Interest Payments, the Demand Deposit and the Hypothesis of the Firms' Liquidity-Maximization : A Basic Revision of Keynes' Interest Theory

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Introduction

Modern economics consists of micro- and macro-economics, the latter of which is said to be originated by J. M. Keynes in his classical work "the General Theory of Employment, Interest and Money" (1936). His framework of analysis on the aggregate economy and economic policies forms the backbone of the macro-economic parts in Samuelson's "Economics," ((5)) which puts a greater stress on the income-expenditure aspect of Keynes' theory than on the liquidity preference theory in it: the propensity to consume, the private investment and the government expenditure determine the equilibrium level of production, and the rate of interest plays the only role of influencing the level of investment, being regarded as a parameter. The LM-IS analysis appears just in an appendix. It is in the LM-IS formulation that the substantial content of his *General Theory of Interest* is said to be neatly expressed, but, unfortunately, the ill-founded liquidity preference theory of Keynes himself

has prevented the interest theory from being fully persuading.

What is the rate of interest?—it is the price that a man earns for parting with liquidity, rightly so says Keynes. But he fails to go on to ask why he can earn yield for doing so: the interest comes from the total profit of the firms. It is kept ambiguous in Keynes' book who pays interest ultimately whom: the firms, which are the payer of interest, *ultimately* pay it not only to the wealth-owners but also to the banks.

Keynes' original explanation of liquidity preference does not clarify at all what is interest: there seems to be few lines to relate it to the contribution of the capital assets (the fixed assets and inventories) to production. He does not quantitatively relate the rate of interest to the total profit. Despite the title of the book "the General Theory of Interest," there seems to be no solid theory of interest to explain its relatively stably positive existence except the psychologically, and too loosely, founded theory in which the rate of interest is determined by the marginal efficiency of capital and the liquidity preference, both of which are highly subjective and lack a satisfactory theoretical foundation in relation to the aggregate production function.

Why interest payment must be stably positive in equilibrium?— because the marginal efficiency of capital and the liquidity preference are both positive—such an answer is obviously unsatisfactory, which would lead to another question *in what way the marginal efficiency of capital, and, if any, the liquidity preference, are related to the productivity of capital assets;* or more fundamentally, why and how the rate of interest differs from the average or marginal product of capital assets. These questions I will consider in this paper.

(1) *Fundamental Ideas*

  (1-2) *The Economic Agent to Demand Money is Mostly the Firms*

In the "General Theory" (Keynes, (1)) an entrepreneur, or a speculator is often called 'a man' or 'an individual'. I call both the entrepreneurs and the speculators simply "the firms" throughout this paper. It will be postulated that the main demander for money is the firms: this notion is the starting point of the whole argument. (See J. Viner, (4), p.260.)

(1–2)   *The Speculators are Identified with the Firms*

The speculative motive for liquidity is not clearly identified as one of *the firms'* motives for money along with the other two by Keynes (Keynes (1)). In this paper it is assumed that *nobody other than the firms demand speculative money*: the speculators are identified with the firms.

(1–3)   *The Firms Maximize Liquidity With Capital Assets Constant*

In the "General Theory" (Keynes, (1)) the liquidity function $L_2(i)$ is founded mainly on the speculators' psychological reaction to a change in the market rate of interest especially in the consol market, and on the auxiliary supposition that men will hold a less (or greater) amount of interest-bearing securities when the market rate falls (or rises.) Thus it is assumed there that the liquidity function describes both the speculators' behavior in financial markets and the 'rentiers' ' asset-choice. In this paper these two classes of economic functions are distinctly discerned into the firms and the 'individuals', respectively; it is assumed that *the firms* do not balance between the interest-bearing and barren forms of assets (which is thought to be the 'individuals' ' job here) but simply *try to maximize liquidity given constant capital assets, subject to their current capacity of paying interest and to the banks' reserve requirement.*

(1–4)   *The Banking System Has to be Considered*

In the "General Theory" (Keynes, (1)) no systematic account is given on the banks' credit balances. In this paper the banking system

is explicity analyzed: it will be shown how the banks can be regarded as an ultimate claimant to a substantial part of the firms' borrowed funds, or specifically their demand deposit.

(1–5) *The Individuals' Asset-Choice Separated From the Firms' Money Demand*

In this paper a balance sheet formulation is made in respect to the asset-choice of the individuals which are regarded as distinct from the firms.

(2) *What is New*

(2–1) *Analysis of Interest Payments is Introduced*

A crucial defect of Keynes' classical work "the General Theory" (Keynes, (1)) is its failure in taking interest payments into consideration in the static framework. There seems to be a theory of determination of the *rate of* interest but not a theory as to who pay interest to whom, or a theory of interest *payments*. This the present paper is going to approach.

(2–2) *Clearing up the Mystery of the Dichotomized Liquidity Function*

Another defect of the "General Theory" (Keynes, (1)) is the over-simplified formulation of the money demand function in it, or the dichotomy of money demand into $L_1(Y)$ and $L_2(i)$: Keynes thinks that this dichotomy is permissible at least conceptually. This paper will present a money demand function which is not based on this conceptual dichotomy but is based on a completely different approach to it, namely on what may be called *"the interest payment approach"*, which I introduce first here.

(2–3) *Illuminating the Ambiguous 'Speculative Demand for Money'*

In the "General Theory" (Keynes, (1)) the 'speculative demand for

money' is not clearly formulated as distinct from *the demand for money to hoard,* remaining to be a most confusing idea, and in this paper it will be clearly discerned that *the speculatively demanded money is not hoarded but active money, or that it is not idle but quickly circulating.* (E.g., remember the term "the *quick* ratio", which means cash plus marketable securities divided by current liabilities: the numerator can be regarded to contain the firms' speculative money!)

(2–4) *Clarifying the Relation between the Interest Rate and the Profit*

In the "General Theory" (Keynes, (1)) there is no clear mention on the relation between the *rate* of interest and profits of the firms: there seems to be overlooked that the total amount of the firms' profits influences their capacity to pay interest. In this paper it is postulated that *the greater the firms' aggregate net profit is, the greater is the firms' capacity to pay interest, and therefore the greater amount of money they will demand* (to borrow from the lenders, irrespective of whether it is actually supplied or not.)

(2–5) *Assuming the Perfect Short- and Long-term Security Markets*

There is not made explicit in the "General Theory" (Keynes, (1)) the existence of the perfectly developed markets of short- and long-term loans or securities among which and the money market the speculators shuttle to equalize the rates of interest in the markets. In this paper this assumption of the perfect security markets will be postulated.

(3) *Other Theoretical Results*

(3–1) *The Banks Ultimately Earn More Than Interest Margins*

The banks are shown to earn *the actual rate of interest multiplied*

by the demand deposit, without repaying it to the depositors, in addition to interest margins.

(3–2) *The Individuals Are Not Claimants To All the Assets of The Firms*

It will be shown that as long as the individuals' balance of bank notes is smaller than the firms' demand deposit, *the individuals' total assets fall short of the total assets of the firms,* or the sum of the capital assets (including inventories) and the money assets of the firms. (See Fig. A–2.)

(3–3) *The LM Curve is Related to the Aggregate Supply Function*

It has been inconceivable in the conventional argument to relate the LM curve to the aggregate supply curve directly, but in this paper the *LM curve is positioned as representing a cost rate confronting the firms which is going to produce, i.e., representing the interest cost per unit of the firms' total assets.* Thus it is related to the aggregate supply function in a simple way.

Keynes' Dichotomized Liquidity Functions are Unreliable

In Keynes' "General Theory" the 'liquidity functions' $L_1(Y)$ and $L_2(i)$ are introduced to play crucial roles in his whole theory. However it has been suspected that this functional separation of the money demand is too much simplified to describe actual circular flows of active money. The 'financial circulation', though conceptually distinct from the 'industrial circulation' (Keynes (2), Chap. 15), cannot be imagined as clearly separated from the latter. The function $L_2(i)$, or the liquidity preference function, is accompanied with ambiguities: as to who prefers liquidity, why it is thought to be negatively sloped, etc.

In this paper I will approach the theoretical explanation of the

money demand function, $L(Y, i)$, from a completely different angle, or from the stock (as against flow) viewpoint: money is regarded as a constituent of the aggregate assets of an economy which costs the holder some positive interest *not* as any kind of opportunity cost but as a cost for the holder of money to have to pay actual interest at a certain positive rate to the banks and the individuals. I will consider money *not* as barren *but* as something that costs its holder actual interest payment.

In reality the holder of money is dominatingly the firms (consider who holds demand deposit); they borrow money from the banks, etc. In this paper, after clearly formulating the balance sheet relationship, the firm-dominated money market is analyzed.

The class of individual speculators, who demand money for the speculative motive, may be regarded as part of businessmen, roughly identified with the firms: so there are only two economic agents which influence the simple economy: the firms and the individual consumers (abbreviated by "the individuals".)

Shortly speaking our new explanation on the money demand function $L(Y, i)$ may be summarized as follows: the firms demand a greater amount of money when their net profit rises, because then they can afford to pay more interest to the lenders of funds. Hence the positive L_Y. Therefore, with a fixed M the firms can afford to pay a higher rate of interest when the net profit rises. Hence the positively-sloped LM curve, or the negativity of L_Y/L_i, and so the negativity of L_i. Behind this, in addition to the assumption that the higher is the level of production, the higher is the level of net profit, there is *the assumption that the firms prefer to hold as much money as possible, so far as they can pay interest from the net profit or the profit after depreciation, interpreting the 'liquidity preference' as the firms' behavioral criterion of maximizing their liquidity at hand;* which sup-

posedly works in statics as the sole objective of the firms in their stock (as against flow) decision making, and therefore which is independent of and consistent with the other criteria, i.e., to maximize the flow of current profit from production, and to maximize net expected return in the future from investment.

Notations

The following notations are defined all in *stock* terms.

$\overline{H}_b$ = (the banks' reserve money)

$\overline{D}_g$ = (the government's loan to the banks)

H_h = (the bank notes of the individuals)

L_h = (the deposits and loans of the individuals)

$\overline{A}_h$ = (the total assets of the individuals)

L = (the total loaned funds through the banks)

$\overline{K}$ = (the capital assets of the firms)

D_f = (the deposit balances of the firms)

H_f = (the bank notes of the firms)

M_f = (the total money balance of the firms)

A = (the total assets of the firms)

The following are in non-stock terms.

N = (the effective work employed)

Y = (the total production)

p = (the general price level of goods)

$\overline{w}$ = (the money wage rate)

i = (the rate of interest)

$\overline{c}$ = (the required reserve ratio)

$\overline{d}$ = (the ratio of depreciation to the capital assets)

q = (the proportion of the individuals' lending through the banks)

Those notations with bars are constant. Definitions of functional relations will be introduced in the text.

§1 The Firms' Dominance in the Money Market

(1) *The Balance Sheets*

(1–1) *The Individuals*

The individuals' balance sheet is written as follows:

$$L_h + H_h = A_h, \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

The variable L_h, or 'the deposits and loans of individuals', includes not only their deposits and short-term loans, but also their holdings of long-term securities, shares, and other assets. The individuals are assumed to hold no real assets for simplicity. They supply L_h in order to earn interest (regarding dividends as a part of interest on the loans to the firms), and their supply of L_h is assumed to be an increasing function of the ex-ante rate of interest (denoted by i), i.e.,

$$L_h = L_h(i), \text{ and } dL_h/di > 0. \dots\dots\dots\dots\dots\dots\dots\dots (2)$$

The individuals hold a proportion (q) of L_h in the form of deposits, and its remaining part $(1-q) \cdot L_h$ in the form of direct holdings of shares, bonds, etc. I assume that $d((1-q)L_h)/di \geqq 0$.

Their total assets, or their accumulated savings, A_h, is assumed constant in the static scheme. So H_h may be written as follows:

$$H_h = H_h(i), \text{ and } dH_h/di < 0, \dots\dots\dots\dots\dots\dots\dots\dots (3)$$

which shows that their demand for bank notes and coins (H_h) is a decreasing function of the rate of interest.

(1–2) *The Banks*

The banks' balance sheet is written as follows:

$$H_b + L = qL_h + D_f + D_g, \dots\dots\dots\dots\dots\dots\dots\dots (4)$$

where H_b is the banks' reserve, and D_g is the government's loan to the banks. The variable L, or 'the total loaned funds through the banks', plays a key role in the following argument, and denotes *the*

total liabilities of the coordinated firms to the banks. (About the coordinated firms see (1–3), below.)

The banks' loan to the firms, or L, is restricted by the required reserve ratio, c, which is assumed constant. (See the eq. (11), below.) The government can change the level of its lending balance to the banks, D_g, at discretion: it can add to, or contract D_g without worrying about its budget or tax revenue. Thus D_g is an independent policy variable, or a parameter.

The individuals are assumed to lend 100q per cent of their funds to the firms through the banks: it means that they, as stock and bond holders, lend their remaining capital to the firms not through the banks.

The banks are assumed to pay interest at the rate i on the individuals' deposits qL_h, and on the government's loan to them, D_g. They do not pay any interest on the demand deposits of the firms, D_f.

They receive interest on the loan, L, at the rate i, from the firms. Since they do not pay interest on the demand deposits, and since D_f is empirically much greater than H_b, or the reserve money, the banks can earn the difference of the interest receipt and the interest payments, $(D_f - H_b) \cdot i$.

(1–3) *the Firms*

Each firm's balance sheet contains lending and/or borrowing balances in relation to other firms, e.g., accounts and notes receivable and payable, holdings of other firms' shares and bonds. When we consider the firms as a whole, or *the coordinated firms,* we aggregate balance sheets over all the firms, and cancel out these stock transactions among firms leaving only the total fixed (tangible and intangible) assets, the total inventory stocks, and the total monetary assets (bank notes, demand deposits, short-term government securities, time deposits, etc.) on the debit side, and the total borrowings from the banks

and the individuals, including the equity capital and surpluses, on the credit side. For simple exposition, it is assumed throughout this paper that the firms' monetary assets consist only of currency and demand deposit and that the surpluses are regarded as a part of the individuals' direct lending to them, any current addition to the surpluses being regarded negligible relative to the surpluses.

The fixed assets and inventory stocks are called in sum "the capital assets" in the following, denoted by K.

The firms' balance sheet is written as follows:

$$K + (H_f + D_f) = L + (1 - q) \cdot L_b, \dots \dots \dots \dots \dots (5)$$

and the firms' money balance, M_f, is defined by

$$M_f = H_f + D_f. \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (6)$$

The firms pay interest on both L and $(1 - q) \cdot L_b$ at the rate i, to the banks and the individuals, resp. The capital assets K is assumed constant in this static model.

(2) The Effect of the Individuals' Asset-Choice on the Firms' Money Balance

By the three balance sheet identities, (1), (4), and (5), and (6), we have

$$K + M_f = (q \cdot L_b + D_f + D_g - H_b) + (1 - q) \cdot L_b$$
$$= (A - H_h) + D_f + D_g - H_b, \dots \dots \dots \dots \dots (7)$$

and also, by this equation and (6),

$$A_h + D_g - H_b = (K + H_f) + H_h. \dots \dots \dots \dots \dots \dots (8)$$

The eq. (8) shows that, since the left-hand side of it and K are constand, the H_h, or the individuals' demand for bank notes can influence the firms' balance through its effect on H_f, or the firms' demand for bank notes, provided that $H_b - D_g = $ (const.) However H_h will be shown below to be able to affect the firms' total assets, or $K + M_f$

also through its effect on D_f.

I assume that the variable H_b can change satisfying the following equation:

$$H_b - D_g = (\text{const.}) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (9)$$

Since the firms' money balance consists not only of H_f but of D_f, the supply of money to the firms can be influenced by the individuals' asset choice simbolized in the function $H_h(i)$ through both H_f and D_f.

(3) *The Liquidity Preference of the Firms*

In this model speculation is considered not as part of individuals' behavior, but as part of the firms' behavior. The 'firms', therefore, are regarded to consist of entrepreneurs and speculators, the latter demand as much liquid assets as possible to make use of opportunities to earn capital gains. The liquidity preference comes from the speculative motive *of the firms,* and not from the individual speculators. Simbolically the liquidity preference of the firms may be expressed as follows: the firms' behavioral criterion in respect to stock variables (as against flow) is assumed to be:

max. M_f by changing M_f itself subject to the restraint (see Fig. 1)

$$(F_K(N(Y), K) - d) \cdot K \geqq i \cdot (K + M_f), \dots\dots\dots\dots\dots\dots (10)$$

where the left-hand side of the inequality (10) denotes the coordinated firms' total profit (see Section 3, below) *less* depreciation $(d \cdot K)$ from which they pay interest (including dividends on equity capital) on their total borrowed funds, which in turn is always equal to $K + M_f$ or their total assets, at the rate i. Behind the formulation of the right-hand side of (10), the perfect market of securities including stocks is assumed to exist. (See p.72 below.)
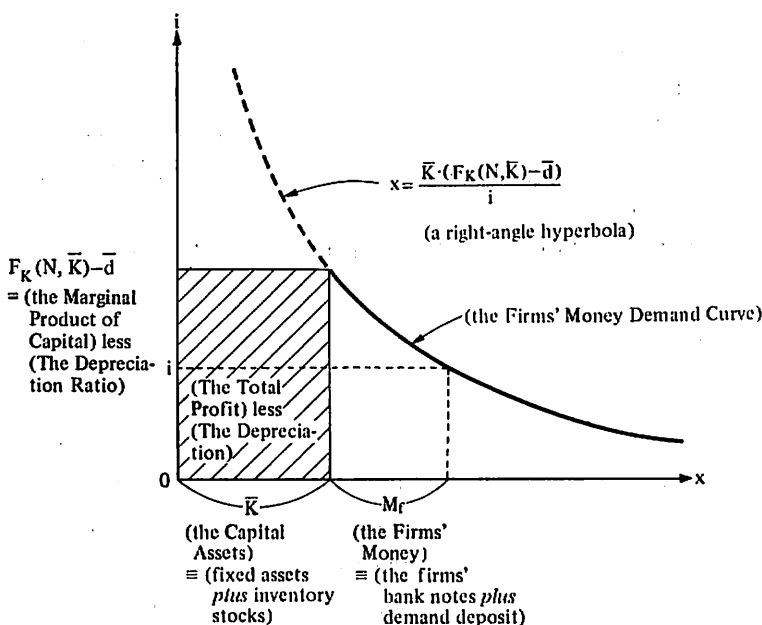
Fig. 1 (The Firms' Money Demand Curve)

(4) *The Firms' Demand Deposit*

See the eq. (7), in which there are only two dependent variables on the right-hand side, H_h and D_f. Now D_f is a major constituent of M_f and appears also in the eq. (4). The firms, or their speculative divisions, behave to maximize M_f subject to the other constraint imposed by the banks and government:

$$c \cdot (D_f + qL_h + D_g) \leqq H_b, \dots\dots\dots\dots\dots\dots\dots\dots\dots (11)$$

in addition to the constraint (10). By this eq. (11), and the eq. (7),

$$K + M_f \leqq (A_h - H_h) + ((H_b/c) - qL_h - D_g) + D_g - H_b$$
$$= ((1 - c)/c) \cdot H_b + (1 - q) \cdot L_h. \dots\dots\dots (12)$$

The first and second terms on the right-hand side of (12) denote the

maximum attainable deposit of the banks less the bank reserve, and the stock holdings and other loans of the individuals not through the banks, resp. Since the firms try to maximize M_f, the inequality (12) will hold with the equality sign.

(5) *The Determination of the 'Earned' Rate of Interest*

The equation (10) can be read as the demand schedule for money of the firms, because it expresses how much money the firms are willing to hold at each given level of the rate of interest, given the total net profit $(F_K - d) \cdot K$. Since K and d are constant the demand curve for money of the firms can be depicted as negatively sloped, given a level of total production Y.

On the other hand the eq. (12) can be read as the supply schedule of the banks and the individuals of money to the firms, because from (12)

$$M_f \leqq ((1 - c)/c) \cdot H_b + ((1 - q(i)) \cdot L_h(i) - K) \ \ldots \ldots (12)'$$

where $d(1 - q)L_h/di \geqq 0$, and H_b and K are constant, as assumed above. The second term on the right-hand side denotes the part of the individuals' assets which they lend to the firms not through the banks less the capital assets of the firms: the right-hand side as a whole represents how much funds the banks and the individuals are willing to lend to the firms at each given level of the rate of interest. It can be interpreted as the supply function of money.

A rate of interest is uniquely determined by the intersection of the demand and supply schedules of money, represented by (10) and (12), resp. I call this rate *"the earned rate of interest"* hereafter. The adjustment speed of the earned rate of interest is assumed infinite, and it means that the desired levels of the demand for money and the supply of money always coincide to each other by the equilibration of the ex-ante earned rate of interest: the actual quantity of the firms' money balance and the actual (ex-post) earned rate of interest
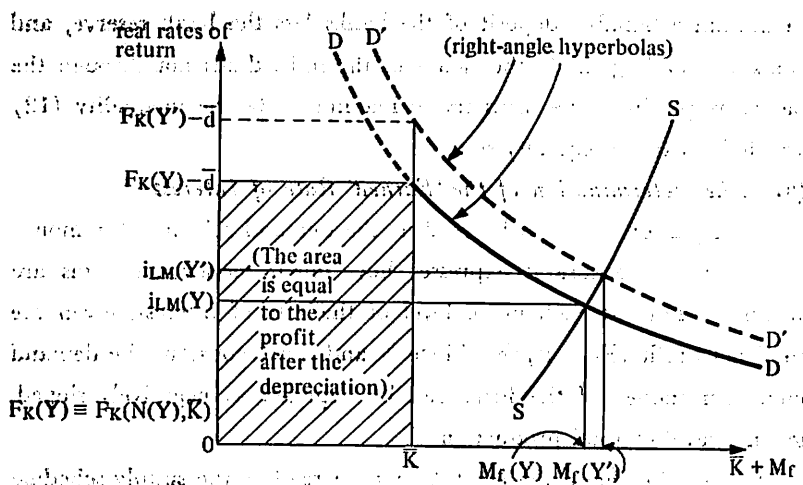
Fig. 2 (The Determination of the actual Earned Rate of Interest (i_{LM}))

are always on both of the demand and supply curves of money.

See Fig. 2. The right angle hyperbola DD depicts the demand curve for money, derived from the eq. (10), and the upward sloping curve SS depicts the supply curve of money, derived from the eq. (12). The coordinate of the vertical axis of the intersection point of the two curves gives the actual earned rate of interest, denoted by $i_{LM}(Y)$: since the demand curve is drawn corresponding to each given Y, the actual earned rate of interest i_{LM} is a function of Y. If Y rises, the total net profit $(F_K - d) \cdot K$ rises, and it enables the firms to demand a greater amount of money *at each level of the interest rate*: if Y rises, therefore, the DD curve shifts to the right, raising the earned rate of interest: Thus $di_{LM}/dY > 0$.

By the eqs. (1), (6) and (7),

$$H_f = L_h(i) + D_g - H_b - K \dots\dots\dots\dots\dots\dots\dots (13)$$

In equilibrium, in view of (12) and (13), it holds that

$$D_f = (1/c) \cdot H_b - D_g - q(i) \cdot L_h(i) \dots\dots\dots\dots\dots\dots (14)$$

So the SS curve is the horizontal sum of the supply curves of H_f and D_f defined by the eqs. (13) and (14), resp. (See Fig. 3).

The government can change H_b by changing D_g keeping the eq. (9). It is easily seen that, when H_b is increased in this way by the government, the supply curve of H_f is constant but that of D_f shifts to the right by the amount $((1 - c)/c) \cdot \Delta H_b$, where $((1 - c)/c)$ is the credit expansion multiplier. Thus the government can control M_f through its effect on the supply curve of D_f: if it raises (reduces) H_b, the SS curve shifts to the right (left), by the multiplier times ΔH_b (See Fig. A–1.) With Y constant, the earned rate of interest i_{LM} falls and M_f in equilibrium rises: the firms' actual money balance will be written as

$$M_f = ((F_K(N(Y), K) - d)/i_{LM}(Y; H_b)) - K \dots \text{(by (10))}$$
$$= M_f(Y; H_b), \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(15)$$

(where $i_{LM}(Y; H_b)$ is the i-coordinate of the intersection of the DD and SS curves, drawn given Y and H_b.)
It holds that $\partial M_f/\partial Y \gtreqqless 0$ and $\partial M_f/\partial H_b > 0$. Given Y, M_f becomes a function of i_{LM}, which in turn is controlled by H_b. $(dM_f/di_{LM} < 0.)$

§2 The Stock and Flow Decisions of the Firms

(1) *The LM Curve, Reconsidered*

The function $i_{LM}(Y; H_b)$ gives the relation between Y and the actual earned rate of interest, i_{LM}, as the locus of all the points satisfying that (1) the firms' quantity of money demanded at each i_{LM} is realized, that (2) the individuals' desired proportions of deposits, lendings and bank notes held out of their accumulated wealth are realized earning the corresponding interest, and that (3) the banks are satisfied in succeeding in 'loaning up' to the required reserve ratio, becoming able to earn the
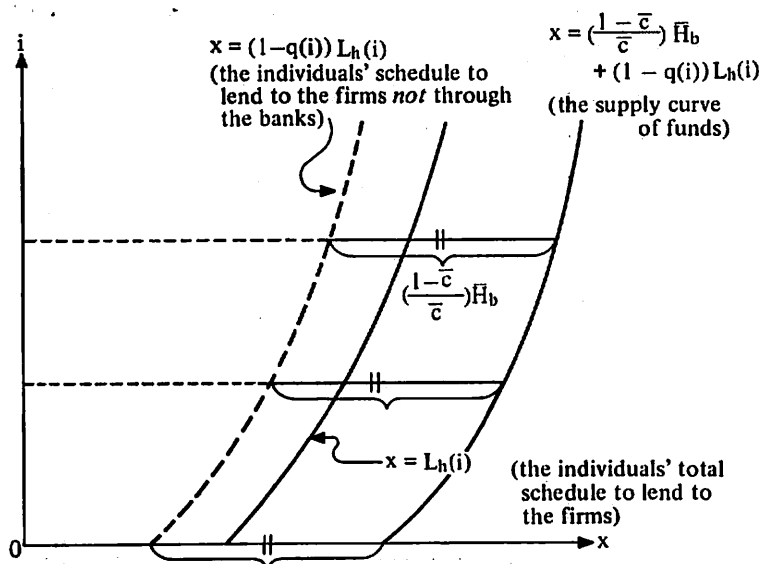
Fig. 3 (The Supply Curve of Funds) ($D_g = H_b$ is assumed.) [The banking system can add to the supply of funds *over and above* the total scheduled lending of the individuals.]

maximum amount of interest receipt subject to that requirement. This locus, corresponding to each fixed level of H_b, may be named here the LM curve. The LM curve thus defined gives an expression of an important aspect of macro-economy, quite different from that expressed by the conventional definition of the usual LM curve. (See Figs. 2, 4 and A–1.)

(2) *The Perfect Markets of Short- and Long-term Securities*

Given Y, the level of i_{LM} is the *actual* earned rate of interest, at which the firms' activities are yielding net profit per unit of assets: they are also speculating, yielding some marginal gains in order to do which
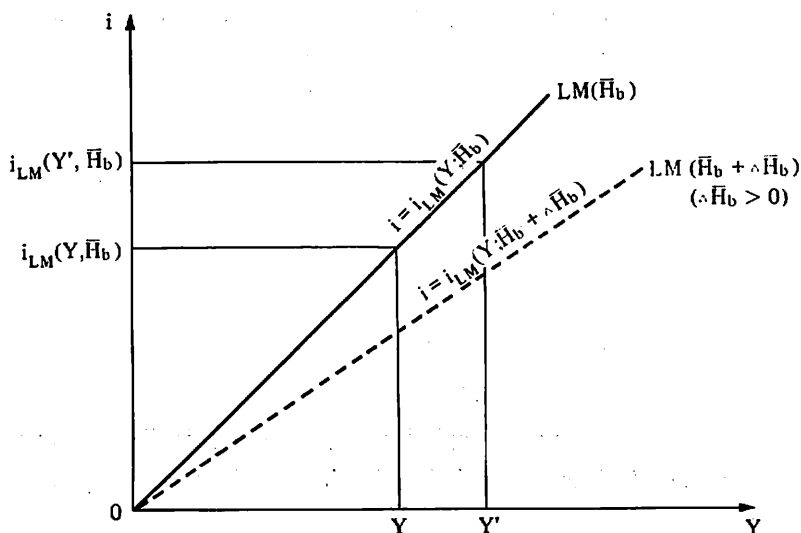
Fig. 4 (The Relation between the Actual Earned Rate of Interest and the Total Production: the LM curve.)

they hold speculative money. Speculation is going on not only in various goods markets but also in financial markets: the firms buy and sell short- and long-term loans, securities and stocks to get interest margins. In so doing they are competing with each other to get as much shares as possible in the total profit after depreciation, $(F_K(Y) - d) \cdot K$: the total net profit accrues not only to the claimants to K but also to the claimants to M_f, and the firms thus have to earn yields on their total assets including the speculative money at the rate i_{LM} on the average: speculation, as long as it is not excessive, serves to raise efficiency in resourse allocation, to stabilize relative (as against absolute) market prices, and therefore to raise stability of distribution and production of commodities. It serves to reallocate funds among firms or industries, among various financial markets.

Thus the firms' money including speculative is an *active* balance,

circulating from one commodity market to another, from one financial market to another, and from a financial market to a commodity market, or the other way around. Without this active circulation of the firms' money, the material production behind the produced value F(N, K) will not have as much value as it has in its presence: the firms' money M_t does not contribute *directly* to the material production, but it works to enhance efficiency in resource allocation, not only in the flows of distribution of commodities, but also in the financial stocks of allocated funds, and thus indirectly works to keep up the value of the material commodity production.

The marginal product of capital assets, $F_K(N, K)$, is different from the marginal efficiency of capital, $i_{IS}(I)$, in that the former represents the real aspect of the relation of capital assets K and production Y, and the latter represents the *financial* relation of (additionally) invested *monetary funds* and the expected future return of investment. On the assumption of the presence of a perfect short-term security market, in which the speculators act, the earned rate of return of these securities will tend to be equal to the policy rate of interest (as distinct from the earned rate) which is determined by the government's monetary policy, because, without uncertainty premium, if the earned rate of return is higher than the policy rate of interest, the firms will come to borrow extra unit of money, and go to buy securities, thus gaining the difference between the two rates, and this action will equalize the rate of return and the policy rate of interest. However *I assume hereafter in this paper that the policy rate of interest is set so as to be always equal to the earned rate of interest.* This assumption is a first approximation of the actual interest rate policy in post-war Japan. (A forthcoming paper of the present writer will empirically examine this and related propositions suggested in this paper.)

   In the presence of perfect short- and long-term security markets, a firm can not only lend excessive liquidity (by buying securities of other firms) but also borrow it from other firms (by selling securities.) To the firm who intends to purchase newly produced investment goods, there are four ways in this simple scheme to raise funds to finance their own investment: (1) by using a part of its own money balance, (2) by borrowing at the money market, (3) by borrowing at the security markets, and (4) by selling securities at the security markets.—In any of these cases, the firm incurs financial cost at the marginal rate of i_{LM}, or the earned rate of interest: (1) the cost of parting with opportunities of getting capital gain at the marginal rate of i_{LM}, (2) $\sim$ (3) the cost of interest payment at the rate i_{LM}, and (4) the cost of parting with interest receipt from other firms.

   The firm therefore compares the marginal efficiency of capital, or the expected rate of return on each marginal unit of gross investment, with the actual earned rate of interest $i_{LM}(Y)$: The firm desires to maximize expected net return by determining the level of gross investment so as to equalize the marginal efficiency to the $i_{LM}(Y)$. However the *actual* level of its investment may *not* coincide with the level at which the marginal efficiency is equal to $i_{LM}(Y)$, since the long-term expectation of the firms may not be realized, because it costs the firms considerable adjustment cost to change the level of gross investment.

(3)   *The Income Determination*

   See Fig. 5. The variable Y, $i_{LM}(Y)$, and $i_{IS}(I)$ denote the *actual* level of production, the *actual* earned rate of interest, and the *actually expected* marginal rate of return (the marginal efficiency of capital), resp.: the actual level Y may be different from that at the intersection of the LM and IS curves; but if so, there is a motive for the

firms to change the actual Y so as to maximize expected net return. The adjustment is stable if, as is safely assumed, the IS curve is less sloped than the LM curve. The intersection of the LM and IS curves is *not* always realized, though it is a point where the firms are maximizing *expected* profit. (See p. 25. 1. 22. of Keynes (1), and also Section 3 below.)

Summary of Sections 1 and 2

The firms have three variables to determine; the levels of production, investment and liquid assets, in statics. As we assume that the multiplier process completes instantaneously, the level of total production is determined at the same time as the level of total (gross) investment is: so *the firms have two variables, or the levels of production and liquid assets, to determine.* When they think about changing the level
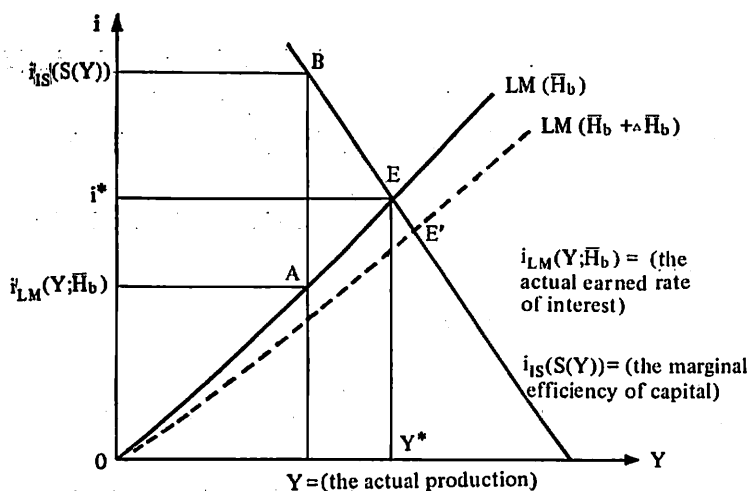

Fig. 5 (The Actual and Equilibrium Levels of Total Production and the Earned Rate of Interest)

of capital assets (i.e., fixed, and inventory assets) it is in dynamics, not in statics: they can do so in the long run by new (as against depreciation) investment. In statics any change in the level of capital assets caused by new investment is assumed to be too small compared with the total capital assets to affect either the productive capacity or the demand schedule of liquid assets.

The firms refer to the ex-ante earned rate of interest, on one hand, and to their total net profit, on the other, when demanding liquid assets: the lower is the former, and the greater is the latter, do they demand the greater stock of liquid assets. (Since the latter, or the total net profit, is theoretically correlated positively with the level of total production, the higher is the total production, they demand the greater stock of liquid assets.)

The ex-post earned rate of interest is determined by the interaction of the firms' demand for money (or liquid assets) on one hand, and the money supply of the banks and the individual wealth-owners who have a not vertical but upward-sloping supply curve of money on the other. Given a level of total production the firms can choose a level of liquid assets only subject to the constraint imposed by the ex-post earned rate of interest, which, in turn, reflects not only the banks' but also the individuals' supply conditions of money: the firms compare the ex-ante earned rate of interest with their ratio of the total net profit to the total assets. At a higher level of the net profit rate they demand a greater stock of money, but then the ex-post earned rate of interest goes up at the money market: so, *at a higher rate of net profit they are also faced with a higher level of the ex-post earned rate of interest, which reflects the higher rate of net profit.*

If the supply curve of money is vertical, as is the case if the individuals are insensitive to any change in the ex-ante rate of interest, a higher rate of net profit will make the ex-post earned rate of interest

to be higher than before by the same proportion as the level of net profit is than before. (See the eq. (10).)

To the firms both the level of production and the level of money have upper bounds: the full-employment ceiling and the money supply curve. Under the hypothesis of the firms' liquidity-maximization, the latter bound is always an effective constraint, but the former is not, since we are here considering the under-employment economy. Therefore the firms can change the level of employment, or of production, without constraints, whereas they are always effectively restrained by the money supply schedule. *Since the money supply curve is generally upward-sloping, the actual level of the firms' money stock does change when the actual level of production changes*: they theoretically correlate positively in the short run, just as the actual level of production and the actual earned rate of interest do so.

As noted above the firms have two variables to determine: the level of production and the level of their money stock. These two are positively correlated in the eq. (15). So the firms have ultimately only one variable to determine, or the level of total production, which, as assumed in this paper, is in one-to-one correspondence to the level of (gross) investment.

How do they determine the level of total production (or employment)? They do so mainly in reference to their own expectation about future profitability schedule of investment, or the schedule of marginal efficiency of capital. More rigidly the level of total investment is determined mainly by the marginal efficiency schedule and the policy rate of interest (which is assumed to be equal to the actual earned rate i_{LM}, in this paper for simplicity), and then it determines, by the assumed instantaneous income multiplier adjustment, the level of total production. (See Fig. 6).
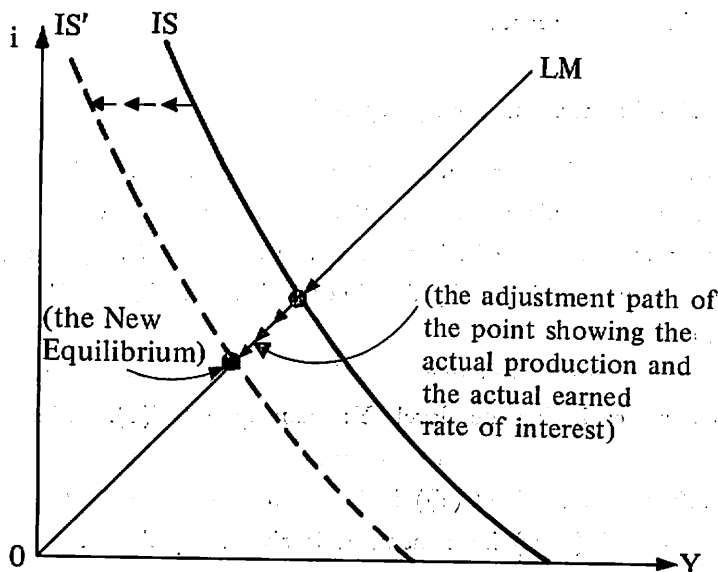
Fig. 6 [The Actual Earned Rate is Adjusted *Along* the LM Curve.]

§3 Production, Incomes and Aggregate Demand (This Section is partly due to Uzawa, (3).)

(1) *Production Function*

The capital assets, K, is defined to consist not only of the firms' fixed (tangible and intangible) assets but also the inventory stock, which are summed over all the private corporations in a national economy.

The effective work employed, N, is defined to be work services employed in all the corporations.

The Neoclassical production function with the well-behaved properties

$$Y = F(N, K), \quad F_N, F_K > 0, \quad F_{NN}, F_{KK} < 0 \text{ and } F_{NK} = F_{KN} > 0$$

is postulated: the linear homogeneity, the twice continuously differ-

entiability are assumed.

(2) *The Incomes and the Aggregate Demand*

By the first of Keynes' so-called "fundamental postulates of the classical theory of employment," i.e., that the real wage rate is equal to the marginal product of labor, the total wage is $F_N(N, K)N$, and the remaining part of Y, or, by the Euler's Theorem, $F_K(N, K)K$, is the total profit.

The consumption function $C(\cdot)$ gives the demand for consumption goods at each income level Y: namely $C=C(Y)$. The $C(Y)$ expresses the *actual* level of consumption, when Y is the actual level. Therefore $S(Y)$ $(=Y - C(Y))$ is the actual level of total savings, and so the actual (gross) investment, I, is always equal to the amount $S(Y)$ when Y is the actual level.

Then, imagine that the actual production is Y, and so the actual investment $I = S(Y)$. The earned rate of interest is actually $i_{LM}(Y)$, with which the firms compare the marginal efficiency of capital at the actual level of investment, $I = S(Y)$. They compare $i_{IS}(I)$ $=i_{IS}(S(Y))$, or the marginal efficiency of the *actual* investment I, with the actual earned rate of interest, $i_{LM}(Y)$. They may be different from each other: there is no guarantee for them to coincide. See Fig. 5. The points A and B are both actual: they will approach the equilibrium point E *along the LM and IS curves, resp.* The firms will increase (decrease) actual investment whenever $i_{IS}(S(Y))$ is greater (less) than $i_{LM}(Y)$, and then the income multiplier process completes instantaneously, the *actual* production rises (falls), finally to attain the full equilibrium level Y^* at the point E. Remark that there is no appearance of unintended changes in inventory: the firms decide to change the actual production level in view of only the difference of

$i_{IS}(S(Y))$ from the actual earned rate of interest $i_{LM}(Y)$.

In summary the full equilibrium condition is simply

$$i_{LM}(Y) = i_{IS}(S(Y)) \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(16)$$

where $i_{LM}(Y) = (F_K(N(Y), K) - d) \cdot K/(K + M_f(Y; H_b))$, and the i_{IS} (I) is the marginal efficiency of capital corresponding to the investment level I. The adjustment criterion of the investment level is postulated as follows;

$$dI/dt = h \cdot (i_{IS}(I) - i_{LM}(S^{-1}(I))) \quad \dots\dots\dots\dots\dots\dots\dots\dots(17)$$

where $S^{-1}(\cdot)$ denotes the inverse function of $S(\cdot)$, and h is a positive constant.

Since there is postulated no investment function, there is no aggregate demand function, as something to appear in the full equilibrium condition(s). Moreover neither Walras' law nor Say's law work anywhere in the model: who changes the actual level of production is the firms, and as long as the propensity to consume stays constant, any shift of the individuals' preference to lend, or $L_h(i)$, will not directly change the actual level of production, but change only what the actual production tends to approach, or only the full equilibrium level of production.

(3) *the Price Level and the Money Balance*

Assuming perfect competition in the goods market, and assuming the money wage rate (w) constant, the general price level, p, is set by the market to satisfy

$$w/p = F_N(N, K), \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(18)$$

because the firms are assumed to maximize profit $p.Y - w.N$, subject to the short-run production function $Y = F(N, K)$, where K is constant. There is the following monotone relation between N and Y

$$Y = F(N, K), \text{ or } N = N(Y), \quad \dots \dots \dots \dots \dots (19)$$

where $N(\cdot)$ is defined by the inverse function of $F(N, K)$ with K constant. By (18) and (19), we have

$$p = p(Y) = w/F_N(N(Y), K), \quad \dots \dots \dots \dots \dots (20)$$

where $dp/dY>0$, since $F_{NN}<0$ and $dN/dY>0$.

The inequality (10) is changed to read

$$(F_K(N(Y), K) - d) \cdot K \gtreqqless i \cdot (K + (M_f/p))$$
$$= i \cdot (K + (M_f/w) \cdot (F_N(N(Y), K))),$$
$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (21)$$

where i is the ex-ante real rate of interest, and (M_f/p) denotes the firms' real money balance. Since (21) always holds as an equality, the actual rate of interest is uniquely determined as a function of Y, which is an increasing function of Y, with M_f constant, and is a decreasing function of M_f, with Y constant. Thus the LM curve has the same properties in this variable price case as in the previous simpler case; the LM curve is positively sloped, and shifts to the right when H_b rises or when the individuals' lending schedule, $L_h(i)$, shifts to the right.

§4 The Aggregate Supply Price

(1) *The Earned Rate of Interest As a Cost of Production*

From the firms' point of view, the actual earned rate of interest (i_{LM}) appears as a cost of production per unit of the total assets $(K + (M_f/p))$, in the production at the level Y: they regard i_{LM} as the rate at which they have to pay interest to the banks and individuals who are claimants to their assets. Thus the total interest payment $(F_K(N(Y), K) - d) \cdot K$ is another cost item, along with the total wage cost $F_N(N(Y), K) \cdot N(Y) = (w \cdot N(Y)/p)$ and the depreciation cost

d·K. In other words, in order to induce the firms to produce Y, it is needed that they can expect as much proceeds as enable them to cover the three cost items: the interest payment $i_{LM} \cdot (K + (M_f/p))$, the depreciation d·K and the wage payment $w \cdot N(Y)/p$.

(2) *The Aggregate Supply Price and the LM Curve*

The aggregate supply price is defined in the "General Theory" as follows: "the aggregate supply price of the output of a given amount of employment is the expectation of proceeds which will just make it worth the while of the entrepreneurs to give that employment" (Keynes, (1), p. 24, l. 6—9.) As noted above, thus defined supply price is seen to be nothing but the cost of production in the wage-unit which is equal to the sum of the interest payment, the depreciation and the wage payment all in the wage-unit: which, in turn, is equal to $F_K K + F_N N$, or Y itself, divided by F_N, though this is different from Keynes' original definition of it in that it includes the depreciation cost. Thus our LM curve can be naturally combined with the aggregate supply price.

Conclusions

It is a rather surprising fact that the "General Theory" of Keynes has been so widely popularized in economics, without such a concrete theoretical foundation of the LM curve as is introduced in this paper. It seems that the conventional explanation of the LM curve has always been based ultimately on the notion (1) that, at a higher rate of interest, 'a man' will prefer to hold a greater proportion of his accumulated saving in the form of interest-bearing assets, with less in the form of the barren asset, or on the peculiar expectational phenomenon (2) that 'a man' prefers to hold a greater proportion of their assets in the form of the barren asset when the

market rate of interest threatens to turn to rise, suggesting an eventual fall in the consol price, after a substantial fall of the market rate below some 'normal' level. The speculative motive to liquidity is mainly combined with this latter explanation. The present writer has not seen any other (apparent) justification of the liquidity demand function than these ill-founded ones.

Worse, Keynes' pretence to comprise the quantity-theoretic aspect, or the proportionality between the stock of money and the flow of total production, into his "General" theory (i.e. in the form of the transaction part of the money demand function, $M_1 = k \cdot Y$) has almost always deceived students in macro-economics, who have often been pursuaded that the 'speculative' part of money forms one circulation and the 'transactionary' money forms another, that the two distinct parts of money demand pull each other corresponding to any changes in the interest rate or the income, with total money supply constant. This dichotomized view of money demand has remained as a fundamental theoretical defect of the book, which has prevented it from becoming a universal scientific scheme which covers any capitalistic national economy.

In this paper it is shown that such a too much simplified assumption of the dichotomized demand for money, or other views similar to it, can be replaced by an approach based on a completely different notion on the aggregate economy: the interest payment approach. It is clarified that the relation between the total income and the actual 'earned' rate of interest can naturally be based on the fact that interest payments including dividends on stocks flow from the profit of the firms, and they, as most widely defined, are identified in magnitude with the total net profit.

On account of market imperfections and uncertainty, rates of in·erest differ in reality in various financial markets, but it does not

deny that the higher the total net profit is, at the higher rates the firms can afford to pay interest on their borrowed liquid or illiquid funds: therefore we may argue that the higher the level of total production is, the higher rates of interest the firms can afford to pay on the liquid funds which they assumedly try to maximize to borrow from the banks, with their capital assets constant. This is the essence of the new foundation of the upward sloping LM curve which is given in this paper.

The 'rentiers' are also separately analyzed in this model: the individuals' asset-choice is explicitly formulated in the shape of their demand to lend to the firms, or $L_h(i)$, $dL_h/di > 0$, which is shown to be in power to influence the actual earned rate, and hence the equilibrium rate, of interest. Therefore they can influence the equilibrium level of income not only through a change in their propersity to consume, but also through a change in their asset-choice.

However it will be obvious that a primary characteristic of Keynes' economic theory is its firm-centered nature: the level of total production is determined ultimately mostly by the firms' expected marginal efficiency of capital, through which the level of investment, and hence the total income, are finally determined. The individuals outside the firms can influence the level of total production only to a minor extent.

Moreover the firms not only organize production but also speculate, demanding as much money as they can pay interest on it: the historical development of the banking system may be reviewed in this line, or in view of the firms' inexhaustible demand for money. The criterion of the firms to maximize liquidity in their total assets with the capital assets constant plays a pivotal role in this paper. Whether it is realistic is an empirical problem; the extreme criterion is postulated as a working hypothesis on which studies on an aggregate economy can be stepped forward.
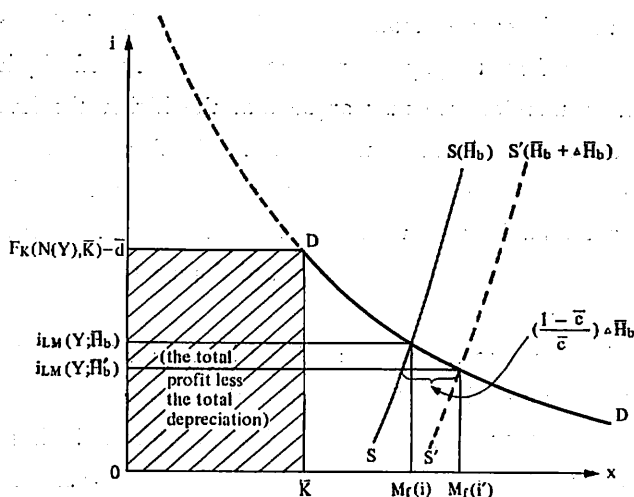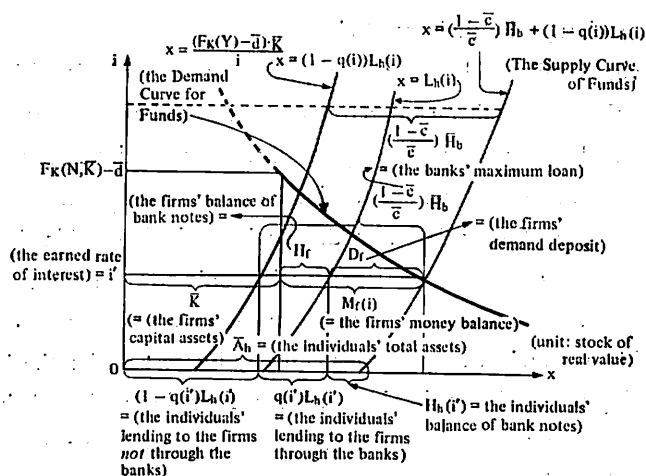
(April, 1979)

Fig. A-1   (The Direct Effect of Changing H_b)

Fig. A-2   (drawn on the assumption that $H_b = D_x$)   (The Relation between the Balance Sheets and the Determination of the Earned Rate of Interest) [Remark that the individuals are not the ultimate claimants to *all* the assets of the firms: the ultimate claimants to the demand deposit are the banks and the Central Bank (or the government.)]
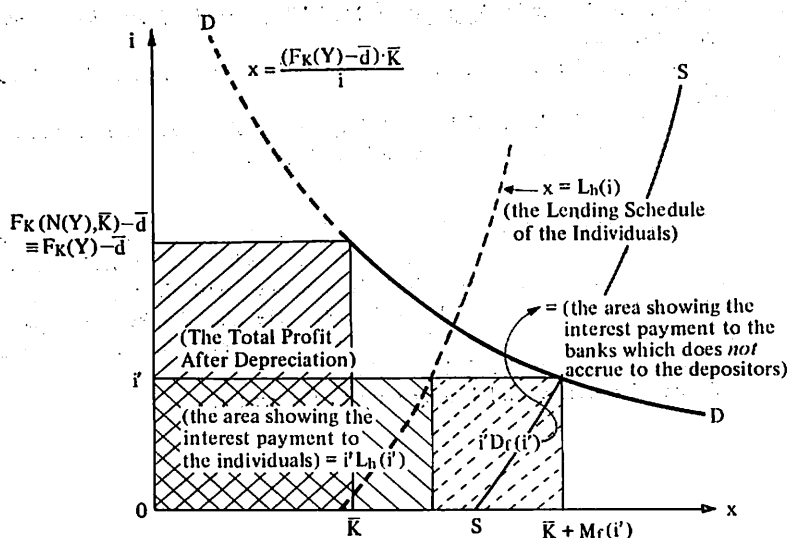
Fig. A-3 (How the Banks Earn Interest Beyond Interest Margins) ⟨$H_h = D_g$ is assumed.) [Since the banks institutionally do not pay any interest on the firms' demand deposit, and they earn interest on their loans to the firms, they are earning $i'D_f(i')$, *besides* possibly their interest margins. However, since they have to pay interest $i'D_g$ to the Central Bank, they ultimately earn $i'(D_f(i') - D_g)$, in addition to any interest margins.]

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