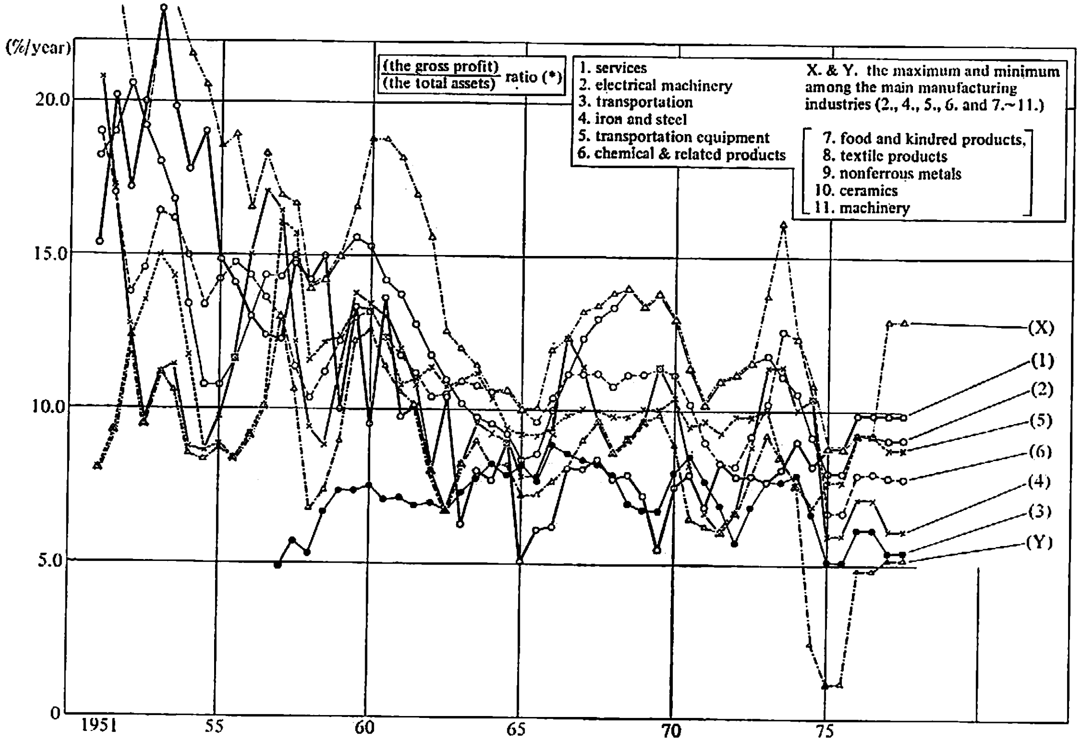


Fig. 2

(*) : (the gross profit *of the present period*)×2 divided by (the total assets *of the present period*).
Source: [10].

It was supposed in the model that some equality among the gross profit rates of various industries would hold, in so far as any differences among them would tend to disappear by some equalization of the profit rates among industries, or by some market mechanism of long-term capital mobility, etc. But the gross profit rates turned out to have kept significantly different among industries. Fig. 2 shows the dispersion of the rates among industries.

Although the dispersion in the first half of 1950's is rather dissapointing, it shows a certain converging tendency of the rates of the various industries into equality especially from the beginning of 1950's to the year 1957, when the *kotei-buai* has been remarkably raised.

There are also graphed the time-series data of the maximum and minimum gross profit rates over the nine manufacturing industries of large corporations. Though the width of the belt delineated by those maximum and minimum time-series data is considerable, it is to be noted that they much approached each other at the points of inflection between the overall up- and down-swings of the belt. The examples of such narrowings are 1956–57, 59–60, 63–64, 69–70 and 73–74. It is to be remarked that these periods are seen to largely correspond to the periods when the *kotei-buai* has been raised significantly, and so it may be said that the narrowings are outcomes of those policy of raising the short rate. These narrowings will be interesting in relation to other business cycle indicators. Moreover existence of such periodical narrowings of the belt seems to show that there is *some* long run tendency of the different manufacturing industries to equalize their gross profit rates, at least after a few years of a boom.

The Short-term Rate of Interest

The short-term rate of interest or of rediscount, or i, is a policy variable, and so it may be regarded as an exogenous variable which determines the money supply as we will assume below.

However there are statistical evidences that it has been changed by the policy maker in response to changes in the levels of employment, external surplus (or deficit) (e. g., see Yajima and Tatemoto, [13]) and the rate of inflation. The at least apparent dependence of the change in the policy rate of interest on especially the turnover ratio is reassured by the result of Tables A 2~A 6.

Most of the data in this paper are tabulated every half a year. Since the short-term rate of rediscount ("the *kotei-buai*") has been changed much more frequently than every half a year, a weighted average has to be calculated from the original time series data to obtain the level of the short rate for each half-yearly unit. (About details see Statistical Appendix.)

(4) The Time Lag of the Short Rate Behind the Gross Profit Rate

The correlation coefficients between the short rate (i) and the gross profit rates (r) of the three categories (00), (01) and (02) of industries are tabulated in Table A 6.

The Short-term Rate and the Profit Rate of the Trading Industry

Fig. 3 shows the lagged short-term rate of interest (the *kotei-buai*) and the gross profit rate of the wholesale and retail trade industry. The short-term rate is a policy variable which has been frequently changed by the central bank. The short rate seems to be much positively correlated with the gross profit rate of the trading industry, and correlation coefficient between them is highest when the short rate is correlated with a lag of 2 years behind the gross profit rate of the industry: the short rate appears to pursue at least 2 years behind the gross profit rate. The correlation coefficient in this case is 84. 8 per cent (see Table A 6.)

Moreover the absolute level (as against the rate of change) of the short rate has been remarkably close to that of the gross profit rate.

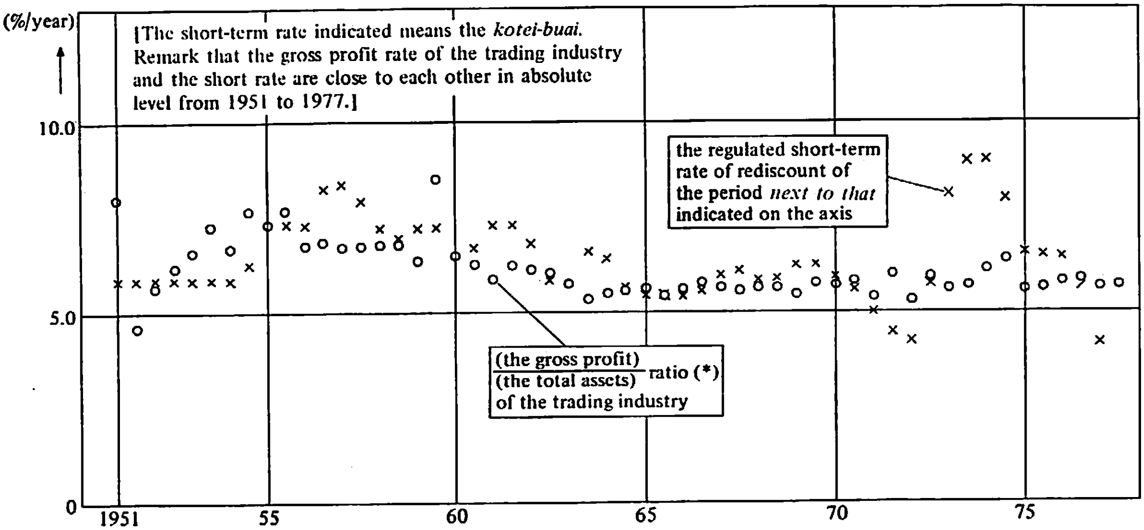


Fig. 3

(*) : (the gross profit *of the present period*)×2 divided by (the total assets *of the present period*).
Source: [10] for the gross profit rate and [9] for the short-term rate of interest which has been processed in the way indicated in Statistical Appendix below.

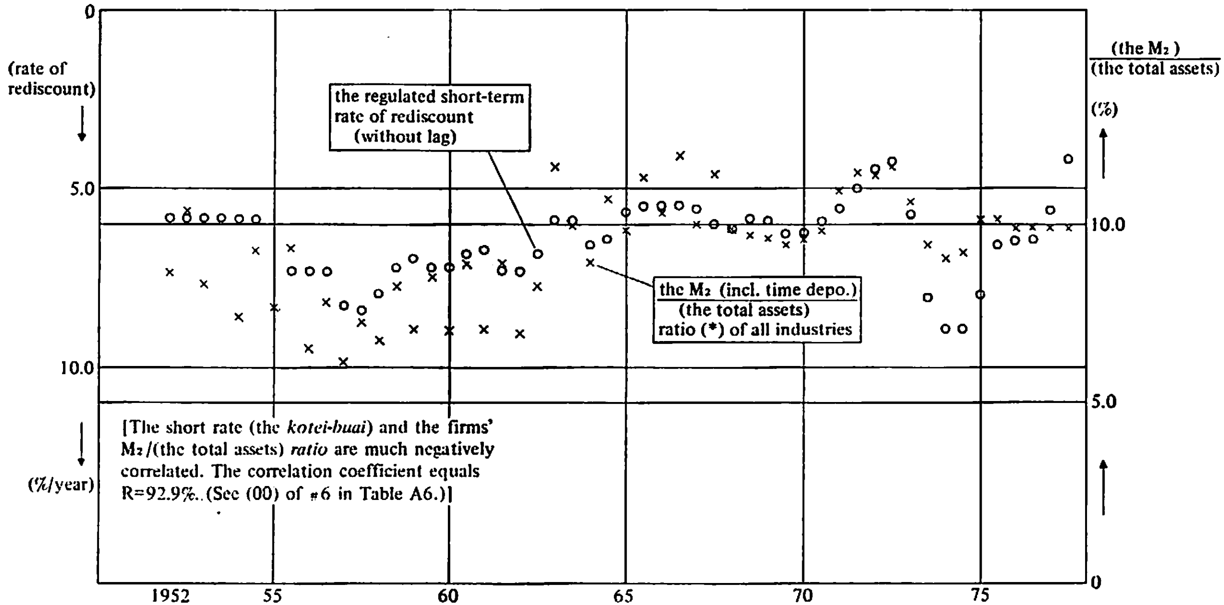


Fig. 4

(*) : (the M_2 *of the present period*) divided by (the total assets *of the present period*).

Source: [10] for the M_2 ratio and [9] for the short-term rate of interest which has been processed in the way indicated in Statistical Appendix below.

(5) The Short Rate and (the Money/the Total Assets) Ratio

The above Fig. 4 shows negative correlation between the short-term rate of interest (i) and [the M_2 (=the bank notes, the demand and time deposits)/the total assets] ratios (m) of all the industries as a whole. In these figures the rate of interest is measured *from above*, so that the diagram of its time-series is depicted upside down, thus enabling us to identify the negative correlation between it and the M_2 ratio.

First, it is obvious by this graph that the negative correlation between the short rate and the M_2 ratio of all industries is very high. Moreover the lag which gives the highest correlation coefficient (92.9%) between them is zero. This is also the case for the manufacturing and trading industries. Table A 6 proves these assertions.

Therefore, since Table A 6 shows that the short-term rate is highly correlated with the gross profit rates of the trading industry and all industries with the lags of 2 and 1/2 years (R=84.8 and 73.8%, resp.), it follows that the M_2 ratio is negatively correlated with the gross profit rate in these cases with the lags. Table A 6 proves this. The R's are −63.8% and −71.9%, resp. However, high correlation with or without a lag does not of course itself indicate any causal order among the three variables, the M_2 ratio (m), the gross profit rate (r) and the short-term rate of interest (i). Some theory is required to determine the causal order of changes in these variables.

Chapter 2 A Theoretical Remodelling

(1) The Assumptions on the Money Market

Assumption 1 We assume in this model that the supply curve of money is horizontal, although it is held to be vertical or upwardsloping in the original model [6]. It amounts to the same thing as assuming that the monetary authority, or the central bank, lets as much money be supplied as the firms demand at the constant level of the policy rate of interest, being perfectly passive in reacting the firms' demand for money without changing the policy rate. However we also assume that:—

Assumption 2 The firms adjust actual money holding to the quantity they demand at the existing policy rate of interest *not* at *infinite* but some finite adjustment speed. (Remark that it is assumed that as soon as a discrepancy occurs in Eq. (1) below, the actual money stock starts to change. In this sense, no *lag of start* is assumed.)

Symbolically the Assumption 2 can be written

$$\frac{dM_f}{dt}=l\left[\frac{(F_K(N(Y),\ K)-d)K}{K+M_f}-i\right], \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(1)$$

where l is some positive constant. In the equilibrium of the money market, the right-hand side of the above equation (1) equals to zero. It was assumed in [6] that this equality was always satisfied, but I relax this assumption here and allow inequality between the firms' desired stock of money signified by

$$\frac{(F_K(N(Y),\ K)-d)K}{i}-K,$$

and their actual level of money stock M_f. This gap between the quantity demanded and the actual holding of money is taken into consideration because of the finiteness of the speed of the firms' money stock adjustment.

Behind the formulation (1) of money stock adjustment, the firms' liquidity-maximization is still presumed: it is assumed here that the firms desire to hold as much money (M_f) as they can afford to pay interest on it (iM_f), so long as their capacity to pay interest on money $(F_K(N(Y),\ K)-d-i)K$ allows.

It should be remarked that thus symbolized capacity for the firms to borrow money does not form any absolute ceiling to their actual borrowing of money: this capacity can be exceeded by their actual money holding. The reason is that they can *pay interest* on some additionally held stock of money *from their already held stock of money itself*. The nature of money which distinguishes it from other less liquid forms of assets enables it to self-liquidate. Part of the firms' money *stock* can at least transitorily be used to pay interest on the excess of their actual stock of borrowed money over and above the level which corresponds to the apparent limit set by the capacity of interest payment determined by the *flow* of their current net profit. Thus the actual holding of money can exceed the quantity of money demanded at the existing policy rate of interest.

Similarly the actual holding of money can fall short of the quantity of money demanded at the existing rate of interest. For example when the firms' demand curve for money shifts to the right with the horizontal money supply 'curve' constant, the actual stock of the firms' money will not instantaneously adjust itself to the level of the money demand at that policy rate of interest but adjust itself to it only gradually: thus the firms' desired stock of money may *actually* be unrealized.

(2) The Investment Adjustment

In the above system of equations we theoretically separate the three rates of return, i. e., (1) the policy rate of interest (i), (2) the average rate of return on the firms' total assets including the money stock (i_{LM}), and (3) the marginal efficiency of capital defined by Keynes (i_{IS}). We separate (1) and (2) in the present paper which have been taken to be always equal to each other in our previous model.

According to the above new Assumption 2, it is the stock of the firms' money that is considered to play the role of diminishing the gap between the policy rate of interest (i) and the average rate of return on the firms' total assets (i_{LM}): the firms increase (or decrease) M_f so as to equalize these two rates when the former is lower (higher) than the latter. Similarly, it is the level of real gross investment that is considered to play the role of eliminating the gap between the average rate of return on the firms' total assets (i_{LM}) and the marginal efficiency of capital (i_{IS}): they increase (decrease) I, the real gross investment, so as to equalize these rates when the former is lower (higher) than the latter.

This conceptual distinction of the rates of return will lead us to the following new interpretation of the schedule of marginal efficiency of capital introduced by Keynes: the term "marginal" implies that the marginal efficiency of capital is defined to be a concept which ought to be quantitatively compared with some "*average* efficiency of capital", or some *average rate of return* on capital. Behind this contrast of the marginal and average schedules, the analogous argument in micro-economics of the representative firm which equalizes the marginal product of labor and average real labor cost may be remembered.

In full equilibrium it is satisfied that $i_{LM}(Y, M_f)=i=i_{IS}(S(Y))$, where $i_{LM}(Y, M_f)$ and $i_{IS}(Y)$, which is the shorthand symbol for $i_{IS}(S(Y))$ in the following, denote the i coordinates of the LM and IS curves, resp. and $i_{LM}(Y, M_f)\equiv\frac{(F_K(N(Y), K)-d)K}{K+M_f}$. The equilibrium condition is of course satisfied if $i_{LM}(Y, M_f)=i$ and $i_{IS}(Y)=i_{LM}(Y, M_f)$. The former equation is required to hold by the assumption of the firms' liquidity-maximization. But why can the latter equation is required to hold?

The Significance of the Marshallian Surplus Below the Marginal Efficiency Curve

The essential point in the following argument is that the marginal efficiency schedule is assumed not to be horizontal, but downward-sloping. On the (I, i) plane, the Marshallian triangular area below the schedule and above the horizontal i-line (ABP in

Fig. 5) corresponds to the excess of the total *additional* profit the investment (BP) will (potentially and expectedly) give in the next period over and above the investment (BP) multiplied by i (=BO), and this excess will provide the firms the additional (and potential) interest payment used to add to their money in the next period. This is because the firms can then leave the excess profit to use as interest payment to *borrow* additional money. They expect that they can earn in the next period the interest payments (in the broader sense) on the additional capital assets (=BP) of the area BP×BO, and still can leave the triangular part (ABP) to pay interest on the additional money they borrow in the next period.

Thus the firms expect in the present period that they can earn

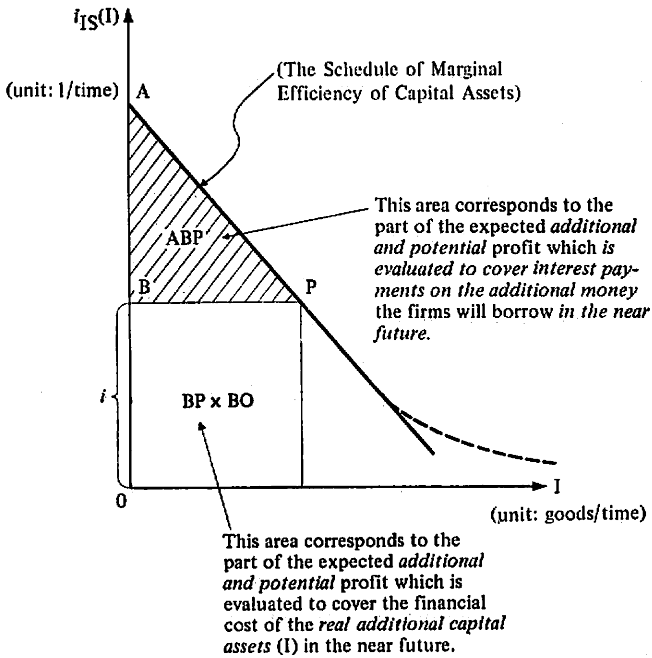


Fig. 5 [the Meaning of the Surplus Area Below the M. E. C.]

by practising the investment BP much greater (additional) profit than sufficient to fill the financial cost of the investment, and that they can borrow as much (additional) money as the excess of the profit over the financial cost of the investment allows them in the next period. As long as the triangular part below the marginal efficiency curve can be assumed to be substantial relative to the financial cost of the investment, the additional money that they expect to be able to borrow will be ordinarily substantial to the additional capital assets or the investment. Thus both the capital assets and the monetary assets are expected to be able to grow side by side in the ordinary circumstances.

Now we assume:—

the Second Hypothesis of the Firms' Liquidity-Maximization: the firms try to invest so as to maximize the ***stock*** of money in the next period ***under the condition*** that the average rate of return on the total assets does not fall in the next period, with auxiliary assumptions that it is expected by the firms that there is a series of infinitesimal units (*dI*'s) of investment opportunities in the present period in order of the expected average rate of profit on each of such units, (this is what Keynes calls the schedule of the marginal efficiency of capital), and that the firms compare this spectrum of the marginal efficiency of capital with the existing average rate of return on the *total* assets (including the money stock). It follows that they practise all of such investment opportunities as are expected to gain net profit at rates which are greater or equal to the existing average rate of return on the total assets, because, by the second hypothesis and the auxiliary assumptions, just above, doing so is the best way for the firms to maximize the money stock they can borrow in the next period. The following two sections give a more rigorous formulation of the investment theory along Keynes' line which is founded on ***this second hypothesis of the firms' liquidity-maximization.***

The Divisibility of Investment Appraisal: A Theory of Long-term Investment

Assume that there are infinite number of investment opportunities, called "molecules", the set of which corresponds one-to-one to the set of all real numbers contained in the segment $[0, 1]$. Each molecule is signified by x, $0 \leqq x \leqq 1$. Each molecule has "the span of investment" or the time of duration of the project x, which is denoted by T_x, $0 \leqq x \leqq 1$.

Each molecule x is assumed by the firms to be practised at a constant pace, denoted by dI_x, its dimension being (goods)/(time). The "size" of each molecule is denoted by $T_x dI_x$, whose dimension is (goods).

To each molecule there corresponds a time profile of its expected rate of return $D(t\,;x)$, whose dimension is $(\text{time})^{-1}$. The flow of return expected from the molecule x at the time t equals $D(t;x)T_x dI_x$, its dimension being (goods)/(time).

Let T signify a "month", with dimension (time), or a fixed time unit. And let T^* signify a "year", or a fixed grand time unit. It is assumed that $T_x \geqq T^*$ for all x, and that $T^* = 12T$, or at least T^* is substantially greater than T.

Any molecule can be divided into shorter subprojects each of which continues for not less than T. The residual part with duration less than T is assumed away for simplicity. At the present moment, the firms have the set of molecules $0 \leqq x \leqq 1$ among which may be included such projects that have partly been carried out already.

The time profile of the expected flow of return $D(t\,;x)T_x dI_x$, with dimension (goods)/(time), is discounted with the discount factor $w(t)$, whose dimension is 1. The present value of the expected flow of return equals

$$\int_0^\infty w(t)D(t\,;x)T_x dI_x dt,$$

with dimension (goods). It is assumed that this is convergent.

Since any molecule confronting the firms has not been completed at the present moment 0, $D(t\,;x)$ is positive only if $t>0$. And since T_x is finite for any x, $D(t\,;x)$ becomes positive only after at least not less than T_x has past from the beginning of each project.

As assumed above the span of investment T_x may be divided into shorter subprojects of x, but each of such subproject is assumed to be carried out at the constant pace dI_x. There may be some blank period between such two subprojects.

Since it is assumed that the actual flow of return is expected to be forthcoming only after the project x is completed, the firms will have to invest without any actual return on each molecule until the whole investment $T_x dI_x$ has been carried out. However, it may be assumed that the firms and the banks who lend liquidity to them appraise each molecule *in advance* and *divisibly*, i. e., evaluate each part of the molecule investment in the light of the present value of the expected flow of return on x, *even if* the whole project of each x has been only partly carried out, or, in other words, *even if* actual flow of return on x has not yet started to be earned.

The banks will be willing to lend additionally as much liquidity at $t=T$ as the potential capacity to return on the total assets is evaluated to rise, even if *actual* flow of return on x has not yet started to be earned. It is assumed that the investment now gives the firms credit to borrow from the banks additionally at $t=T$, because it raises their *potential capacity of interest payment*. The *actual* interest payment on the additional money borrowed from $t=0$ to $t=T$ can be safely assumed to be done from the *stock* of borrowed money at $t=T$. *The self-liquidating property of money* works effectively here.

In this sense it may be assumed that, since each molecule confronting the firms has at least unit time length of the project still left ahead, the firms and the banks at time 0 (or at present) expect an increase in the firms' potential capacity to earn on the total assets between $t=0$ and $t=T$. More precisely, it is assumed that they expect at $t=0$ that the potential flow of return will be increased during the month, or the unit time period $[0, T]$, by

$$\left(\frac{T}{T_x}\right)\left(\frac{\int_0^\infty w(t)D(t\,;\,x)T_x dI_x dt}{\int_0^\infty w(t)dt}\right)=\frac{\int_0^\infty w(t)D(t\,;\,x)T dI_x dt}{\int_0^\infty w(t)dt},$$

with dimension (goods)/(time). The project of x will usually be unfinished at $t=T$, but since the whole present value of flow of return on x is expected to amount to the value as shown above, this present value will give the banks or the firms themselves certain credit for the rise in their potential earning capacity in terms of flow of return, by the proportion T/T_x of the *overtime-average* flow of return on x,

$$z(x)=\frac{\int_0^\infty w(t)D(t\,;\,x)T_x dI_x dt}{\int_0^\infty w(t)dt},$$

with dimension (goods)/(time), which is expected to accrue on the whole size $T_x dI_x$ of each molecule. We may thus write

$$V(T\,;\,x)-V(0\,;\,x)=\left(\frac{T}{T_x}\right)z(x),$$

where

$$\int_0^\infty z(x)w(t)dt=\int_0^\infty w(t)D(t\,;\,x)T_x dI_x dt,$$

and $V(t\,;\,x)$ denotes the potential *overtime-average* flow of return on the part of the project x which has been carried out until the time t, defined for $0 \leqq t \leqq T$.

The firms and the banks assume at $t=0$ that the (sub)project of x continues till $t=T$ without interruption. They assume therefore that the whole project x will proceed by TdI_x from $t=0$ to $t=T$. However $V(T\,;\,x)-V(0\,;\,x)$ means the potential overtime-average flow of return on *the whole of the part* TdI_x of the size $T_x dI_x$, and *not* on each unit of the project. The potential overtime-average flow of return on *each unit of* the part TdI_x of the project x will be

$$b(x)=\frac{V(T\,;\,x)-V(0\,;\,x)}{TdI_x}=\frac{z(x)}{T_xdI_x}=\frac{\int_0^\infty w(t)D(t\,;\,x)dt}{\int_0^\infty w(t)dt},$$

with dimension (time)$^{-1}$. We may call $b(x)$ "*the average efficiency* of the molecule x."

The marginal efficiency of capital assets for each level of investment I is defined as the inverse function of the function $I(i_{IS})$ which is defined as

$$I(i_{IS})=\int_{(b(x)\geqq i_{IS})}dI_x.$$

Now by the definitions of $b(x)$ and $V(t\,;\,x)$, the increase in the potential overtime-average flow of return on the firms' total assets owing to dI_x continuing from $t=0$ to $t=T$ will be $TdI_xb(x)$ with dimension (goods)/(time). Therefore, the total increase in the potential overtime-average flow of return on the firms' total assets owing to $I(i_{IS})$ continuing from $t=0$ to $t=T$ will be

$$R=R(i_{IS})=\int_{(b(x)\geqq i_{IS})}Tb(x)dI_x,$$

with dimension (goods)/(time). R corresponds to the familiar trapezoid area under the schedule of the marginal efficiency of capital.

For simplicity let us assume that the function $I(i_{IS})$ has inverse function. Then we can write $i_{IS}=i_{IS}(I)$ and

$$R=R^*(I)=\int_0^I Ti_{IS}(q)dq.$$

$R^*(I)$ denotes the increase in the potential earning capacity *at* T in terms of flow of return the firms and the banks will expect at the present moment, compared with that at 0 from the investment at the pace I from $t=0$ to T.

The Second Hypothesis of the Firms' Liquidity-Maximization and Keynes' Investment Theory

Given the marginal efficiency schedule $i_{IS}(I)$, the present money stock $M_f{}^\circ$ and the present level of production Y°, the total (potential) flow of return expected to be earned at $t=T$ will be

$$K(F_K(N(Y^\circ),\ K)-d)+\int_0^I i_{IS}(q)T\,dq,$$

and we denote the expected average rate of return at $t=T$ by r^T. Then, since the above total profit expected to be earned at T must be equal to the product of r^T and the expected total assets at T, or $K+M_f^\circ+\Delta(K+M_f)$, we have the identity

$$K(F_K(N(Y^\circ),\ K)-d)+\int_0^I i_{IS}(q)T\,dq$$
$$=r^T(K+M_f^\circ+TI+(M_f^T-M_f^\circ)). \quad \cdots\cdots\cdots\cdots\cdots\cdots (2)$$

We assume that the firms are subject to the constraint that r^T should be greater than or equal to the average rate of return at the present, or $i_{LM}(Y^\circ,\ M_f^\circ)$, namely that

$$r^T \geqq i_{LM}(Y^\circ,\ M_f^\circ). \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (3)$$

The problem becomes: maximize M_f^T by changing I, subject to (3), or in words, maximize the stock of money held at $t=T$ under the condition that the average rate of return on the total assets at $t=T$ is not less than that at the present.

Since the constraint (3) is equivalent with

$$r^T(K+M_f^\circ+TI+\Delta M_f) \geqq i_{LM}(Y^\circ,\ M_f^\circ)(K+M_f^\circ+TI+\Delta M_f),$$

and since

$$i_{LM}^\circ\cdot(K+M_f^\circ)=K(F_K^\circ-d),$$

the problem will become: maximize M_f^T by changing I subject to

$$K(F_K^\circ-d)+\int_0^I i_{IS}(q)T\,dq \geqq i_{LM}(Y^\circ,\ M_f^\circ)(K+M_f^\circ+TI+\Delta M_f)$$
$$=K(F_K^\circ-d)+i_{LM}(Y^\circ,\ M_f^\circ)(TI+\Delta M_f),$$

or maximize ΔM_f by changing I sub. to

$$\int_0^I (i_{IS}(q)-i_{LM}(Y^\circ,\ M_f^\circ))T\,dq \geqq i_{LM}(Y^\circ,\ M_f^\circ)\Delta M_f.$$

The solution to this problem will be the I at which the term on the left-hand side is maximized, or $i_{IS}(I)=i_{LM}(Y^\circ,\ M_f^\circ)$ is satisfied.

The firms will try to change I so as to equalize i_{IS} to i_{LM}. If i_{IS} is greater (smaller) than i_{LM}, I will grow (decrease). Then by the multiplier process which is assumed to complete instantaneously, Y will grow (decrease) and i_{LM} grow (decrease) accordingly. Thus i_{IS} and i_{LM} will approach each other. In this process, the *function* $i_{LM}(Y, M_f)$ is assumed to be unknown to the firms. The value of it appears to them simply as the general economic condition of the average rate of return which they do not think that they can change directly: it is only the level of investment that the firms think they can directly change. It is out of their immediate concern whether the average rate of return grows or falls as a result of their changing the level of investment.

The adjustment formula for I will thus be

$$dI/dt = v \cdot (i_{IS}(I) - i_{LM}(S^{-1}(I), M_f)), \quad \cdots\cdots\cdots\cdots\cdots (4)$$

with v positive constant, which will constitute the whole adjustment system together with Eq. (1). Thus, we may formulate as follows.

*Assumption 3** We keep the assumption in the previous model that the firms change the level of total production (Y) so as to eliminate any gap between the $i_{LM}(Y)$ and the $i_{IS}(Y)$, i. e.,

$$\frac{dY}{dt} = j \cdot (i_{IS}(S(Y)) - i_{LM}(Y, M_f)), \quad \cdots\cdots\cdots\cdots\cdots (5)$$

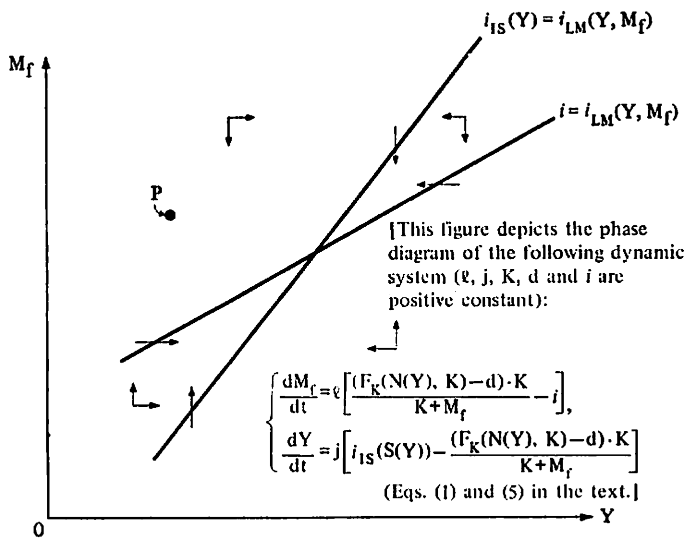


Fig. 6

where j is a positive constant, $di_{IS}/dY<0$ and

$$i_{LM}(Y,\ M_f)\equiv\frac{(F_K(N(Y),\ K)-d)K}{K+M_f}.$$

The system of the differential equations (1) and (5) can be illustrated in the above phase diagram (Fig. 6).

(3) Self-liquidation of Money

Let us depict the demand and supply curves of money which were introduced in the previous paper. (Fig. 7)

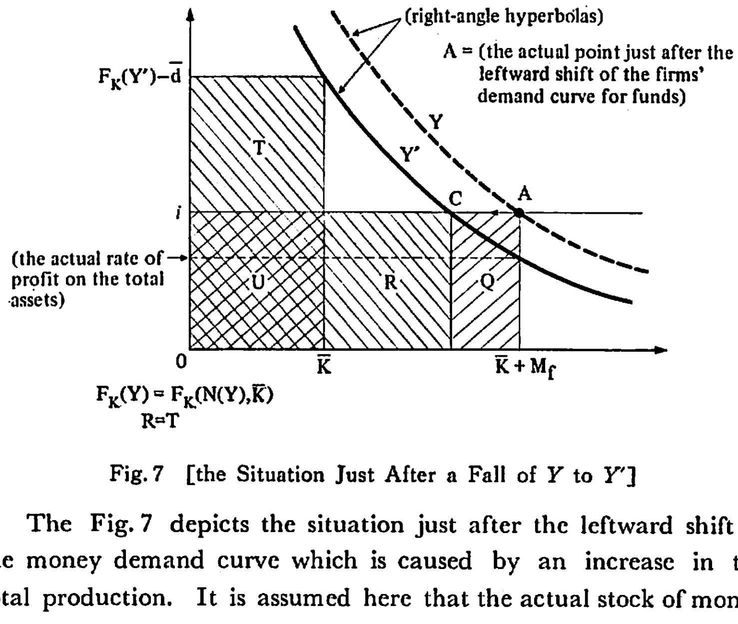


Fig. 7 [the Situation Just After a Fall of Y to Y']

The Fig. 7 depicts the situation just after the leftward shift of the money demand curve which is caused by an increase in the total production. It is assumed here that the actual stock of money held by the firms stays at the level of the point A just after the shift. The point A has started to change leftward. It does *not* stay on the new money demand curve but pursues it from the right *with a lag*. Why is it an actual situation? Was not it assumed that $T+U=U+R$, or, in words, that the total net profit $(=T+U=K(F_K(Y')-d))$ is *always* equal to the total interest payments ($=U+R+Q$)? So, must not it hold that $Q=0$?—The answer is: No. Q *can* be positive, because the firms *can* pay a part of interest using their borrowed money itself! The money they have borrowed can be used to pay *as interest on itself:* the self-liquidating nature of money allows the area Q to be positive which signifies

Q = (the total interest payments) − (the total net profit) $\gtreqless 0$.

This argument can be applied to the opposite case where the money demand curve shifts to the left from an equilibrium in the money market.

The buffer role of the firms' money works to keep such a point like P in Fig. 7 to be an actual point.

(4) The Stabilizing Role of Changing the Policy Rate

See Fig. 8. When the IS curve shifts upwards in the (Y, i) co-ordinates, the IS–LM curve shifts to the right in Fig. 8. It holds that $a<b$. The equilibrium point will move to the right *by a greater magnitude than that of the shift of the IS–LM curve* as long as the policy rate of interest stays constant. The new equilibrium is certainly stable, but in that sense the total production changes to a greater extent than in the case where the i–LM curve is flatter or less sloped. In this sense there is room for the monetary authority to help 'stabilize' the relatively cumulative change of Y.

In order to stabilize the change in Y the policy rate of interest (i) should be *raised* so as to shift the i–LM curve *downwards.* Then, the new equilibrium E' will be nearer to E, (or that before the shift of IS curve) than in the case without the rise of the policy rate.

Similarly when the IS–LM curve shifts to the left, the policy rate of interest should be lowered so as to alleviate the cumulative downward change in Y.

This is a main role of changing the policy rate. The monetary

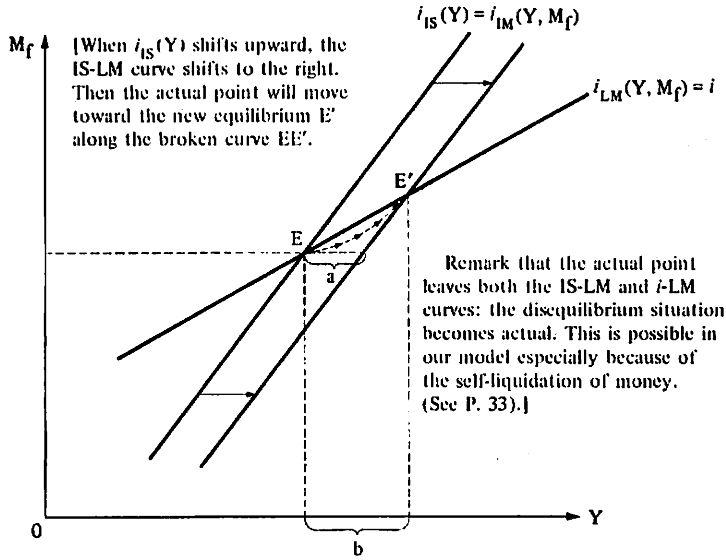


Fig. 8

authority has changed the policy rate *in response to* a change in the general economic activity level. When we see the past data, this causal order can empirically be verified by Table A 6. The policy is stabilizing in the above sense.

(5) The Positive Correlation Between the Turnover Ratio and the Rate of Profit

See Fig. 7. We assume the constant policy rate (i). The locus $A \rightarrow C$ which goes to the left horizontally indicates the gradual (not discrete) movement of M_f some time after a discrete reduction in Y. The locus shows that M_f gradually falls when the total production falls, and that the policy rate of interest stays constant. Remark that this locus $A \rightarrow C$ does *not* depict the locus of the profit rate.

The locus of M_f and the profit rate just after the discrete reduction in Y is shown by the arrow $A \rightarrow B$ in Fig. 9. The profit rate *falls* when the production level falls. The essential reason for this phenomenon is as follows. First, as noted just above, if the actual money held by the firms were to decrease without any time lag with the shift of the money demand curve, then the profit rate of the firms would stay *constant* as long as the money supply curve stays horizontal. But in reality the actual stock held by the firms is sticky in response to the fall of the turnover ratio: it does not actually fall very fast when the turnover ratio has started to fall.

Fig. 11 shows that the profit rate of the manufacturing is very much positively correlated with their turnover ratio without any lag.

In Fig. 9 the locus $(B \rightarrow C)$ indicates the locus of M_f and the profit rate in the case where the policy rate of interest stays at the same level as before some time after the leftward shift of the money demand curve, which is assumed to have stopped corresponding to the production level Y'. Remark that the locus of M_f and the profit rate does not leave the money demand curve (but always

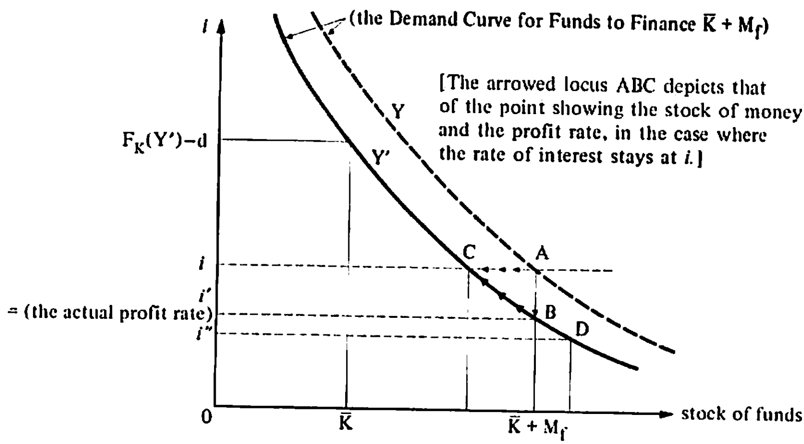


Fig. 9

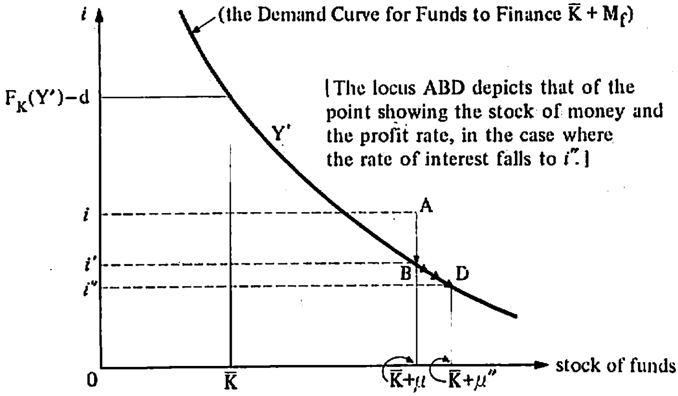


Fig. 10

keeps on it) all the way from A to B and B to C. Also remark that at the point B the firms are still paying the interest at the rate of i, but that the part $iA \times AB$ of the total interest payment $(iO \times iA)$ is being paid not from the net profit but by self-liquidating the firms' money stock itself, as explained in page 33 above. The locus $B \rightarrow C$ is traced because the firms want to diminish this excess of the interest payment (or the policy-set 'normal' profit) over and above the profit. This is how the profit rate and the (constant) policy rate diverge when Y changes.

In Fig. 9, M_f falls sooner or later as Y falls to Y'. But the statistical investigation (Table A 6) shows that there is not any positive correlation between the turnover ratio and (the money/the total assets) ratio of the firms.

The Stabilizing Role of Changing the Short Rate, Again

This is the case where the policy rate of interest is assumed to be constant for fairly long time. But the fact is that the authority changes the policy rate downwards (upwards) about 1/2 to 1 year after the leftward (rightward) shift of the money demand curve

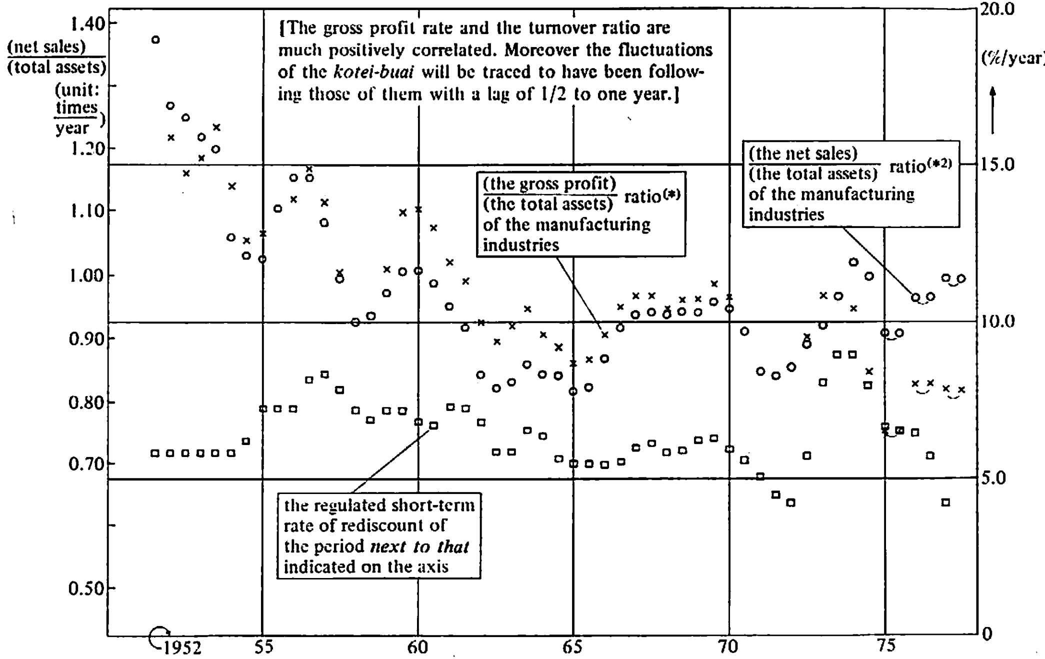


Fig. 11

(*) : (the gross profit *of the present period*)×2 divided by (the total assets *of the present period*).

(*2) : (the net sales *of the present period*)×2 divided by (the total assets *of the present period*).

Source: [10] for the gross profit rate and the turnover ratio, and [9] for the short-term rate of interest which has been processed in the way indicated in Statistical Appendix.

(Table A 6). This realistic case is depicted in Fig. 10. It is assumed in Fig. 10 that the policy rate is decreased to i'' just when the profit rate is at the level i' of the point B. The policy rate is assumed to fall to i'' below the level i'. By this fall of the policy rate, the actual stock of money rises from the level of μ to μ''. This change in the money stock is due to the liquidity-maximization of the firms introduced in the previous paper: the firms try to maximize to borrow the stock of money as far as they can afford to pay interest on it from the profit.

These changes in the money stock held by the firms and the policy rate of interest correspond to the data of Fig. 4. In fact 1/2 to 1 year after the economy starts to fall in the total production, the policy rate starts to fall and correspondingly the stock of money held by the firms rises relative to the total assets of the firms.

Remark that this result of expanding money is just in the reverse direction to the result in the alternative case when the policy rate is kept constant. If the policy rate were kept constant at the level of i, the stock of money held by the firms would *decrease* to the level of the point C. The policy of changing the short rate in the appropriate direction works to break this cumulatively unstabilizing consequence. In terms of Fig. 13 below, the lowering of the rate turns the latent locus EF of (Y, M_f) at j towards E'. If it were not for the policy, the point would head towards F.

(6) The Relation Between the Model and the Data

As noted in the beginning, the above theory in this paper is originally built so as not to contradict the empirical facts I collected: relations among these facts will be explained in view of the above model. Before doing so, it will be necessary to clarify relations between the notation of the variables appearing in the above model and the statistical variables which I calculated.

There are four variables whose time-series I am here concerned. They are

(1) the short-term policy rate of interest (the *kotei-buai*), denoted by i,

(2) the firms' gross profit rate, r,

(3) the firms' turnover ratio, e and

(4) the firms' (money/the total assets) ratio, m.

Among them (3) and (4) can be regarded to be closely associated with the notation we are already familiar with in the analytical model above, i. e.;

$m=m(m^*)$ and $\dot{m}>0$, where $m^*=M_f/(K+M_f)$, $e=e(n)$, and $\dot{e}>0$, where the dots denote the derivatives of m and e in respect of m^* and $n=N/K$, resp. N/K signifies the firms' (effective work/capital assets) ratio. Then we immediately have $n=n(e)$ and $dn/de>0$, and that $i_{LM}=(F_K(n(e), 1)-d)(1-m^*)$, by the linear homogeneity of the production function.

Though the variables whose movement the phase diagram analyzes in the Last chapter are Y and M_f, each of these can be understood to correspond one-to-one to the variables e and m above, respectively, by the following consideration:—

The adjustment system (1) and (5) may be generalized to the case where K is no longer constant. It is assumed that m^* rises (falls) as $i_{LM}>i$ ($i_{LM}<i$), and that n rises (falls) as $i_{IS}>i_{LM}$ ($i_{IS}<i_{LM}$). I. e. it is assumed that, with K variable, the firms change $M_f/(K+M_f)$ by currently changing money stock M_f so as to diminish the gap between i_{LM} and i, and that they change Y/K by currently changing Y, and hence N/K, so as to diminish the gap between i_{IS} and i_{LM}. Behind this assumption, of course, the same considerations as those behind Eqs. (1) and (5), i. e. the firms' liquidity-maximization and the consequences derived therewith are presumed. Thus we have

$$\frac{dm^*}{dt}=q\cdot[(F_K(n(e), 1)-d)(1-m^*)-i],$$

$$\frac{de}{dt} = g \cdot [i_{IS}^*(e) - (F_K(n(e), 1) - d)(1 - m^*)],$$

where $i_{IS}^*(e)$ denotes the marginal efficiency of capital assets rewritten as a decreasing function of the turnover ratio, e. The phase diagram of the point (e, m^*) in the (e, m^*) plane is similar to that of the system (1) and (5) in the (Y, M_f) plane and so is that of the point (e, m) in the (e, m) plane. (See Fig. 6.) Moreover in view of the highest correlation coefficients between r and m in the no-lag case among other lagged cases (Table A 6), the statistical locus of the point (r, m) will have topologically similar natures as that of the point (e, m) has.

Chapter 3 The Cyclical Locuses of (e, m) and (r, m)

(1) A Model of The Business Cycle

In this section we build a business cycle model which endogenizes the authority's policy of changing the short-term rate of interest (the policy rate). In doing so, we do not need any other additional assumptions than those made above: the following model is an outcome of the above model, and corresponds to the statistical facts, as will be shown below.

In Fig. 12 the *IS-LM* curve has shifted to the left, and the actual point (Y, M_f) will start to move to the left (along the horizontal line *Ea*). The point will then move along the broken curve *EF*, finally to arrive at the new equilibrium *F*.

This is the simple case. But what will happen if the *i-LM* curve shifted to the left after the shift of *IS-LM* curve and *before* the actual point arrives at *F*? This case is depicted in Fig. 13.

In Fig. 13 the actual point turns at *j*, to the northwest to reach *E'*. This locus simulates the actual movement of (e, m). The statistical data shows as noted above that the policy rate of interest is lowered 1/2 to 1 year after e and r started to fall (Table A 6).

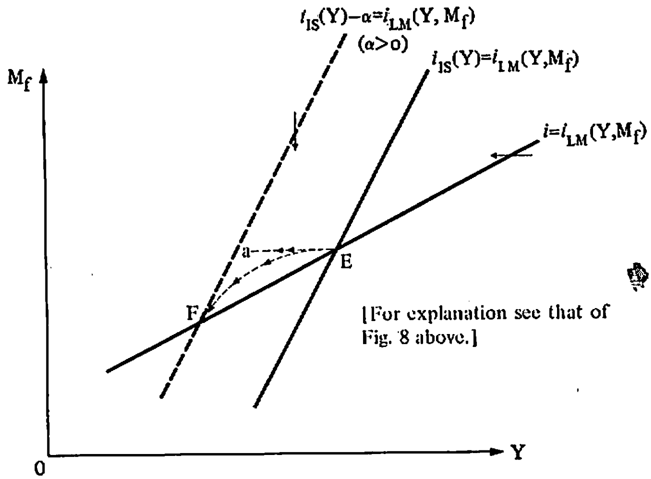


Fig. 12

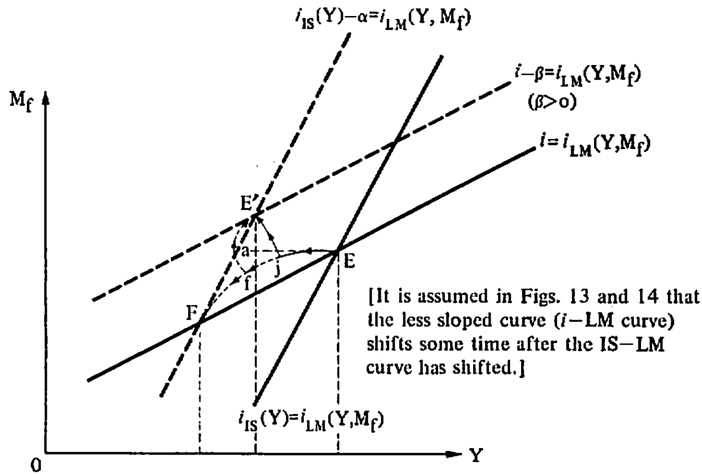


Fig. 13

This lag corresponds to the lag of the shift of the *i-LM* curve (which shifts up if the policy rate, *i*, is lowered) behind the shift of the *IS-LM* curve (which shifts if the *IS* curve shifts.)

Comparing the locus *EjE′* with *EjF*, it is clear that the policy of lowering the rate results in a reversal of direction of change in

M_f. By this policy m turns to rise which must have fallen to the level at F if there had not been this policy. Moreover by this policy the latent fall of the variable Y to the level of F is prevented, and the variable Y falls only to the level of E'.

Thus the policy of lowering i a little after a recession has two simultaneous effects: to break the fall of e to F, and to make m turn to rise.

Remark here that, in Fig. 13, EfE' (instead of EjE') is also a possible locus. This is the case where the interest-lowering policy is a little more lagged.

Similarly Fig. 14 depicts the locus $E'j'E''$ (and $E'f'E''$) of the point when the economy is in the prosperity phase. The policy rate is assumed to be raised a little after the shift of the *IS–LM* curve to the right.

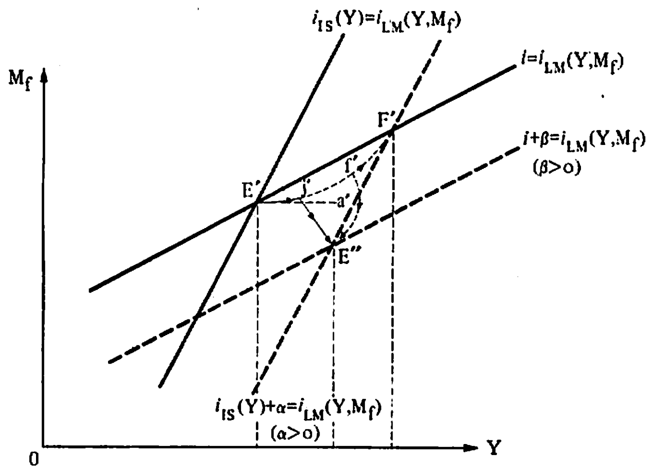


Fig. 14

Under the assumption that the *IS–LM* curve fluctuates cyclically, and that the policy rate is changed accordingly with a lag, the resulting trade cycle will be shaped like $EjE'j'E$ in Fig. 15 or some similar closed locus. Other possible shapes are, e. g. $EfE'f'E$, $EjE'f'E$, and $EfE'j'E$.

However the correlation coefficients between e and m in Table A 6 show that e and m have moved much more often in the opposite direction than in the same and this is the case for various values of lags. Therefore it will be safe for us to think that the standard case is $EjE'j'E$ rather than the other cases, because this locus contains relatively little part where e and m moves in the same direction.

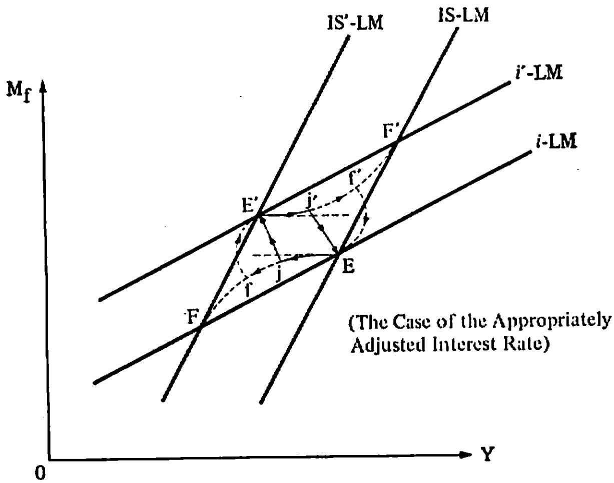


Fig. 15 (the *Clockwise* Cycle)

(2) Statistical Evidence of the Cycle

If it were not for the policy of changing the short rate in the appropriate directions, the movement of the point (Y, M_f) will be like the locus EE^*E in Fig. 16. The locus $EjE'j'E$ or other locuses in Fig. 15 are directed clockwise, whereas the locus EE^*E in Fig. 16 is directed counterclockwise. This difference in the direction of turning between the two theoretical cases is very clear. As explained above the movement of the point (Y, M_f) corresponds to that of the point (e, m), and the movement of the actual point (e, m) will deserve investigation. The Figs. 17 and 18 depict the movement of

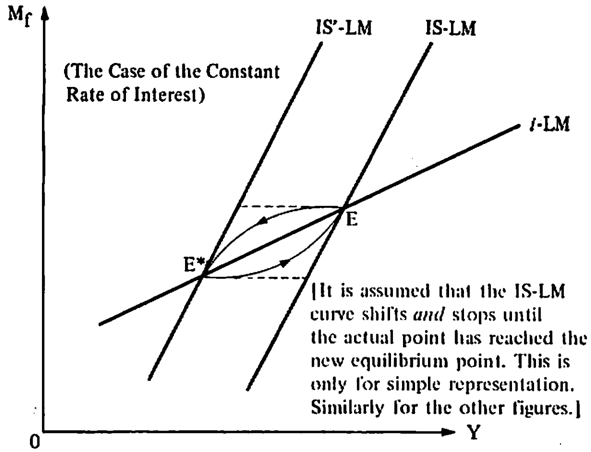
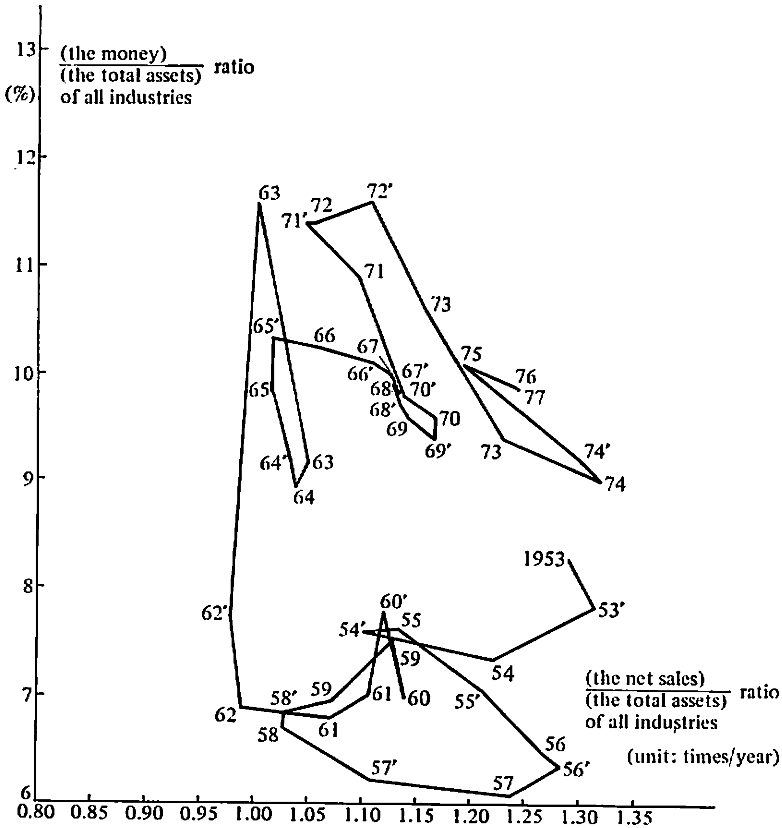


Fig. 16 (the ***Counterclockwise*** Cycle)

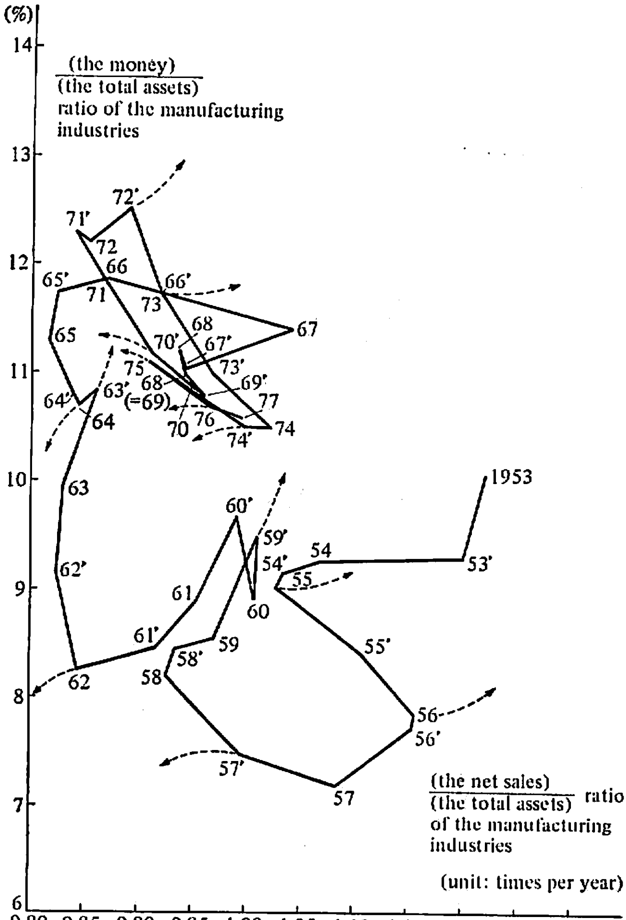
this point for all industries and for the manufacturing industries, resp. It will be seen in these graphs that the locuses of the point (e, m) turn successively in the clockwise direction for the most part. It may be argued that this indicates that the interest rate policy significantly affected the course of the point in the manner indicated in the above model.

Moreover the Figs. 19 and 20 show such clockwise turnings of the point (r, m) for all industries and for the manufacturing industries. The clockwise cycles are especially impressive in Fig. 20 for the orbits from 1964 on. Since r is shown above to be highly correlated with e without lag, the same explanation as that for the cycles of (e, m) will be possible for the cycles of (r, m). Furthermore, the effect of the interest rate policy can easily be traceable by drawing the potential cumulative movement of the points in the case when there were not for the appropriate interest rate policy at each point of turning. These potentitally unstable movements of the points are drawn with the broken curves in Figs. 18 and 20.



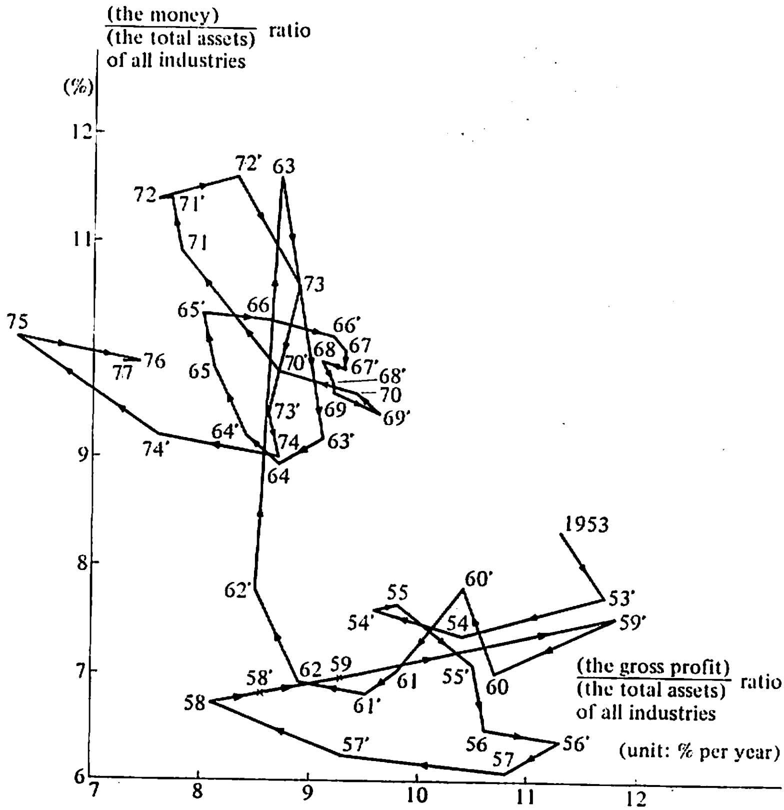
(Figs. 17, 18, 19 and 20 have been drawn by use of the calculated data in Table A7.) (Source: [10])

Fig. 17



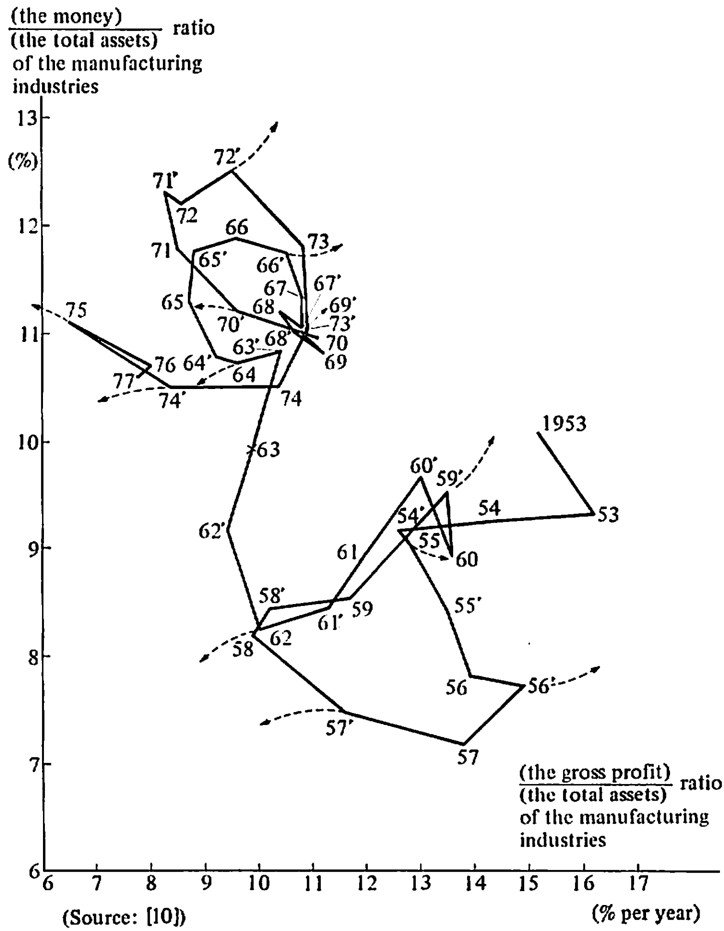
(Source: [10])

Fig. 18



(Source: [10])

Fig. 19


(the money) / (the total assets) ratio of the manufacturing industries
13
(%)
12
11
10
9
8
7
6
72'
71'
72
66
73
71
65'
66'
67'
67
65
68
69'
75
73'
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58'
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55'
61'
62
58
56
56'
57'
57
(the gross profit) / (the total assets) ratio of the manufacturing industries
6
7
8
9
10
11
12
13
14
15
16
17
(Source: [10])
(% per year)

Fig. 20

Statistical Appendix

The Weighted Average of the Short-term Rate

The ***kotei-buai***, or the short-term discount (or interest) rate which is statistically treated in the text has been obtained from the serial data of (1) the rate of "discounts of commercial bills and others of similar rating" from 1951 to the first third of August 1955, (2) the rate of discounts of "commercial bills" from the second half of August 1955 until August 1969, (3) the "discount rate on commercial bills and interest rate on loans secured by government securities or specially designated debentures" from September 1969 until the end of 1971 and (4) the "discount rate on commercial bills and interest rates on loans secured by government securities, specially designated depentures and bills corresponding to commercial bills" from January 1972 until the end of 1977, all of (1) to (4) being the rates on the most favored bills and loans among the various bills and loans on which "basic money rates of the Bank of Japan" are cost.

The data (1)~(4) above has been processed to obtain the weighted average of it for each half-yearly period in the following way. (1) Specify a half-yearly period as either the April-to-September or the October-to-March period. (2) Specify the raw figures of the above data for the specified period. (3) Calculate how long, (specifically for how many 1/3 months), each of the raw figures was kept constant in a continuous time span. (4) Calculate the weight for each of the raw figures by dividing the length of duration of each figure by 18 ($=3\times6$) 1/3 months. And (5) calculate the weighted average of the raw figures using this set of weights for each period.

The numerical result of this weighted average is tabulated in Table A7 below.

Table A1: the Definition of n and the Lagged Time Series of A and B

which lags, A or B?	B lags behind A by n (−n), n=4, 3, 2 and 1		A lags behind B by n (n), n=0, 1, 2, 3 and 4	
corresponding pairs of A and B	$\begin{cases} A_{55F-n} \\ B_{55F} \end{cases}$ ……	$\begin{cases} A_{72L-n} \\ B_{72L} \end{cases}$	$\begin{cases} A_{55F} \\ B_{55F-n} \end{cases}$ ……	$\begin{cases} A_{72L} \\ B_{72L-n} \end{cases}$

Remark: A_{55F} means the variable A for the *first half* of 1955, and B_{72L} means B for the last half of 1972. A_{55F-n} means A for the period n *half-years before* the *first* half of 1955.

Table A2: List of Empirical Variables

r	the gross profit (of the present period)×2/the total assets (of the present period)=the gross profit rate,
e	the net sales (of the present period)×2/the total assets (of the present period)=the turnover ratio,
m	the currency and deposits (of the present period)/the total assets (of the present period)=(the money/the total assets) ratio,
i	the weighted average of the *kotei-buai* (of the present period)=the rate of rediscount.

Table A3: List of Industries

all industries (00)
- *Manufacturing (01)*
- Mining
- Construction
- *Wholesale and Retail Trade (02)*
- Real Estate
- Transportation
- Electric Power and Gas
- Services

Table A4: the 6 Cases of Lagged Regressions

A \ B	r	e	m
r	/	/	/
e	1	/	/
m	2	3	/
i	4	5	6

Table A5: List of Industrial Categories

(00)	all industries
(01)	the manufacturing of all kinds
(02)	the trading, wholesale and retail

Table A6: The Correlation Coefficients (R) for the Lagged Time Series of A and B*

Case #*4	1			2		
A vs.** B	A=e, B=r			A=m, B=r		
	(00)*3	(01)	(02)	(00)	(01)	(02)
$n=-4$	1.3	45.1	87.9	−39.8	−39.6	− 5.5
$n=-3$	− 4.2	47.3	89.3	−39.8	−44.1	− 3.7
$n=-2$	19.1	63.6	87.8	−44.3	−49.3	− 6.6
$n=-1$	54.1	83.2	87.1	−52.0	−58.9	−22.4
$n=0$	72.6	90.6	89.0	−63.2	−69.8	−41.7
$n=1$	60.4	79.4	91.3	−71.9	−80.1	−50.6
$n=2$	30.2	58.0	90.0	−67.4	−83.1	−59.7
$n=3$	3.7	41.1	91.2	−61.7	−80.0	−63.1
$n=4$	− 3.9	39.1	86.3	−56.5	−77.5	−63.8

Case #	3			4		
A vs. B	A=m, B=e			A=i, B=r		
	(00)	(01)	(02)	(00)	(01)	(02)
$n=-4$	− 9.8	−18.8	−17.2	19.5	16.2	27.5
$n=-3$	− 6.0	−27.5	−20.8	19.3	20.5	34.0
$n=-2$	− 1.8	−36.4	−31.0	21.6	27.8	44.1
$n=-1$	−14.0	−49.7	−46.4	35.2	42.3	57.2
$n=0$	−30.5	−61.9	−64.2	58.2	63.8	69.5
$n=1$	−42.5	−73.0	−69.0	73.8	80.3	67.3
$n=2$	−43.4	−76.6	−70.0	71.6	80.7	70.7
$n=3$	−35.7	−74.9	−69.5	58.5	69.2	77.3
$n=4$	−34.6	−71.6	−70.6	51.1	62.4	84.8

Case #	5			6		
A vs. B	A=i, B=e			A=i, B=m		
	(00)	(01)	(02)	(00)	(01)	(02)
$n=-4$	−37.4	−10.8	35.2	−46.8	−42.6	−55.9
$n=-3$	−29.8	−25.2	45.2	−53.6	−56.7	−62.1
$n=-2$	−17.9	11.4	55.6	−65.6	−71.5	−71.4
$n=-1$	1.2	30.6	68.3	−81.5	−85.8	−82.0
$n=0$	25.3	51.6	80.7	−92.9	−93.3	−85.9
$n=1$	45.3	68.5	83.9	−90.1	−90.5	−78.2
$n=2$	48.9	74.6	85.4	−79.1	−83.5	−61.6
$n=3$	34.6	64.8	84.8	−72.2	−75.7	−38.8
$n=4$	29.6	57.4	82.9	−68.4	−68.8	−17.1

*) See Table A 1. **) See Table A 2. *3) See Tables A 3 and A 5. *4) See Tables A 2 and A 4. Source: [10]. Unit: %.

Table A7: The Calculated Data of r, e, m and i

variable*	the gross profit rate (r) (unit: %/year)			the turnover ratio (e) (unit: times/year)			(the money/the total assets) ratio (m) (unit: %)			(i)**
year	(00)*	(01)*	(02)*	(00)	(01)	(02)	(00)	(01)	(02)	
1951	17.5	24.5	8.02	1.469	1.422	4.649	7.18	8.46	8.91	5.84
	14.7	20.3	4.63	1.453	1.376	4.734	8.09	9.36	10.80	5.84
1952	12.0	15.9	5.58	1.381	1.271	4.466	8.60	8.60	10.41	5.84
	10.9	14.7	6.15	1.281	1.251	3.638	8.83	8.83	8.74	5.84
1953	11.3	15.2	6.60	1.291	1.218	3.942	8.31	10.06	7.99	5.84
	11.7	16.2	7.29	1.313	1.204	4.027	7.82	9.31	8.48	5.84
1954	10.4	14.3	6.70	1.122	1.064	3.968	7.37	9.25	9.75	5.84
	9.6	12.6	7.74	1.102	1.033	3.821	7.60	9.17	10.51	5.84
1955	9.8	12.8	7.34	1.133	1.027	3.663	7.63	9.03	10.91	6.25
	10.5	13.5	7.69	1.210	1.105	3.834	7.09	8.42	9.12	7.30
1956	10.6	13.9	6.83	1.265	1.157	3.892	6.50	7.84	6.94	7.30
	11.3	14.9	6.85	1.282	1.155	3.686	6.38	7.74	6.03	7.32
1957	10.8	13.8	6.77	1.236	1.084	3.793	6.09	7.19	5.81	8.24
	9.3	11.6	6.82	1.108	0.995	3.524	6.23	7.48	6.59	8.40
1958	8.1	9.9	6.76	1.026	0.925	3.402	6.73	8.20	7.54	7.93
	8.6	10.2	6.77	1.028	0.934	3.399	6.86	8.45	8.01	7.22
1959	9.3	11.7	6.37	1.070	0.973	3.465	6.97	8.54	7.50	6.94
	11.8	13.5	7.08	1.127	1.008	3.479	7.53	9.50	7.81	7.18
1960	10.7	13.6	6.50	1.139	1.009	3.602	7.02	8.92	6.52	7.20
	10.4	13.0	6.28	1.120	0.989	3.319	7.80	9.66	6.42	6.82
1961	9.8	11.9	5.90	1.105	0.952	3.123	7.03	8.88	6.02	6.71
	9.5	11.3	6.31	1.107	0.918	3.016	6.82	8.45	5.77	7.30
1962	8.9	10.0	6.16	0.989	0.843	2.760	6.93	8.25	6.44	7.30
	8.5	9.4	6.10	0.979	0.823	2.727	7.76	9.17	7.41	6.73
1963	8.7	9.9	5.79	1.001	0.830	2.776	11.59	9.95	8.54	5.88
	9.1	10.4	5.42	1.050	0.860	2.732	9.16	10.82	8.40	5.88
1964	8.7	9.6	5.49	1.038	0.844	2.676	8.94	10.71	7.74	6.57
	8.4	9.2	5.55	1.033	0.839	2.624	9.20	10.79	9.01	6.41
1965	8.1	8.7	5.61	1.017	0.817	2.588	9.85	11.29	10.26	5.66
	8.0	8.8	5.48	1.017	0.823	2.574	10.33	11.74	11.53	5.48
1966	8.6	9.6	5.62	1.057	0.869	2.660	10.28	11.86	11.29	5.48
	9.2	10.5	5.85	1.110	0.918	2.744	10.11	11.74	11.20	5.48
1967	9.3	10.8	5.71	1.125	0.937	2.731	10.00	11.41	11.45	5.52
	9.3	10.8	5.60	1.132	0.941	2.628	9.83	11.04	11.32	6.00
1968	9.1	10.4	5.74	1.129	0.936	2.660	9.89	11.17	10.71	6.11
	9.2	10.7	5.67	1.134	0.941	2.627	9.70	11.03	10.04	5.84
1969	9.2	10.7	5.50	1.141	0.941	2.596	9.63	11.04	9.28	5.91
	9.6	11.2	5.74	1.166	0.958	2.573	9.42	10.80	8.81	6.25
1970	9.4	10.8	5.67	1.169	0.949	2.522	9.63	11.01	8.65	6.25
	8.7	9.6	5.77	1.139	0.910	2.472	9.83	11.18	8.87	5.94
1971	7.8	8.5	5.42	1.096	0.869	2.367	10.85	11.81	11.19	5.47
	7.7	8.3	5.97	1.046	0.840	2.259	11.38	12.27	12.44	5.00
1972	7.6	8.5	5.33	1.056	0.853	2.237	11.41	12.25	12.40	4.47
	8.3	9.5	5.92	1.107	0.890	2.236	11.60	12.50	12.12	4.25
1973	8.9	10.8	5.58	1.157	0.921	2.390	10.58	11.79	9.93	5.75
	8.6	10.9	5.65	1.230	0.967	2.511	9.44	10.95	7.66	8.11
1974	8.7	10.4	6.13	1.319	1.021	2.760	8.97	10.50	7.36	9.00
	7.6	8.4	6.38	1.300	0.998	2.771	9.22	10.48	8.19	9.00
1975	6.3	6.5	5.62	1.191	0.909	2.561	10.10	11.14	9.97	8.03
	–	–	–	–	–	–	–	–	–	6.61
1976	7.4	8.0	5.81	1.240	0.964	2.575	9.94	10.73	10.27	6.51
	–	–	–	–	–	–	–	–	–	6.44
1977	7.3	7.8	5.66	1.244	0.994	2.634	9.91	10.63	11.07	5.64
	–	–	–	–	–	–	–	–	–	4.21

*) See Tables A2, A3 and A5. Source: [10].
**)See *the weighted average of the short rate* in Statistical Appendix. Source: [9]

References

[1] J. Barth, A. Kraft and J. Kraft, Estimation of the Liquidity Trap Using Spline Functions, *the Review of Economics and Statistics,* May 1976,

[2] P. Davidson, Keynes' Finance Motive, *Oxford Economic Papers,* March 1965,

[3] J. R. Hicks, Mr. Keynes and the "Classics"; A Suggested Interpretation, *Econometrica,* April 1937,

[4] H. G. Johnson, et al, "Readings in British Monetary Economics", (1972), Clarendon Press, Oxford,

[5] J. M. Keynes, "The General Theory of Employment, Interest and Money", (1936), the reprint ed. Macmillan & Co. Ltd. and Maruzen Co. Ltd.

[6] K. Miyazaki, Interest Payments, the Demand Deposit and the Hypothesis of the Firms' Liquidity-Maximization: A Basic Revision of Keynes' Interest Theory, *Keizai-Shirin* (the Hosei University Economic Review), September 1979,

[7] O. E. C. D., "OECD Financial Statistics", (1977-11),

[8] P. A. Samuelson, "Economics" (the 10-th ed.), (1976), McGraw-Hill Book Company,

[9] Statistics Dept., the Bank of Japan, "Economic Statistics Annual", (1977),

[10] Statistics Dept., the Bank of Japan, "Shuyo Kigyo Keiei Bunseki (Financial Statistics Analysis of Large Corporations)", (in Japanese), (1951-1977),

[11] Y. Suzuki, "Money and Banking in Contemporary Japan—Theoretical Setting and Its Application—" (in Japanese), (1974), Toyo Keizai Shimpo Sha Co. Ltd.,

[12] Y. Suzuki, On Changes in Interest Rates, and Their Influences on Business Investment and Bank Loans in Japan (in Japanese), *The Economic Studies Quarterly* (The Journal of the Japan Association of Economics and Econometrics), March 1968,

[13] A. Yajima and M. Tatemoto, An Econometric Study of Monetary Sector (in Japanese), *The Economic Studies Quarterly,* September 1966.