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The incentive for reducing the expenditures in local governments and resident migration

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Abstract

This paper examines the incentives for local governments to improve their own fiscal balances in an environment of interregional resident migration, and shows that whether a local government will make its fiscal balances efficient depends on the condition of population distribution. As residents migrate between regions to get a higher utility, their utility will become equal at a migration equilibrium. We obtain the following results from this property of resident migration: a local government will improve its own fiscal balance efficiently when the utility of residents in the other region increases with emigration from there. If the residents' utility in the other region decreases, a local government will have a incentive to deteriorate its own fiscal balance. However residents' utility will not decrease under this incentive.

key words: Resident migration; Reducing public expenditures; Local governments

JEL classification numbers H72, H11

1 Introduction

Do local governments have incentives to reduce their expenditures on local public goods? The efficiency of local governments is as important as that of the other sectors. [1] This is particularly important considering that the fiscal balance of Japanese local governments have deteriorated since the latter half of the 1990s. The central government has promoted decentralization to the local governments in order to reform the balance of public finance. In concrete terms, the central government will transfer tax sources to local governments and reduce subsidies. The purpose is to make all sections of the public sector more efficient by transferring authority from the central government to local governments which have more information concerning their own regions.

Incidentally, it has been traditionally said that if residents migrate freely under a decentralized public finance system, local governments have incentives to be efficient (Tiebout (1956)). However many researches have clearly shown that resident migration does not always bring efficiency. [2] Even if it is assumed that local governments are benevolent and that there is no rent seeking by politicians and bureaucrat, resident

[1] See Oates (1999) and Inman & Rubinfeld (1997). These papers surveyed researches on local governments.

[2] Refer to Itaba (2002) which surveyed these researches in detail.

migration does not guarantee Pareto efficient allocation. Other studies have looked at the efficiency of the population distribution and the provision of local public goods, but there are no studies looking at the reduction of public expenditures under resident migration. [3] The purpose of this paper is to show the conditions where local governments have incentives for improving its own public balance under resident migration. By showing these conditions, it will be possible that the central government will arrange these conditions where local governments make efficient efforts to improve the public balance. This analysis contributes to the efficiency of Japanese local governments.

With regard to local public good in particular, if there are no externalities and local governments do not take account of resident migration, then local governments provide local public good efficiently (Boadway & Flatters (1982)). However the population distribution does not always become Pareto efficient in this case. If local governments take account of migration, local public goods may also not be provided efficiently because local governments cannot control resident migration directly in order to make the population distribution efficient. In this case, local governments may distort the provision of local public goods in order to improve the distribution of the population. If so, the device of interregional transfers resolves this problem. Even if the local public good has externalities, this device makes the provision of local public goods Pareto efficient. Myers (1990), Wellisch (1994) and Caplan, Cornes & Silva (2000) showed the effect of interregional transfers in cases where a local public good has no externalities or has externalities. [4] On the other hand, Mitsui & Sato (2001) showed that interregional transfers distributed by the central government without commitment brought a concentration of residents into one region. As a result, the population distribution became inefficient. Thus interregional transfers do not always bring about Pareto efficiency.

However these researches lacked the perspective of examining the efficiency of local governments. More precisely, they did not analyze whether local governments have an incentive to reduce their own public expenditures under residents migration. The reduction of public expenditures means that local governments reduce the cost of local public goods and improve the revenues of public services such as water, gas and traffic services by increasing productivities. [5]

By reducing their own public expenditures, local governments can attract immigra-

[3] There are studies on the incentive for policy innovation in local governments. Rose-Ackerman (1980) and Strumpf (2002) showed that under federalism, local governments became risk-averse and free-riders who imitate new technologies which other governments develop. These studies are similar to this paper because local governments need new skills or know-how in order to reduce public expenditures without decreasing the level of public services. However neither study looked at the environment of resident migration, but rather at the uncertainty of policy innovation.

[4] Myers analyzed the effect of interregional transfers managed by each local government in the case of local public goods without externalities. The result was that this transfer made the population distribution Pareto efficient, so there was no need for the central government. Wellisch analyzed the case of local public goods with externalities. In this case, the population distribution would lead to Pareto efficiency, too. Caplan et al. analyzed the effect of the different timing, meaning that each local government was a leader and the central government a follower. In this case, the distribution would become Pareto efficient. These studies showed that interregional transfers were needed in order to achieve efficiency.

[5] These services often are in deficit. For example, many water services in Japan which are managed by local governments are in deficit. The average of the price in each region is 174 yen/m^3. On the other hand, the average cost is 178 yen/m^3 (data for 2004 from Somu-syo (2006)). This deficit, of 4 yen/m^3, is compensated with taxes. Thus local governments can reduce their public expenditures by improving their revenue or reducing these costs.

tion from other regions. The reason for this immigration is that the reduction of public expenditures brings larger utility to residents in these regions where local governments reduce expenditures than in other regions. Since residents migrate to a region that provides the largest utility, all residents get same utility at a migration equilibrium. Taking this into account, whether residents' utility increase at a Nash equilibrium depends on the distribution of population. If a region where a local government reduces its public expenditures is congested, that means an overpopulated region, the increase of residents' utility will be weakened because of worsening this congestion. At this point in time, if the other region is also congested, the utility will rise because the emigration from the other region will mitigate its congestion. As a result, the reduction of public expenditures will increase all residents' utility when both of regions are congested. In this case, the efforts to reduce public expenditures will be Pareto efficient at Nash equilibrium. On the other hand, when the other region is sparse, the reduction of expenditures in a congested region will cause residents to migrate from the sparse region to this congested region. Since the sparseness and congestion become worse in both of region, residents' utility decrease by this reduction. In this case, the local government in the congested region does not make Pareto efficient efforts at a Nash equilibrium. But, the local government in the sparse region always makes Pareto efficient efforts because immigration mitigates the sparseness. To sum up, whether local governments make Pareto efficient efforts depends on the condition of population in other regions. Since local governments do not have the authority to improve the public balance in the other region, they will raise their own residents' utility by making impacts on migration by controlling their own public expenditures. As a result, the utility of the residents in their own region will not be higher than the utility at the optimal population scale in the other region. Thus no local government has an incentive to improve the public balance if the residents' utility in the other decreases due to emigration from the region. Previous researches have paid little attention to the relation between the population distribution and the distortion of provision of local public goods.

The outline of this paper is as follows. Section 2 describes the basic model structure and the main result. Section 3 describes the case in which local governments simultaneously decide to provide local public goods and make efforts to reduce public expenditures. The conclusion is given in Section 4. The proofs of the propositions and lemma are provided in the Appendix.

2 Model

A nation consists of two regions, named $i = 1, 2$, and a local government exists in each region. The population in region i is n_i, and the aggregate population is $\bar{n}$, thus $n_2 = \bar{n} - n_1$. We assume continuous residents. Each resident has homogeneous preference and is able to chooses either region 1 or 2 to reside in. The utility function of a representative resident in region i is $u^i(x_i, y_i)$. x_i is the consumption of a private good and y_i is the consumption of a local public good which has no externalities to the other region and is provided by the local government in region i. We assume that u^i is strictly quasi-concave,

[6] and that y_i is a normal good. [7]

Each resident is endowed with one unit of homogeneous labor which is supplied to firms in region i. Firms produce the private good and pay labor a wage equal to the marginal product. The collective production function for the private good in region i is assumed to be $f_i(n_i)$ [8] which is concave, $f_i' \geq 0$, $f_i'' \leq 0$ and $f_i(0) = 0$. The firms in region i are assumed to be owned by the residents of region i. Hence, the profits of the firms in region i are equally distributed to each resident of region i. We also assume there are no transfers between regions. Each local government collects a resident-based head tax in order to produce the public good. The marginal cost of the public good is c_i which is fixed for y_i. We consider that each local government makes efforts to improve the fiscal balance, which is public revenue minus public expenditure. In this paper, improving this balance means that each local government decreases its net public expenditure. Thus the constraint of resources in region i is $f_i(n_i) = n_i x_i + c_i y_i + F - a_i + d(a_i)$. a_i is efforts and $d(a_i)$ is the cost of efforts. One unit of effort leads to a reduction of one unit of public expenditure, but the effort cost is generated. F is the fixed cost of the local public good. We assume that $F - a_i + d(a_i) > 0$ for all a_i. This means that F is larger than the surplus of the cost reduction effort, [9] and $F = 0$ when $y_i = 0$. We assume that $d(a_i)$ is strictly convex, $d(0) = 0$, $d'(a_i) > 0$, $d'(0) = 0$, $d'(+\infty) = +\infty$, and $d''(a_i) > 0$. In addition, we describe a_i satisfying $a_i - d(a_i) = 0$ as $\bar{a}$. We assume that local governments choose $a_i \in [0, \bar{a}]$. This assumption means that local governments do not make efforts in deficit. We assume that each local government chooses the effort level a_i and the quantity of local public good y_i to maximize the utility of a representative resident in region i.

Pareto Efficiency Following Wellisch (1994), Pareto efficient allocation is defined as feasible allocations at which it is impossible to increase u^i without reducing u^j ($i \neq j$, $i, j = 1, 2$). But governments cannot compel residents to migrate from one region to another region because residents have the right of free migration. [10] Residents migrate to get higher utility. If they have the same utility level between region 1 and 2, then they will not migrate. Thus Pareto efficiency holds under $u^i = u^j$. In addition, Pareto efficiency, which is defined as the impossibility of increasing u^i without reducing u^j under $u^i = u^j$, is characterized by maximizing a linear combination of u^1 and u^2 subject to the following (2) and (3).

$$\max_{x_1, x_2, y_1, y_2, n_1, a_1, a_2} \delta u^1(x_1, y_1) + (1 - \delta) u^2(x_2, y_2) \tag{1}$$

[6] The sufficient condition for strictly quasi-concavity is $2u_y^i u_x^i u_{xy} - (u_x^i)^2 u_{yy} - (u_y^i)^2 u_{xx}^i > 0$. u_x^i, u_y^i and u_{xy}^i mean $\partial u^i / \partial x_i$, $\partial u^i / \partial y_i$ and $\partial^2 u^i / \partial x_i \partial y_i$, respectively.

[7] The sufficient condition for a normal good for y_i is $1/u_x^i (u_x^i u_{xy}^i - u_y^i u_{xx}^i) > 0$. We assume this condition.

[8] We assume that productivity may be different between regions. Precisely, product function depends on the land, $f(n_i, T_i) \equiv f_i(n_i)$. T_i means the land scale in region i. The larger T_i, the larger the productivity.

[9] If F is small, the effort surplus may be larger than the public cost in the region. More accurately, $c_i y_i + F - a_i + d(a_i) < 0$ for some a_i. In this case, the local government returns this surplus to residents. As a result, production of the private good increases in the region. This case is not intrinsically different from the following analysis. In addition, it may not be actually returned to private goods because the effort surplus is not always pecuniary revenue. In this paper, since it is not the purpose to clarify this, we assume that $F - a_i + d(a_i) > 0$ for all a_i.

[10] If a social planner as a central government exists, he (she) cannot do it.

$$s.t.\ f_1(n_1)+f_2(\bar{n}-n_1)-n_1x_1-(\bar{n}-n_1)x_2-c_1y_1-F+a_1-d(a_1)-c_2y_2-F+a_2-d(a_2)=0 \quad (2)$$

$$u^1 = u^2 \quad (3)$$

$$0 \le \delta \le 1, \ \ x_1, x_2, y_1, y_2 \ge 0$$

(2) is the constraint of resources.

We use the Lagrange function, and define λ_1 as the Lagrange multiplier of the resources constraint (2) and λ_2 as the multiplier of the migration equilibrium constraint (3). We assume on interior solution to y_i and x_i.

We achieve the following first-order conditions:

$$x_i \ : \ (\delta+\lambda_2)u^i_x - \lambda_1 n_i = 0, \quad (4)$$

$$y_i \ : \ (\delta+\lambda_2)u^i_y - \lambda_1 c_i = 0, \quad (5)$$

$$n_1 \ : \ f_1'(n_1) - x_1 = f_2'(\bar{n}-n_1) - x_2 \quad (6)$$

$$a_i \ : \ \lambda_1(1-d'(a_i)) = 0, \quad i = 1, 2. \quad (7)$$

From (4) and (5) , we obtain

$$(\delta+\lambda_2)\frac{u^i_x}{n_i}(n_i\frac{u^i_y}{u^i_x} - c_i) = 0. \quad (8)$$

λ_2 is positive as long as $u_1 = u_2$ at the optimal solution. We assume there exists an interior solution of the migration equilibrium, $\lambda_2 > 0$. Thus (8) means that Pareto efficient y_i is satisfied when the sum of the marginal rate of substitution for y_i equals the marginal cost of the local public good, which satisfies the Samuelson condition.

We denote Pareto efficient a_i as a_i^*. Then, a_i^* is satisfied with

$$1 - d'(a_i^*) = 0. \quad (9)$$

because $\lambda_1 = (\delta+\lambda_2)u^i_x/n_i > 0$ from (4) and $\lambda_2 > 0$. This means that the socially optimal effort of each local government is to make the marginal effect of the reduction in public expenditure equal to the marginal cost of efforts.

From (6), a distribution of population is socially efficient when the net marginal product in each region becomes equal. If (6) is not equal, more private goods can be produced in the nation by having the residents migrate from the region of the smaller net marginal product to the region of the larger. We denote a Pareto efficient population in region i as n_i^*.

The purpose of this paper is to analyze the incentive for improving the balance of public revenues and expenditures. This implies efforts to reduce the costs and to increase the revenues of public services. We can consider the following methods for cutting costs and increasing revenues; outsourcing and restructuring public services, reducing staff, introducing a system for evaluating public policies to avoid waste, computerization of office work, improving the revenue of or privatizing water, gas, traffic and child care services, and so on. It is difficult for local governments to introduce these methods in the short term because they need to implement organizational and institutional changes. In addition, these methods entail costs. Thus, once local governments introduce such devices to improve this balance, it is difficult to withdraw them quickly because of long-term contracts and the needs to change the public institutions. For example, Ota city

[11] made a contract for entrusting water service to a private firm. [12] The contract term is from 1 April 2002 to 31 March 2007. This contract contains a clause for a penalty if either of the parties cancels the contract. Thus Ota city is committed to this system of providing the water service in trust with the private firm. [13] From these viewpoints, reducing public expenditures requires a commitment. Local governments must commit themselves to these methods if they wants to reduce their public expenditures.

Following Mitsui & Sato, we assume that residents can freely migrate to the region in which they want to live, and that after they choose a region to live in, they cannot move between regions. This assumption shows the following fact. Residents practically choose their occupations when they decide where to live. It is not easy for individual to change his (her) job because of spending time to search new jobs. On the other hand, providing many local public goods is a daily task. For example, it includes these are police, fire fighting, library, health care services, and so on. In order to reflect these features, the time structure in this model adopts the following three stages. First, local governments set their own effort level, a_i, which is how they will reduce public expenditures, ex-ante. Second, residents decide where to reside. Third, local governments provide local public goods, y_i. The cases where local governments take migration into account and do not take it into account are analyzed in the next section.

Subgame perfect equilibrium We will find a sub-game perfect equilibrium in this model. The model will be solved using back-word induction.

Stage 3 At stage 3, local governments set y_i given n_i and a_i in order to maximize a representative resident in its own region. Specifically, our maximization problem is

$$\max_{y_i} u^i(x_i, y_i) \tag{10}$$

$$s.t. \quad f_i(n_i) = n_i x_i + c_i y_i + F - a_i + d(a_i).$$

Then the first order condition is

$$\frac{u_x^i}{n_i}\left(n_i \frac{u_y^i}{u_x^i} - c_i\right) = 0. \tag{11}$$

Therefore y_i satisfies the Samuelson condition, namely that y_i is Pareto efficient. [14] But y_i is set given n_i and a_i, so y_i is a function of them. If they are not Pareto efficient, the *quantity of* y_i will not be equal y_i of (8).

[11] Ota city is in Gunma prefecture in Japan.

[12] Komiyama (2003) explained this entrusting contract in detail.

[13] Another example is the outsourcing of simple routines. Shizuoka prefecture established a center for general-affairs office work (Somu-zimu center) and introduced the outsourcing of general-affairs office work (Somu-zimu) in 2002. Shizuoka prefecture has saved about 97 million yen every year. Wakasugi & Kobayashi (2006) explained this.

[14] As the population in i increases under $f_i' - x_i \geq 0$, y_i becomes larger because y_i is a normal good. Specifically we obtain

$$\frac{dy_i}{dn_i} = \frac{1/u_x^i(f_i' - x_i)(u_x^i u_{xy} - u_y^i u_{xx}^i) + u_y^i}{1/(u_x^i u_y^i)(2u_x^i u_y^i u_{xy}^i - (u_x^i)^2 u_{yy}^i - (u_y^i)^2 u_{xx}^i)} > 0 \tag{12}$$

by differentiating (8) or (11) from the quasi-concavity of u_i and the normal good of y_i. However dy_i/dn_i may be negative when $f_i' - x_i < 0$. This property affects the second order condition for n_i of the indirect utility function.

We denote the indirect utility of the resident in region i as

$$V_i(n_i, a_i) \equiv \begin{cases} u^i\left(\frac{f_i - c_i y_i - F + a_i - d}{n_i}, y_i\right) & \text{if } f_i - n_i x_i - c_i y_i - F + a_i - d \geq 0 \\ 0 & \text{if } f_i - n_i x_i - c_i y_i - F + a_i - d < 0. \end{cases} \quad (13)$$

Because of the fixed cost $F > 0$, the resources constraint in (10) may be in a breach in the neighborhood of $n_i = 0$. In this case, we assume $x_i = y_i = 0$ and $V_i = 0$.

Stage 2 At stage 2, residents choose either region 1 or 2 depending on which gives them a larger utility. If $V_i(n_i, a_i) > V_j(n_j, a_j)$ for given a_i and a_j, residents in region j will emigrate from j to i. If the utility in region 1 and 2 is equal, residents will not migrate. This situation is defined as the following migration equilibrium conditions, either 1 or 2.

1. If $n_1, n_2 > 0$, then $V_1(n_1, a_1) = V_2(n_2, a_2)$.

2. If $n_i = 0$, then $V_i(0, a_i) \leq V_j(\bar{n}, a_j)$.

Condition 1 means that the migration equilibrium is an interior solution. Condition 2 means that it is a corner solution. This definition is a Nash equilibrium in stage 2. Because the weight of each resident is zero, even if a resident who chooses region 1 migrates to region 2, he (she) cannot have an influence on the others and on the productivity in each region. Therefore, under this migration equilibrium, each resident lacks the incentive for the migration given the others' strategies, meaning a Nash equilibrium in this stage.

The migration equilibrium depends on the level of efforts, (a_1, a_2), which is determined at stage 1. Thus, the migration equilibrium becomes a function of (a_1, a_2). But the migration equilibrium may acquire multiple equilibria at (a_1, a_2). [15] In this case, each equilibrium is not always continuous at each value of (a_1, a_2). However this discontinuity complicates the analysis in this paper. In order to keep the discussion from this complexity, we denote one migration equilibrium as $n_1(a_1, a_2)$, as the case in which the migration equilibrium varies continuously with the change of (a_1, a_2), meaning that the residents' strategy is $n_1(a_1, a_2)$. [16] We consider the migration equilibrium in this limited class. However a migration equilibrium $n_1(a_1, a_2)$ may disappear at some (a_1, a_2). This fact depends on the configuration of V_1 and V_2. In this case, the migration equilibrium becomes another equilibrium with the change of (a_1, a_2).

We obtain the effect of a unit of resident migrating from the region j to i by differentiating (13), [17] that is

$$\frac{\partial V_i}{\partial n_i} = \frac{u_x^i}{n_i}[f_i'(n_i) - x_i] \quad (14)$$

$$\frac{\partial V_j}{\partial n_i} = -\frac{u_x^j}{\bar{n} - n_i}[f_j'(\bar{n} - n_i) - x_j]$$

by using the envelop theorem on (11). Whether a flow of population into region i increases their utility or not depends on the sign of $[\,\cdot\,]$. V_i may have multiple peaks for n_i generally

[15] In this case, the migration equilibria will not be a function but a correspondence of (a_1, a_2).

[16] Mayers, Wellisch and Caplan et al. researched models that assumed that each local government takes resident migration into account in the framework of the game. However these studies did not consider the possibility of multiple migration equilibria and of the discontinuity of a migration equilibrium, so they dealt with the migration equilibrium as a function of some variables. This paper follows them.

[17] A unit of resident does not mean one resident but one weight resident.

as Atkinson & Stiglitz (1980) suggested in their discussion about residents migration in Ch. 17. [18] We define the following condition.

Definition 1 *$\forall n_i \in (\underline{n_i}, \bar{n}_i)$ for some $\underline{n_i}$ and $\bar{n}_i$ in $[0, \bar{n}]$,*

1. *if $f_i'(n_i) - x_i > 0$, then region i is locally sparse,*
2. *if $f_i'(n_i) - x_i = 0$, $f_i'(n_i - \delta) - x_i > 0$ and $f_i'(n_i + \delta) - x_i < 0$ for all $\delta > 0$ such that $(n_i - \delta, n_i + \delta) \subseteq (\underline{n_i}, \bar{n}_i)$, then region i is locally optimal, and*
3. *if $f_i'(n_i) - x_i < 0$, then region i is locally congested.*

1 means that a flow of population into region i will bring increasing utility to the region from (14). Thus the population in i is still sparse. 2 and 3 are parallel logic. But, as we stated above, V_i may have multiple peaks for n_i, so this definition is only local. If V_i has a single peak for n_i, this definition is global.

Next, we consider the stability condition of a migration equilibrium. If a migration equilibrium is disturbed for some reason, it may diverge, for example to $n_i = 0$ or $n_i = \bar{n}$. To avoid this divergence, we look only at the case of a locally stable migration equilibrium. This stability means that when some residents migrate from region 1 to region 2 at a migration equilibrium, the utility of residents in region 2 becomes higher than in region 1, and vice versa. As a result, the residents who migrated come back. Hence we define $\partial V_i/\partial n_i - \partial V_j/\partial n_i < 0$. [19] The stability condition is used in Atkinson & Stiglitz (1980), Boadway & Flatters, Wellisch, Caplan et al. and Mitsui & Sato. [20] In this model, the stability condition is

$$D \equiv \frac{u_x^1}{n_1}(f_1'(n_1) - x_1) + \frac{u_x^2}{\bar{n} - n_1}(f_2'(\bar{n} - n_1) - x_2) < 0. \tag{15}$$

The population distribution at the migration equilibrium under this stability condition is categorized by one of only the following three cases. Specifically,

Case 1 $f_i' - x_i < 0$ and $f_j' - x_i < 0$,

Case 2 $f_i' - x_i = 0$ and $f_j' - x_i < 0$, and

Case 3 $f_i' - x_i < 0$, $f_j' - x_i > 0$, and $|\frac{u_x^i}{n_i}(f_i' - x_i)| > |\frac{u_x^j}{n_j}(f_j' - x_j)|$, $\quad i \neq j, \quad i, j = 1, 2$.

Under this stability condition, V_1 and V_2 do not become tangent or overlap each other at any of the migration equilibria.

[18] The second order condition of V_i for n_i is

$$\frac{\partial^2 V_i}{\partial n_i^2} = \left[\frac{V_i'(n_i)}{u_x}\frac{u_{xx}^i(f_i' - x_i)}{n_i} + \frac{u_x^i f_i''}{n_i} - \frac{V'(n_i)}{n_i}\right] + \left[\frac{V_i'(n_i)}{(u_x^i)^2}(u_x^i u_{xy}^i - u_y^i u_{xx}^i) + \frac{(u_y^i)^2}{u_x^i c_i}\right] y_i'(n_i)$$

The first bracket on the right-hand side is negative if $f_i' - x_i > 0$. But the second bracket is positive because $y_i'(n_i) > 0$ when $f_i' - x_i > 0$. Thus, we cannot identify the sign of the second order condition even if $f_i' - x_i > 0$. Of course, we cannot identify it in the case of $f_i' - x_i < 0$, either.

[19] Boadway & Flatters find that both regions tend to have a unique stable migration equilibrium under a condition of overall overpopulation and that both tend to have an unstable migration equilibrium if there is underpopulation.

[20] Mitsui & Sato defined it more generally without using differential calculus.

Stage 1 At stage 1, each local government chooses an effort level. First, we consider how residents migrate when a local government increases its effort level. In other words, how does the migration equilibrium vary when a local government slightly reduces public expenditures. To see this, we differentiate $V_1(n_1, a_1) = V_2(\bar{n} - n_1, a_2)$ by a_i. When the migration equilibrium $n_1(a_1, a_2)$ is continuous about a_i, we obtain

$$\frac{\partial n_i}{\partial a_i} = -\frac{1}{D}\frac{u_x^i}{n_i}(1 - d'(a_i)) \tag{16}$$

from (15). This means that increasing efforts to reduce public expenditures brings immigration when the effort is less than the Pareto efficient level, $1 - d'(a_i) > 0$. However if a local government makes a major change in its effort, then the migration equilibrium may become discontinuous or move to the corner solutions, $n_i = \bar{n}$ or 0.

Now if residents do not migrate when government i increases its effort, the change of the utility will be

$$\left.\frac{\partial V_i}{\partial a_i}\right|_{n_i \text{ is fixed.}} = \frac{u_x^i}{n_i}(1 - d'(a_i)). \tag{17}$$

This means that V_i is an increasing function of a_i till a_i^* and is a decreasing function of a_i beyond a_i^* when n_i is fixed. In other words, the indirect utility is maximized at the Pareto efficient effort a_i^* at each n_i, that is to say $V_i(n_i, a_i) \leq V_i(n_i, a_i^*)$ for all n_i and a_i.

Next, we consider each local government's decision regarding its own efforts, a_1 and a_2. We differentiate V_i by a_i and substitute (16) into it. We obtain

$$\frac{\partial V_i}{\partial a_i} = \frac{u_x^i}{n_i}(f_i'(n_i) - x_i)\frac{\partial n_i}{\partial a_i} + \frac{u_x^i}{n_i}(1 - d_i'(a_i)) = \frac{u_x^1 u_x^2}{D n_1(\bar{n} - n_1)}(f_j'(n_j) - x_j)(1 - d'(a_i)) \tag{18}$$

If $\partial V_i / \partial a_i > 0$, then the local government i will get a larger payoff by slightly increasing its effort as long as the migration equilibrium varies continuously. If $\partial V_i / \partial a_i < 0$, then local government i will get a larger payoff by slightly decreasing its effort as well. We get the following proposition.

Proposition 1 *If the strategy chosen by residents becomes a migration equilibrium $n_1(a_1, a_2)$ which is satisfied by a differentiable function, $f_1' - x_1 < 0$ and $f_2' - x_2 < 0$ for all a_1 and a_2, then (a_1^*, a_2^*) becomes each local government's behavior in the strategy of a subgame perfect equilibrium.*

In the case of this proposition, two regions are congested at any (a_1, a_2). We can interpret this feature as the case in which the effect of each local government's effort is relatively small and total population is relatively large because local governments have hardly any influence on the congested population. Then (a_1, a_2) becomes Pareto efficient, but the population distribution may not be Pareto efficient because (6) does not always follow from (18). Therefore an effort to reduce the public expenditure does not have the effect of making the population distribution efficient. Of course, since proposition 1 is a sufficient condition for the efficiency effort, the other conditions may exist, too.

Here we assume that V_i has a single peak for n_i. [21] This assumption is plausible in practice. Hayashi (2002) measured a minimal efficient scale of population in Japanese

[21] The second order condition of V_i for n_i is not always negative even if u_i is strictly quasi-concave. Boadway & Flatters assumed that the graph of V_i for n_i is single peaked.

local governments. In Hayashi's analysis, local governments have a U-form with regard to the local public expenditure per capita as population increase because the fixed cost of the local public good brings a scale economy to local governments. This means that there exists a optimal population level for local governments. Thus, it is thought that this assumption is plausible. We denote the optimal population level in region i as n_i^{**} when V_i has a single peak.

Assumption 1 *The following conditions about $V_i(n_i, a_i)$ for all a_i are assumed.*

1. *There exists a unique $n_i^{**} \in (0, \bar{n})$ that satisfies* $\left.\frac{\partial V_i}{\partial n_i}\right|_{n_i = n_i^{**}} = \frac{u_x^i}{n_i^{**}}[f_i'(n_i^{**}) - x_i] = 0.$

2. $\forall n_i \in [0, n_i^{**})$, $\frac{\partial V_i}{\partial n_i} > 0$ *and* $\forall n_i \in (n_i^{**}, \bar{n}]$, $\frac{\partial V_i}{\partial n_i} < 0.$ [22]

Using this single peak assumption, we obtain the following facts about each optimal population (n_i^{**}, n_j^{**}): $n_i^{**} < n_i(a_1, a_2)$ and $n_j^{**} < \bar{n} - n_i(a_1, a_2)$ in Case 1, $n_i^{**} = n_i(a_1, a_2)$ and $n_j^{**} < \bar{n} - n_i(a_1, a_2)$ in Case 2, and $n_i^{**} < n_i(a_1, a_2)$ and $n_j^{**} > \bar{n} - n_i(a_1, a_2)$ in Case 3. See Figure 1, 2 and 3. With regard to the three cases under Assumption 1, the following lemma is obtained.

Lemma 1 *Under Assumption 1, there are no stable, interior and multiple migration equilibria satisfying the combinations of Case 1 - Case 1, Case 1 - Case 2 and Case 2 - Case 2.*

However, a multiple migration equilibria satisfying Case 3 and another case may exist. As a result, the following proposition is obtained.

Proposition 2 *Under Assumption 1, local governments i and j choose the following effort level in a subgame perfect equilibrium.*

1. *If the migration equilibrium $n_1(a_1, a_2)$ satisfies $f_1' - x_1 < 0$ and $f_2' - x_2 < 0$ at (a_1^*, a_2^*), then (a_1^*, a_2^*) is a Nash equilibrium at stage 1.*

2. *If the migration equilibrium $n_1(a_1, a_2)$ satisfies $f_i' - x_i = 0$, $f_j' - x_j < 0$ and $V_j(\bar{n}, a_j^*) \leq V_j(\bar{n} - n_i, a_j)$ at (a_i^*, a_j), then (a_i^*, a_j) is a Nash equilibrium at stage 1.*

3. *If the migration equilibrium $n_1(a_1, a_2)$ satisfies $f_i' - x_i < 0$, $f_j' - x_j > 0$, $|\frac{u_x^i}{n_i}(f_i' - x_i)| > |\frac{u_x^j}{n_j}(f_j' - x_j)|$ and is continuous for all (a_1, a_2), then $(0, a_j^*)$ and $(\bar{a}, a_j^*)$ is a Nash equilibrium at stage 1.*

a_j may not become a_j^*, and $n_i = n_1(a_1, a_2)$ if $i = 1$ and $n_i = \bar{n} - n_1(a_1, a_2)$ if $i = 2$.

1 and 2 in proposition 2 state that (a_i^*, a_j^*) and (a_i^*, a_j) are the behaviors in SPE if the migration equilibrium is either Case 1 or Case 2 at (a_i^*, a_j^*) and (a_i^*, a_j) respectively. On the other hand, 3 means that the severer condition, meaning the existence of Case 3 for all (a_i, a_j), is required in order that it can be a strategy of SPE.

In 1, the migration equilibrium which satisfies $f_i' - x_i < 0$ and $f_j' - x_j < 0$ at (a_1^*, a_2^*) is interpreted as the case where the total population is large and where each region's

[22] We can consider the example case of a single peak. See appendix.

potential gap is similar. The large total population in this paper means that $n_1^{**}+n_2^{**} < \bar{n}$ at (a_1^*, a_2^*), that is to say each region cannot absorb the total population at the optimal population level. The potential gap in this paper means the difference of resident's utility at the optimal population level and at (a_1^*, a_2^*) in each region. In other words, each local government has the similar technology and know-how to provide the public good, and the productivity of the private good is similar in each region. In this case, when the total population is large, both local governments choose a Pareto efficient effort level. The logic is the following. When local government i increases its effort in order to decrease its public expenditures, residents' utility will increase in i. As a result, residents will migrate from region j to i. Since both regions are congested, this migration will dilute the increase of utility. But the congestion in region j will be mitigated, and the utility of residents in j will be improved. As a result, at the new migration equilibrium, the utility will increase. If local government i decreases its effort, the opposite will occur. Hence, the condition where both regions are congested promotes efforts by each local government to make efforts at efficiency.

In 2, the migration equilibrium which satisfies $f_i' - x_i = 0$ and $f_j' - x_j < 0$ at (a_i^*, a_j) is interpreted as the case where the potential gap is large and where the total population is large. In this case, the residents' utility at the optimal population level in region j is larger than it is in region i, when both local governments choose an efficient effort. Local government j, which has large potentiality, does not make efforts in spite of having room to reduce public expenditures, while local government i chooses an efficient effort. The reason is the following. If j increases its effort, residents in i will migrate from i to j. Then j's congestion will worsen and i will become sparse. On the other hand, if j decreases its effort, residents in j emigrate from j to i. Then, although j's congestion will be mitigated, i will become congested and this congestion will bring a larger deterioration than the mitigation. As a result, residents' utility will decrease. The condition $V_j(\bar{n}, a_j^*) \leq V_j(\bar{n} - n_i, a_j)$ at (a_i^*, a_j) means the utility level when the number of residents migrating to j at a Pareto efficient effort is not very large. In this case, the total population is large, and the effort effect is small. If $V_j(\bar{n}, a_j^*)$ is large, on the contrary, local government j may choose a Pareto efficient effort level and gather all residents. In 1 and 2, the total population is large because $n_1^{**} + n_2^{**} < \bar{n}$. Specifically, when the total population is large, the equilibrium tends to appear at either Case 1 or 2.

In 3, the total population level is small because $n_i > n_i^{**}$ and $\bar{n} - n_i < n_j^{**}$. Besides, if there is neither stable nor unstable migration equilibrium, except the corner equilibrium, the potential gap between regions is large, because residents' utility at the optimal population level is region j is larger than it is in region i. In this case, local government i chooses no effort or excessive effort, while local government j chooses the efficient effort. If i makes the efficient effort, residents will migrate from region j to region i. This migration will cause a worsening of the congestion for i and sparseness for j. This will invite a decline in the utility. Hence, j chooses the smallest or largest effort level at the equilibrium.

In conclusion, it is better for the total population is large and the difference of the potentiality in each region to be small in order to elicit Pareto efficiency from each local government. Incidentally, the population distribution at the equilibrium may not become Pareto efficient. The efficient distribution is the net marginal product equalized in each region. However (18) does not guarantee this efficiency.

3 The simultaneous decision case

In the previous section, we considered the behavior when neither local government could change at stage 2 or 3. However we can also consider the case where y_i and a_i are decided simultaneously. This case is divided into the following two cases.

The case where migration is not taken into account First we consider the case in which neither local government takes migration into account. In this case, each local government maximizes

$$\max_{a_i, y_i} u^i(x_i, y_i)$$
$$s.t.\ \ f_i(n_i) = n_i x_i + c_i y_i + F - a_i + d(a_i)$$

given n_i. The first order conditions are

$$y_i \ : \ \frac{u_x^i}{n_i}[n_i \frac{u_y^i}{u_x^i} - c_i] = 0 \tag{19}$$
$$a_i \ : \ \frac{u_x^i}{n_i}(1 - d'(a_i)) = 0. \tag{20}$$

The Samuelson condition and Pareto efficient effort level are fulfilled in (19) and (20) given n_i. However we do not know how the population is distributed because neither local government does not considers the residents' migration at all in this case. In other words, Pareto efficiency of the population distribution is not guaranteed by (19) and (20) at all. Besides, residents' utility in this case is not any greater than it was in the previous section's case. At first sight, it seems that the utility in the case where no account is taken of migration is larger than it in the case where it is taken into account because the effort is always Pareto efficient in the former case. However, when V_i is either 2 or 3 in Proposition 2, there exists some $a_i \neq a_i^*$ which makes V_i larger. This result applies in this case. Hence the decision made without taking account of migration is not more efficient than that taking account of migration at least under the single peak assumption.

The case where migration is taken into account Second, we consider the case where each local government takes migration into account. Then each local government maximizes

$$\max_{a_i, y_i} u^i(x_i, y_i)$$
$$s.t.\ \ f_i(n_i) = n_i x_i + c_i y_i + F - a_i + d(a_i)$$

taking account of the migration equilibrium $n_1(a_1, a_2, y_1, y_2)$ which is a function of a_1, a_2, y_1 and y_2. [23] In the case where the residents migration is taken into account, we do not know whether this maximized problem fulfills the second order condition or not. Thus we consider the following. We differentiate u_i for y_i and a_i. Then

$$y_i \ : \ \frac{u_x^i}{n_i}(f_i'(n_i) - x_i)\frac{\partial n_i}{\partial y_i} - \frac{u_x^i}{n_i}c_i + u_y^i \tag{21}$$
$$a_i \ : \ \frac{u_x^i}{n_i}(f_i'(n_i) - x_i)\frac{\partial n_i}{\partial a_i} + \frac{u_x^i}{n_i}(1 - d'(a_i)). \tag{22}$$

[23] If $i = 1$, then $n_i = n_1(a_1, a_2, y_1, y_2)$ and if $i = 2$, then $n_i = \bar{n} - n_1(a_1, a_2, y_1, y_2)$. In this case, residents will migrate after they see (a_1, a_2, y_1, y_2), so the migration equilibrium becomes a function of these variables.

When y_i changes slightly under $u_1 = u_2$, the change of the migration equilibrium $n_1(a_1, a_2, y_1, y_2)$ will be

$$\frac{\partial n_i}{\partial y_i} = -\frac{1}{D}\frac{u_x^i}{n_i}(n_i\frac{u_y^i}{u_x^i} - c_i). \tag{23}$$

If there are fewer local public goods than the amount under Samuelson's rule, increasing y_i will bring immigration. We substitute (23) and (14) for (21) and (22) respectively. Then these equations are

$$y_i \ : \ \frac{u_x^1 u_x^2}{Dn_1(\bar{n} - n_i)}(f_j' - x_j)(n_i\frac{u_y^i}{u_x^i} - c_i) \tag{24}$$

$$a_i \ : \ \frac{u_x^1 u_x^2}{Dn_1(\bar{n} - n_i)}(f_j' - x_j)(1 - d'(a_i)). \tag{25}$$

(25) is as the same as (18). This means that Proposition 1 applies to this case about the effort level and local public good, [24] namely if $n_1(a_1, a_2, y_1, y_2)$ satisfies $f_1' - x_1 < 0$ and $f_2' - x_2 < 0$ for all a_1, a_2, y_1, y_2, then y_i and a_i will be Pareto efficient. In the other cases where $n_1(a_1, a_2, y_1, y_2)$ does not satisfy both $f_1' - x_1 < 0$ and $f_2' - x_2 < 0$, the effort and public goods may not be Pareto efficient.

The population distribution does not always become Pareto efficient, that is (6). If the population distribution is Pareto efficient and stable, then a_i and y_i will become Pareto efficient, too, because $f_1' - x_1 = f_2' - x_2 < 0$. On the other hand, if local governments do not take migration into account when they provide the local public goods, the Samuelson condition will be satisfied at the migration equilibrium. At first sight, it seems more efficient for each local government not to take resident migration into account. However the residents in the case where migration is taken into account may be better off than in the case where it is not taken into account because the population distribution may be more inefficient and each local government will not adjust it in the former case.

4 Conclusion

This paper analyzed the incentives for local governments to improve the public balance under resident migration. The paper showed that whether a local government makes efforts or not depends on the population condition in the other region. Even if a local government intends to reduce public expenditure in order to improve the residents' utility in its own region, the utility will become as same as that of residents in the other region, because residents will migrate until their utility becomes equal. Then if the outflow of population from one (congested) region improves the residents' utility, the local government in the other region will make efficient efforts. However if the utility worsens due to the outflow of population from one (sparse) region, the local government in the other region will not

[24] $f_i' - x_i$ is different from the previous section, because each local government maximizes for a_i and y_i given the other government. Then the second order condition of u_i about n_i given a_i and y_i is

$$\frac{\partial^2 u_i}{\partial n_i^2} = \frac{f_i' - x_i}{n_i^2}(u_{xx}^i(f_i' - x_i) - 2u_x^i) + \frac{u_x^i}{n_i}f_i''.$$

If $f_i' - x_i \geq 0$, then $\partial^2 u_i / \partial n_i^2 < 0$. However, when $f_i' - x_i < 0$, this condition may become positive. Hence u_i may not fulfill the second order condition.

make efforts. Then this local government has the incentive to worsen its public balance in order to promote migration into its own region. Thus local governments do not always make efforts efficiently under decentralization. Besides, the population distribution is not guaranteed to be Pareto efficient. However if migration is taken into account when making this decision, it has the effect of making the population distribution efficient to a certain degree. Of course, if migration is not taken into account, this effect does not exist. Thus even if there is no device like a interregional transfer mechanism under the decentralized local public finance system, the effort of improving the public balance with migration taken into account has the effect of making the population distribution efficient to a certain degree.

Appendix

The example of a single peak population In this section, we consider the example of a single population peak. For example, we define one case, where $u_i(x_i, y_i) = x_i y_i$, $f_i(n_i) = n_i^\alpha$ $(0 < \alpha < 1/2)$, $d(a_i) = a_i^2$, and $\bar{n} = 1$. Then the constraint of resources in region i becomes $n_i^\alpha = n_i x_i + c_i y_i + F - a_i + a_i^2$.

We calculate (11) in this example, and get

$$x_i = \frac{n_i^\alpha + a_i - a_i^2 - F}{2n_i} \text{ and } y_i = \frac{n_i^\alpha + a_i - a_i^2 - F}{2c_i}.$$

Of course, if $n_i^\alpha + a_i - a_i^2 - F < 0$, then $x_i = y_i = 0$. Thus the indirect utility function becomes

$$V_i = \begin{cases} \frac{(n_i^\alpha + a_i - a_i^2 - F)}{4n_i c_i} & \text{if } n_i \geq (F + a_i^2 - a_i)^{1/\alpha} \\ 0 & \text{if } n_i < (F + a_i^2 - a_i)^{1/\alpha} \end{cases} \quad (26)$$

When n_i is small, $n_i^\alpha + a_i - a_i^2 - F < 0$ in (26) because of the assumption of a large F. In this case, the utility becomes zero. [25]

Next, we prove that the case of the single peak for the population exists in this example. We differentiate V_i by n_i, then

$$\frac{\partial V_i}{\partial n_i} = \frac{(n_i^\alpha + a_i - a_i^2 - F)[(2\alpha - 1)n_i^\alpha - a_i + a_i^2 + F]}{4n_i^2 c_i}.$$

For instance, when the parameters are $\alpha = 1/3$ and $F = 4/15$, V_i has only one peak at $n_i^{**} = 27(a_i^2 - a_i)^3 + 64/125$. The net benefit of the local government's effort is $0 \leq a_i - a_i^2 \leq 1/4$, so the range of the population of the peak is $721/8000 \leq n_i^{**} \leq 64/125]$.

Proof of Proposition 1 *On (18), a_i which maximizes V_i satisfies $1 - d'(a_i^*) = 0$ because $f_1' - x_1 < 0$ and $f_2' - x_2 < 0$ for any a_1 and a_2 and $D < 0$. a_j does also. Hence (a_1^*, a_2^*) is the strategy in a subgame perfect equilibrium.*

Q.E.D.

[25] See footnote 9.

Proof of Lemma 1 *Supposing there are two or more migration equilibria at the same a_i and a_j, these satisfy one of the three cases. We denote two of these migration equilibria as $(n_i, V_1 = V_2)$ and $(\tilde{n}_i, \tilde{V}_1 = \tilde{V}_2)$.* [26]

1. We suppose that one migration equilibrium n_i is Case 1 and the other migration equilibrium $\tilde{n}_i$ is also Case 1. From Assumption 1, if $\tilde{n}_i > n_i$, $\tilde{V}_i < V_1 = V_2 < \tilde{V}_j$ and if $\tilde{n}_i < n_i$, $\tilde{V}_i > V_1 = V_2 > \tilde{V}_j$. But these contradict $\tilde{V}_1 = \tilde{V}_2$ which is an interior condition.

2. We suppose that one migration equilibrium n_i is Case 1 and the other migration equilibrium $\tilde{n}_i$ is Case 2. Then $\tilde{n}_i = n_i^* < n_i$ under Assumption 1 because of $f_i' - x_i < 0$ at n_i and $f_i' - x_i = 0$ at $\tilde{n}_i$. Therefore $\tilde{V}_i > V_1 = V_2$. In addition, $\tilde{V}_j < V_1 = V_2$ because $\bar{n} - n_i^* > \bar{n} - n_i$. But these contradict $\tilde{V}_1 = \tilde{V}_2$.

3. We suppose that one migration equilibrium n_i is Case 2 and the other migration equilibrium $\tilde{n}_i$ is also Case 2. Then $n_i = n_i^* = \tilde{n}_i$ because $f_i' - x_i = 0$. From Assumption 1, there is only one n_i^*. Therefore there are no multiple equilibria satisfies only Case 2.

Q.E.D.

Proof of Proposition 2 *The proof consists of three parts, 1, 2 and 3.*

1. Let us assume the local government i deviates from a_i^* to $a_i \neq a_i^*$ given a_j^*. We denote the migration equilibrium as n_i^d [27] *at this deviation (a_i, a_j^*).*

1) First we consider the case of a stable and interior migration equilibrium that varies continuously from a_i^* to a_i. (18) is 0 at a_i^* because $1 - d'(a_i^*) = 0$. For all $a_i > a_i^*$, V_i is a decreasing function under Assumption 1 because residents migrate from i to j if (16)< 0 and $f_j' - x_j < 0$ for all $a_i > a_i^*$. For all $a_i < a_i^*$, V_i is an increasing function because of the same logic. Then V_i is maximized at a_i^*.

2) Next we consider the case where the migration equilibrium does not vary continuously from a_i^* to a_i. This deviation brings the migration equilibria to either $n_i = 0$, $n_i = \bar{n}$ or Case 3 because of lemma 1. Then, at $n_i = 0$ and $n_i = \bar{n}$, $V_i(0, a_i) \leq V_i(0, a_i^*) < V_i(n_i, a_i^*)$ and $V_i(\bar{n}, a_i) < V_i(\bar{n}, a_i^*) < V_i(n_i, a_i^*)$ because of (17) and Assumption 1. Thus this deviation decreases i's payoff.

In Case 3, region i becomes either $f_i' - x_i < 0$ or $f_i' - x_i > 0$ at n_i^d. When $f_i' - x_i < 0$ at n_i^d, $f_j' - x_j > 0$ in region j. Then $n_i < n_i^d$ because of Assumption 1 and $f_j' - x_j < 0$ at n_i. Thus $V_i(n_i^d, a_i) < V_i(n_i^d, a_i^*) < V_i(n_i, a_i^*)$ because $f_i' - x_i < 0$ in $[n_i, n_i^d]$. Hence local government i does not have the incentive for this deviation.

When $f_i' - x_i > 0$ at n_i^d, $f_j' - x_j < 0$ in region j. Then $n_i^d < n_i$ because if $n_i^d \geq n_i$, $V_i(n_i, a_i^*) = V_j(\bar{n} - n_i, a_j^*) \leq V_j(\bar{n} - n_i^d, a_j^*) = V_i(n_i^d, a_i) < V_i(n_i^d, a_i^*)$ from $f_j' - x_j < 0$ and (17). But this contradicts $f_i' - x_i < 0$ in $[n_i, n_i^d]$ at a_i^* under Assumption 1. Therefore $n_i^d < n_i$. Then $V_i(n_i^d, a_i) = V_j(\bar{n} - n_i^d, a_j^*) < V_j(\bar{n} - n_i, a_j^*) = V_i(n_i, a_i^*)$ because $f_j - x_j < 0$ in $[n_i^d, n_i]$ and because of Assumption 1. Hence local government i does not have the incentive for this deviation.

2. 1) Let us assume local government i deviates from a_i^* to $a_i \neq a_i^*$ given a_j. We denote the migration equilibrium of this deviation as n_i^d. When the migration equilibrium varies continuously from a_i^* to a_i given a_j, V_i is maximized at a_i^*, as is 1-1) in this proof.

We consider the case where the migration equilibrium does not vary continuously from a_i^* to a_i. This deviation brings the migration equilibria to either $n_i = 0$, $n_i = \bar{n}$ or Case

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[26] If $i = 1$, then $n_i = n_1(a_1, a_2)$, and if $i = 2$, then $n_i = \bar{n} - n_1(a_1, a_2)$.

[27] n_i^d means that $n_i^d = n_1^d(a_1, a_2)$ if $i = 1$ and $n_i^d = \bar{n} - n_1^d(a_1, a_2)$ if $i = 2$. $n_1^d(a_1, a_2)$ is the migration equilibrium when a local government deviates.

3 because of lemma 1. Then, at $n_i = 0$ and $n_i = \bar{n}$, $V_i(0, a_i) \le V_i(0, a_i^*) < V_i(n_i, a_i^*)$ and $V_i(\bar{n}, a_i) < V_i(\bar{n}, a_i^*) < V_i(n_i, a_i^*)$ because of (17) and Assumption 1. Thus this deviation decreases i's payoff.

In Case 3, region i becomes either $f_i' - x_i < 0$ or $f_i' - x_i > 0$ at n_i^d. When $f_i' - x_i < 0$ at n_i^d, $f_j' - x_j > 0$ in region j. Then $n_i < n_i^d$ because of Assumption 1 and $f_j' - x_j < 0$ at n_i. Then $V_i(n_i^d, a_i) < V_i(n_i^d, a_i^*) < V_i(n_i, a_i^*)$ because $f_i' - x_i < 0$ in $(n_i, n_i^d]$ at a_i^*. Hence local government i does not have the incentive for this deviation.

When $f_i' - x_i > 0$ at n_i^d, $f_j' - x_j < 0$ in region j. Then $n_i^d < n_i$ because if $n_i^d \ge n_i$, $V_i(n_i, a_i^*) = V_j(\bar{n} - n_i, a_j) \le V_j(\bar{n} - n_i^d, a_j) = V_i(n_i^d, a_i) < V_i(n_i^d, a_i^*)$ from $f_j' - x_j < 0$ and (17). But this contradicts $f_i' - x_i = 0$ at n_i at a_i^* under Assumption 1. Therefore $n_i^d < n_i$. Then $V_i(n_i^d, a_i) = V_j(\bar{n} - n_i^d, a_j) < V_j(\bar{n} - n_i, a_j) = V_i(n_i, a_i^*)$ because $f_j - x_j < 0$ in $[n_i^d, n_i]$ and because of Assumption 1. Hence $V_i(n_i^d, a_i) < V_i(n_i, a_i^*)$, namely local government i does not have the incentive for this deviation.

2) Let us assume that local government j deviates from a_j to $\hat{a}_j \neq a_j$ given a_i^*. First, we consider that the migration equilibrium moves to the other interior equilibrium from a_j to $\hat{a}_j$. The migration causes this deviation decreases the utility of residents in region j because V_i is the optimal scale of population at n_i. Hence j does not have the incentive for this deviation. Second, we consider the case where the migration equilibrium moves to the corner due to the change from a_j to $\hat{a}_j$. In this case the migration equilibrium becomes either $n_i = 0$ or $n_i = \bar{n}$. Then $V_j(0, a_j^*) \le V_i(\bar{n}, a_i^*) < V_i(n_i, a_i^*) = V_j(\bar{n} - n_i, a_j)$ because of the definition of the migration equilibrium and $f_i' - x_i = 0$ at n_i, and $V_j(\bar{n}, a_j^*) \le V_j(\bar{n} - n_i, a_j)$, which is the condition in this proposition. Hence (a_i^*, a_j) is a Nash equilibrium.

3. 1) V_i is a decreasing function for a_i in $[0, a_i^*]$ because $f_j' - x_j > 0$ and $1 - d'(a_i) > 0$ in (18). Then an optimal effort for j is $a_i = 0$. On the other hand V_i is an increasing function for a_i in $[a_i^*, \bar{a}]$ because $f_j' - x_j > 0$ and $1 - d'(a_i) < 0$ in (18). Thus a_i which maximizes V_i is $a_i = 0, \bar{a}$.

2) V_j is an increasing function for a_j in $[0, a_j^*]$ because $f_i' - x_i < 0$ and $1 - d'(a_j) > 0$ in (18). On the other hand V_j is a decreasing function for a_j in $[a_j^*, \bar{a}]$ because $f_i' - x_i < 0$ and $1 - d'(a_j) < 0$ in (18). Thus a_j which maximizes V_j is $a_j = a_j^*$.

Q.E.D.

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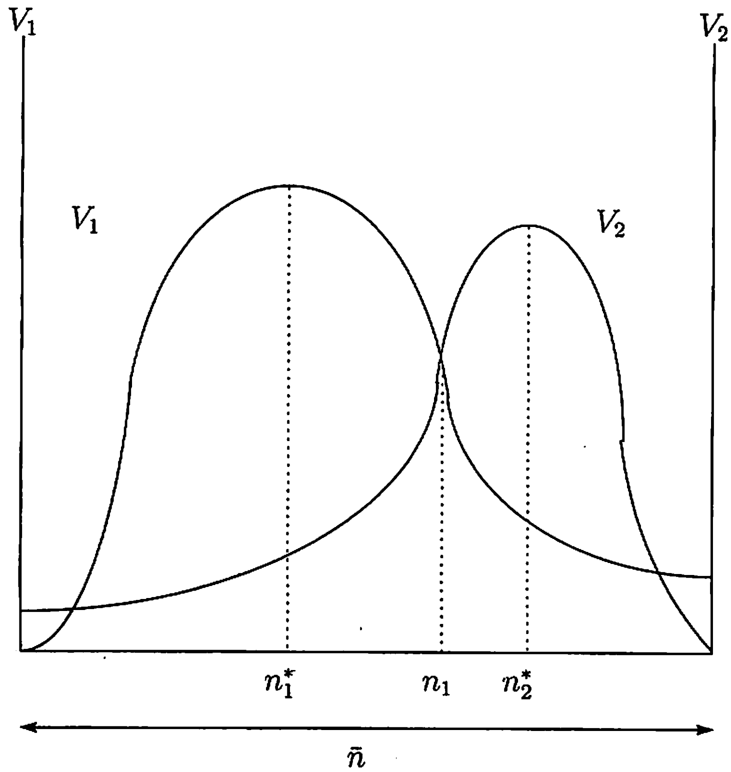
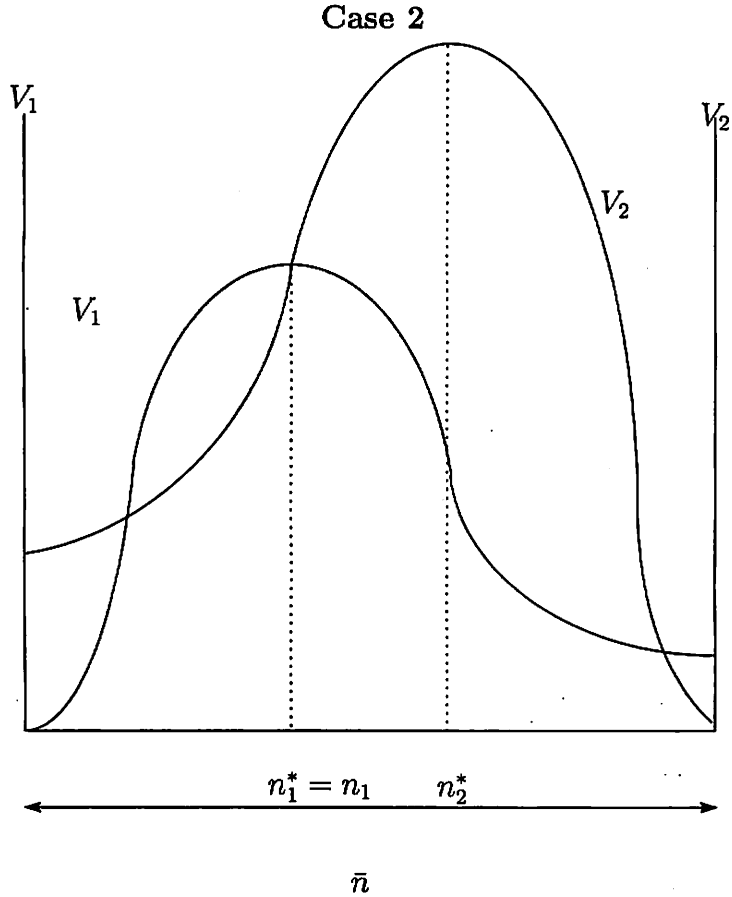
Case 1

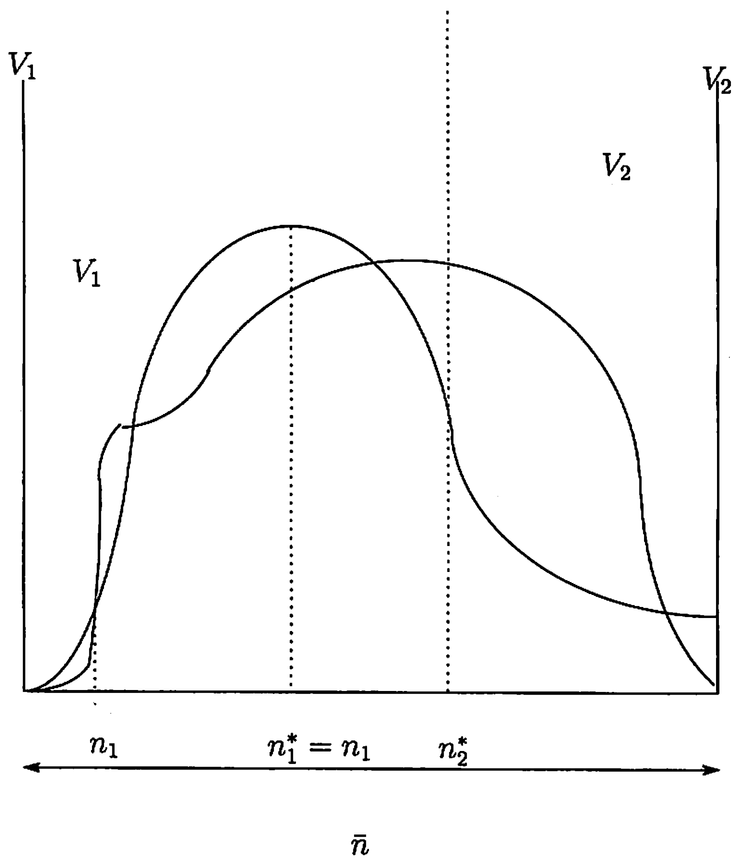


n_1 is the migration equilibrium at $(\bar{a}, \bar{a})$.

Case 2



Case 3



n_1 is the migration equilibrium of Case 3.