Reexamination of Stackelberg Leadership : Fixed Supply Contracts under Demand Uncertainty

SASAKI, Dan / HIROKAWA, Midori

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by

Midori Hirokawa & Dan Sasaki

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Midori Hirokawa
Faculty of Economics
Hosei University
4342 Aihara, Machida-shi
Tokyo 194-02 Japan
mhiro@mt.tama.hosei.ac.jp

Dan Sasaki
Economics Department
Faculty of Economics and Commerce
University of Melbourne
Parkville, Victoria 3052 Australia
dsasaki@cupid.ecom.unimelb.edu.au

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Abstract

This paper shown an endogenous Stackelberg leader-follower relation that stems purely from commitment, not from chronological order of entry. We consider a symmetric oligopoly game with *a priori* demand uncertainty. Each supplier can choose to make either a short-term or a long-term contract with retailers. Once a contract is signed, it mandates the supplier to supply a constant quantity throughout the contractual term, whether short or long. The demand uncertainty resolves by the time when a short-term contract expires. At that stage, a supplier on a short-term contract can re-adjust its supply quantity based upon the observed demand. On the other hand, a long-term contract can serve as a quantity commitment device. Thereby a supplier's choice between short- and long-term contracts depends upon the degree of demand uncertainty as well as time preferences. There exist cases where only one supplier makes a long-term quantity commitment and thereby exercises a Stackelberg leadership, even though the suppliers indeed play a pure strategy subgame perfect (i.e., best-response) equilibrium.

Keywords : Stackelberg-like equilibrium, discount factor, relative demand uncertainty, dynamic market.

JEL classification : L13, D43, C72.

* Preliminary and incomplete. Comments welcome. The authors assume the sole responsibility for any remaining errors.

1 Introduction

Stackelberg leader-follower relations have often been modelled in association with the chronological order of oligopolists' entries. Two conceptual problems inhere in such a framework. One is that the temporary *monopoly* by the leader *prior* to the Stackelberg oligopoly market needs to be explicitly modelled, since a firm's incentive to become a leader consists of two concurrent effects: the temporary monopoly profit *before* a follower's entry, and then the strategic pre-emption which comes in effect *after* the follower's entry. Curiously, many existing theoretical studies on endogenous Stackelberg behaviour focus exclusively on the latter, entirely ignoring the former.

The other problem is how to justify the key assumption that only the leader, not the follower, can commit with its action. Even when an incumbent and an entrant compete in the same industry, as long as the incumbent remains able to change its action (be it a price or a quantity) after its opponent's entry, there can be no *strategic* leader/followerships. The feasibility of commitment is not automatically guaranteed by asynchronous entries.

This paper models strategic leader/followerships explicitly as a matter of commitment, instead of a matter of the order of moves. In our attempt, we take into account the institution that most commodities are sold not literally in an open "market" but through retailers, with whom a supplier firm has to make a supply *contract* that stipulates a constant quantity of supply every period throughout the contractual term.[1] Instead of focusing on asynchronous entries, in this paper we consider the strategic bearings of contractual term lengths (as for the former, see Hirokawa and Sasaki, 1997).

Two quantity-competing firms enter simultaneously into an industry where the demand is uncertain. The game lasts two periods. Upon entry, each firm decides whether to make a long-term contract that stipulates the same quantity of supply in both periods, or a short-term contract without such commitment. The demand uncertainty resolves after one period. Then, only a firm which has previously been on a short-term contract can revise its output quantity for the second period, making use of the observed demand level. At the same time, if only one of the firms chooses a long-term contract whilst the other doesn't, the committed firm can exercise a Stackelberg leadership in the second period. We shall learn in this paper that such a trade-off between precautionary flexibility and

[1] To keep the analysis as simple as possible, we assume that there are many retailers so that they can exercise no oligopsonistic bargaining power.

strategic pre-emptive advantage can give rise to *a posteriori* asymmetric Stackelberg-like behaviour, even though firms are *a priori* symmetric.

We avail of a sizeable literature on endogenous determination of Stackelberg leader-follower relations. In a classical quantity competition without demand uncertainty, for instance, being a leader is unambiguously profitable whilst being a follower makes a firm worse off as opposed to a simultaneous Cournot-Nash equilibrium. This implies that, if the leader/followerships could be endogenously chosen by individual firms, then no firm would choose to be a follower (Henderson and Quandt, 1971). In general, in a supermodular game (such as Bertrand, where actions are strategic complements), it is more profitable to be the follower than to be the leader. In a submodular game (such as Cournot, where actions are strategic substitutes), the converse holds (Gal-Or, 1985).

Subsequently, endogenous leader/followerships have been modelled as a timing game played by potential entrants. Namely, each firm faces the choice between "enter immediately" and "wait," which leads to a normal-form game as the following matrix.

Firm i	Firm j	
	Enter now	Wait
Enter now	(Simultaneous Nash)	(Stackelberg)
Wait	(Stackelberg)	(Simultaneous Nash)

Along this framework, originated by Hamilton and Slutzky (1990), a number of models to explain the possibility of a Stackelberg equilibrium have been constructed. Robson (1990) imposes costs associated with an early action. Matsumura (1995) and Nishijima (1995) discusses the qualitative relation between strategic complementarity and/or substitution of actions and the choices of timing for such actions. In Nishijima (1997), an *a priori* better informed player tends to take a lead, whilst a less informed player tends to wait until the opponent's action reveals information. Some of the recent contributions on this subject have started to take demand uncertainty into consideration. Sadanand and Sadanand (1996) and Hirokawa (1997) derive the possibility of endogenous Stackelberg equilibria when the oligopolists are *a priori* uncertain about the market demand.

In almost all of these preceding theoretical models, in spite of their supposedly dynamic framework, the market is modelled statically: it is assumed to operate only in the oligopoly phase, disregarding the transient *monopoly* by the leader before the follower enters. The absence of an explicitly multi-stage market model also serves to obscure the other problem: how to make a quantity commitment.

It is true that there exist a number of Stackelberg models using an explicitly multi-stage game. These contributions can also be viewed as attempts to clarify the process of commitment. A precommitted capacity investment or advance production can possibly serve as a device for quantity commitment. Saloner (1987) and Pal (1991; 1996) construct models consisting of two feasible production periods, where a firm can earn the status of a Stackelberg leader by choosing to produce in the former period. Note that, even though they model dynamic *production*, they do not model a dynamic *market*. Their market operates only after the second production period, not between the two production periods. In addition, the reason why advance production automatically entitles the firm to Stackelberg leadership is not explicitly derived, but assumed *ad hoc.*[2]

In this paper, we model an explicitly dynamic *market*, as is laid out in section 2. Section 3 presents an equilibrium comparative statics analysis to identify those condition under which leader-follower behaviour can be endogenised. Section 4 conducts a welfare comparative statics analysis. Finally, section 5 summarises our main findings.

2 The Model

Two *a priori* identical firms simultaneously enter the same industry. The game lasts through two marketing periods. At the beginning of the game, each firm decides whether to make a short-term supply contract or a long-term supply contract. If a firm makes a long-term contract, then the firm must supply the same quantity in both periods. This restriction does not apply if a firm makes a short-term contract in the beginning.

In each period, the two firms sell homogeneous products (perfect substitutes). The inverse demand function

$$P = A - Q\,,$$

[2] The strategic purpose of advance production is not necessarily to earn Stackelberg leadership. Even in the absence of the leader-follower relation, firms may still have a pre-emptive incentives for advance production, as shown in Poddar and Sasaki (1997). Note that advance production cannot automatically preclude the possibility that the firm may still choose not to sell all the quantity produced. In other words, advance production *alone* cannot guarantee any quantity commitment.

where $Q = q_1 + q_2$ is the sum of the two firms' outputs, has a stochastic intercept A of which the prior c.d.f. $F(a)$ is commonly known to the two firms, with finite mean $E[A]$ and variance $\mathrm{Var}[A]$. This intercept stays unchanged throughout the two marketing periods. Production costs are c per unit, where $0 < c < E[A]$. The discount factor is δ, where $0 \leq \delta \leq 1$ (see Appendix A.1).

The intercept A is unobservable to the firms until the price and quantities realise at the end of the first period. Therefore, once the state of demand has been observed, then a firm can make use of this information to optimise its second-period supply quantity if and only if the firm has chosen a short-term contract in the beginning. On the other hand, if only one of the firms has made a long-term contract whilst the other has not, then the long-term commitment entitles the firm to Stackelberg leadership in the second marketing period. Therefore, at the beginning of the game, firms face the trade-off between the strategic advantage of commitment and the loss of adjustability to the demand realisation.

The main purpose of this theoretical exercise is to inspect firms' incentives to make a long-term commitment under the presence of demand uncertainty. To keep this analysis as succinct as possible, we would like to concentrate on firms' optimal output decisions, taking for granted their incentives for *entry*. To ensure with certainty the positivity of firms' output quantities and resulting market prices, assumptions

$$\inf[A] \geq c + \frac{1+\delta}{3+2\delta}\,(E[A]-c) \qquad \text{IR[2]}$$

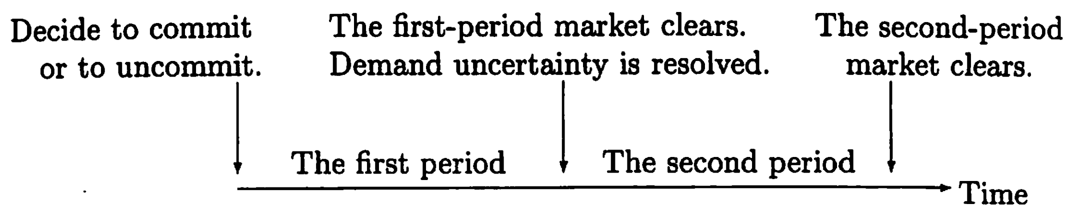
and

$$c \geq E[A] - \frac{2(3+2\delta)}{4+3\delta}\,\inf[A] \qquad \text{PP[2]I}$$

shall be imposed henceforth throughout this paper (see Appendix A.1).

The profit maximisation problems for each firm i ($i = 1, 2$) are as follows, depending upon the two firms' commitment decisions. q_i^I, $q_i^{II}[A]$ and q_i^L denote an uncommitted (on a short-term contract) firm's quantity in first and second marketing stages, and a committed (on a long-term contract) firm's quantity throughout the game, respectively. Note that only an uncommitted firm's second-period quantity can be made contingent upon the state A.

Figure 1 : Time structure of the game.



Discounted weight of each period :	1	$\delta \quad (0 \leq \delta \leq 1)$
Market size :	A (unobservable)	A (observable)
Production level if commit :	q_i^L	q_i^L
Production level if uncommit :	q_i^I	$q_i^{II}[A]$

[1] If both firms commit:

$$\max_{q_i^L} E\left[(A - q_1^L - q_2^L - c)\, q_i^L\right] \qquad i = 1, 2.$$

[2] If firm i commits whilst the other firm j doesn't, then the game is solved backward. In the second marketing stage, firm j solves:

$$\max_{q_j^{II}[A]} \left[(A - q_i^L - q_j^{II}[A] - c)\, q_j^{II}[A]\right].$$

Let $q_j^{II}[A, q_i^L]$ denote the solution for this maximisation. Back in the first stage, the two firms solve

$$\max_{q_i^L} E\left[(A - q_i^L - q_j^I - c)\, q_i^L + \delta\,(A - q_i^L - q_j^{II}[A, q_i^L] - c)\, q_i^L\right],$$

$$\max_{q_j^I} E\left[(A - q_i^L - q_j^I - c)\, q_j^I\right]$$

respectively.

[3] If neither firm commits, in the second stage:

$$\max_{q_i^{II}[A]} \left[(A - q_i^{II}[A] - q_j^{II}[A] - c)\, q_i^{II}[A]\right] \qquad i = 1, 2,$$

and back in the first marketing stage:

$$\max_{q_i^I} E\left[(A - q_1^I - q_2^I - c)\, q_i^I\right] \qquad i = 1, 2.$$

3 Equilibrium Outcomes

The game specified in the previous section can be summarised into the following payoff matrix. See Appendix A.1 for explicit calculation of the expected discounted profits, denoted by $\bar{\pi}$'s in the matrix.

Firm	Opponent	
	Commit	Not commit
Commit (long contract)	π^{LL}	π^{LS}
Not commit (short contracts)	π^{SL}	π^{SS}

Hence, firm's *equilibrium* choices of contractual term lengths yield those three outcomes specified in the previous section, [1], [2] and [3], under the following parametric conditions.

Proposition I :

[1] (Commit, Commit) is the dominant strategy equilibrium iff

$$\bar{\pi}^{LL} > \bar{\pi}^{SL} \text{ and } \bar{\pi}^{LS} > \bar{\pi}^{SS} \quad \Longleftrightarrow \quad \frac{\mathrm{Var}[A]}{(E[A]-c)^2} < \min\left\{\frac{(1+\delta)(12+7\delta)}{9(3+2\delta)^2}, \frac{(1+\delta)(3+\delta)}{2(3+2\delta)^2}\right\}.$$

[2] (Commit, Not commit) is a pure strategy subgame perfect equilibrium iff

$$\bar{\pi}^{LL} \leq \bar{\pi}^{SL} \text{ and } \bar{\pi}^{SS} \leq \bar{\pi}^{LS} \quad \Longleftrightarrow \quad \frac{(1+\delta)(12+7\delta)}{9(3+2\delta)^2} \leq \frac{\mathrm{Var}[A]}{(E[A]-c)^2} \leq \frac{(1+\delta)(3+\delta)}{2(3+2\delta)^2}.$$

[1][3] Both (Commit, Commit) and (Not commit, Not commit) are pure strategy subgame perfect equilibria iff

$$\bar{\pi}^{LL} \geq \bar{\pi}^{SL} \text{ and } \bar{\pi}^{SS} \geq \bar{\pi}^{LS} \quad \Longleftrightarrow \quad \frac{(1+\delta)(3+\delta)}{2(3+2\delta)^2} \leq \frac{\mathrm{Var}[A]}{(E[A]-c)^2} \leq \frac{(1+\delta)(12+7\delta)}{9(3+2\delta)^2}.$$

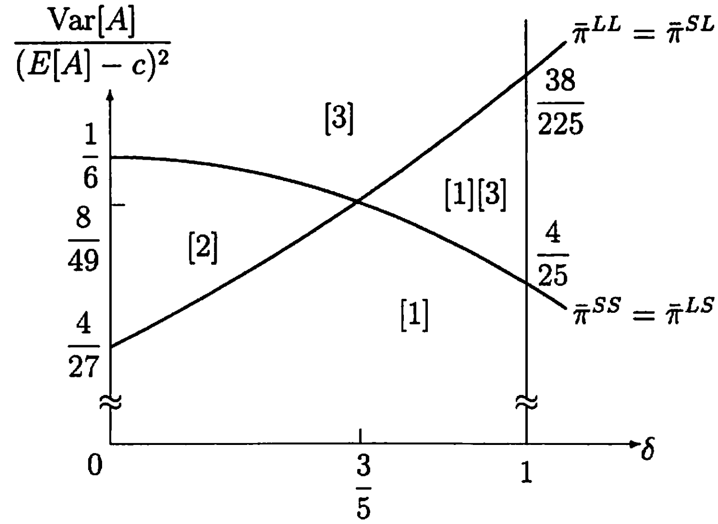
[3] (Not commit, Not commit) is the dominant strategy equilibrium iff

$$\bar{\pi}^{SL} > \bar{\pi}^{LL} \text{ and } \bar{\pi}^{SS} > \bar{\pi}^{LS} \quad \Longleftrightarrow \quad \frac{\mathrm{Var}[A]}{(E[A]-c)^2} > \max\left\{\frac{(1+\delta)(12+7\delta)}{9(3+2\delta)^2}, \frac{(1+\delta)(3+\delta)}{2(3+2\delta)^2}\right\}.$$

Proof follows directly from the payoff matrix (also see Appendix A.1). Note that there is also a mixed strategy equilibrium whenever the game is not dominance solvable (the middle two cases: [2] and [1][3]).

The entire parameter space is thereby devided into four regimes, as illustrated in figure 2. Among the three outcomes of pure strategy subgame perfect equilibria, [1] and [3] are in the spirit of simultaneous Cournot-Nash, while [2] has an inclination towards Stackelberg.

Figure 2 : Pure strategy subgame perfect equilibria.



This comparative statics result can be interpreted as follows. Toward the south-east of this diagram, where the demand uncertainty is small and the relative importance of the second period is high, firms have strong incentives for strategic advantage by committing with a long-term contract. This leads to equilibrium [1] to the lower-right of the locus $\bar{\pi}^{LL} = \bar{\pi}^{SL}$. On the other hand, toward the north-east, where the demand uncertainty is large and the second period is relatively important, firms' incentives are not to commit so that they can adjust their second-period supply quantities flexibly. This entails equilibrium [3] to the upper-right of the locus $\bar{\pi}^{SS} = \bar{\pi}^{LS}$. The pure Stackelberg-like outcome [2] arises over the remainder of the parameter space, which spans only $0 \leq \delta \leq \frac{3}{5}$. The Stackelberg-like behaviour can also arise as a consequence of mixed strategy equilibria in regimes [1][3] and [2].

Economic Implication : A low δ describes the situation where either (a) firms are short-sighted, or interest rates are high, or (b) the periods are long, i.e., demand uncertainty clears slowly. A high δ describes the converse.

The above analysis indicates that a pure strategy Stackelberg-like equilibrium arises only when the weight of the second marketing period δ is low, which is either (a) when firms are relatively myopic or (b) when the damand uncertainty clears very slowly.

Also, in order for a Stackelberg-like outcome to be sustainable either as a pure strategy equilibrium or as a consequence of a mixed strategy equilibrium, the degree of uncertainty (measured by the variation of the demand relative to the expected size of demand) has to lie within an intermediate range. This is intuitively because high variation of the demand tends to discourage commitment, entailing [3] no commitment Cournot-Nash equilibrium, whilst low variation of the demand does the opposite, entailing [1] commitment Cournot-Nash equilibrium.

Obviously, the degree of uncertainty $\frac{\mathrm{Var}[A]}{(E[A]-c)^2}$ is determined by the demand distribution $F(\cdot)$. To facilitate intuition, some numerical examples are given in Appendix A.2.

4 Social Welfare

In quantity competition, Stackelberg outcomes are generally welfare superior to Cournot-Nash outcomes in the absence of uncertainty. This is simply due to the fact that the *total supply quantity* is higher in the former than in the latter. This effect of quantity enhancement continues to hold in our model.

Lemma : The expected *total* supply quantity in each period is higher in the Stackelberg-like equilibrium [2] than in simultaneous Cournot-Nash equilibria [1] and [3].

Proof : The expected total supply is

$$E\left[q_1^L + q_2^L\right] = E\left[q_1^I + q_2^I\right] = E\left[q_1^{II}[A] + q_2^{II}[A]\right] = \frac{2}{3}\left(E[A] - c\right)$$

in Cournot-Nash outcomes [1] and [3], as opposed to

$$E\left[q_i^L + q_j^I\right] = E\left[q_i^L + q_j^{II}[A]\right] = \frac{4+3\delta}{2(3+2\delta)}\left(E[A] - c\right)$$

in the Stackelberg-like ourcome [2].

In terms of welfare, however, there is again a trade-off between the benefit from quantity commitment and adjustability to the randomness of demand.

Proposition II : In expected discounted social welfare,

- the symmetric commitment outcome [1] is unambiguously inferior to other two outcomes [2] and [3] ;

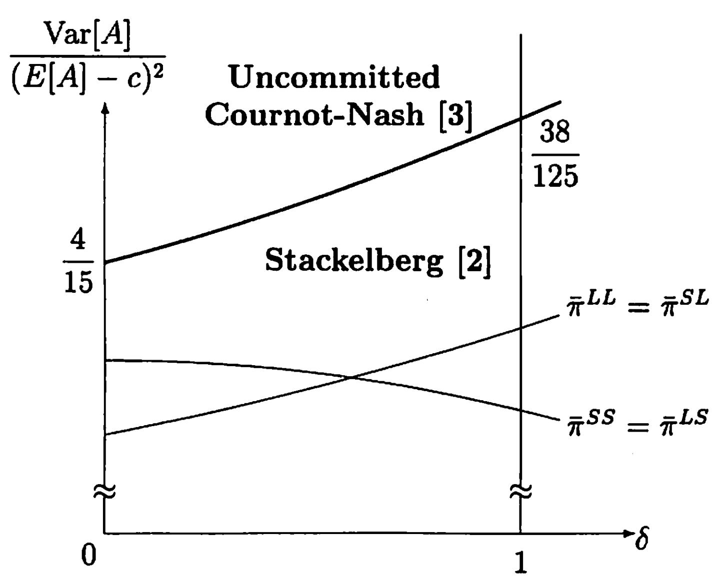
- the Stackelberg-like outcome [2] dominates the symmetric non-commitment outcome [3] iff
$$\frac{\mathrm{Var}[A]}{(E[A]-c)^2} \leq \frac{(1+\delta)(12+7\delta)}{5(3+2\delta)^2}$$

See Appendix A.1 for proof. In conjuction with Proposition I, the following is implied.

Corollary : The parametric range over which the Stackelberg-like outcome [2] is *strategically* sustainable is a *proper subset* of that range where outcome [2] is *socially* superior to other outcomes [1] and [3].

In figure 3, outcome [2] is *socially* desirable over the whole region below the thick locus, though it is *strategically* sustainable (either as a pure strategy equilibrium or as a consequence of a mixed strategy equilibrium) only in those two pieces of areas between the two thin loci.

Figure 3 : Socially optimal commitment profiles.



These observations suggest that some kind of policy intervention to encourage the asymmetric profile [2] *beyond firms' private incentives* is socially desirable.[3]

[3] Such intervention is feasible when the public authority can control the lengths of supply contracts but not the quantity. An example of such a situation is the (multi-unit) auction for public utility supply, such as energy or water. The policy question here is whether some, not all, of the supply slots are allocated for long-term suppliers and others are for shorter-term suppliers.

5 Conclusion and Summary

In this paper, we have shown:

- A Stackelberg-like outcome can arise as a pure strategy subgame perfect equilibrium when the market is dynamic (i.e., lasts multiple periods and clears every period) and each firm has a choice between short-term and long-term supply contracts.
- A long-term supply commitment becomes a strategic substitute between firms when and only when the relative importance of the first stage (i.e., the shortest contractual term admissible at the beginning of the game) is substantial. This condition is necessary in order to sustain endogenous Stackelberg-like behaviour as a pure strategy equilibrium.
- For social welfare, the Stackelberg-like outcome is desired when, but *not only* when, it is endogenously sustainable as a strategic equilibrium. Hence some policy or legislative measures may be desired to encourage socially optimal Stackelberg-like behaviour.

Contrary to some of the earlier beliefs, in endogenising Stackelberg-like behaviour, the market *before* the arrival of the leader/followerships plays a non-negligible role. In our two-period model we have learned that, for a Stackelberg-like outcome to be a pure strategy equilibrium, the weight of the first period (which is one since it is not subject to time discounting), in which there is not yet a leader or a follower, is required to be sufficiently high relative to that of the second period (which is parametrised by the discound factor δ), in which there are a leader and a follower.

Appendix

A.1 Computational Details

We start from an implicit assumption that firms' optimal supply quantities and resulting market prices are always positive. This maintains their objective (profit) functions linear-quadratic, which makes $E[A]$ and $\mathrm{Var}[A]$ sufficient statistics in solving firms' optimisation problems, thereby enables us to derive explicit-form solutions for firms' market behaviour without narrowly specifying the prior demand distribution $F(\cdot)$. Then we shall rediscover the positivity conditions for quantities and prices.

[1] If both firms commit, then each produces

$$q_i^L = \frac{E[A]-c}{3}$$

in both marketing stages. This quantity is *always* positive as long as $E[A] > c$. The market price

$$p_{[1]} = A - 2q_i^L = A - \frac{2}{3}(E[A]-c)$$

is positive as long as

$$c \geq E[A] - \frac{3}{2}\inf[A]. \qquad \text{PP[1]}$$

Each firm's discounted profit is thereby

$$\pi^{LL} = \frac{1+\delta}{3}(E[A]-c)\left(A-c-\frac{2}{3}(E[A]-c)\right),$$

and the resulting social welfare is

$$w^{[1]} = (1+\delta)\int_{q=0}^{2q_i^L}(A-q-c)\,dq = \frac{2(1+\delta)}{9}(3A-E[A]-2c)(E[A]-c).$$

[2] If one commits whilst the other doesn't, then the committed firm produces

$$q_i^L = \frac{(1+\delta)(E[A]-c)}{3+2\delta}$$

throughout the two marketing stages, whilst the uncommitted firm produces

$$q_j^I = \frac{(2+\delta)(E[A]-c)}{2(3+2\delta)}$$

in the first marketing stage, and

$$q_j^{II}[A] = \frac{1}{2}\left(A-c-\frac{1+\delta}{3+2\delta}(E[A]-c)\right)$$

in the second marketing stage. The former two, q_i^L and q_j^I, are both unambiguously positive as long as $E[A] > c$. The latter, q_j^{II}, is assuredly positive under the proviso that

$$\inf[A] \geq c + \frac{1+\delta}{3+2\delta}\left(E[A]-c\right) \qquad \text{IR[2]}$$

The market price in the first stage

$$p_{[2]}^I = A - q_i^L - q_j^I = A - \frac{4+3\delta}{2(3+2\delta)}\left(E[A]-c\right)$$

is positive whenever

$$c \geq E[A] - \frac{2(3+2\delta)}{4+3\delta}\inf[A], \qquad \text{PP[2]I}$$

and the second stage price

$$p_{[2]}^{II} = A - q_i^L - q_j^{II}[A] = A - \frac{1}{2}\left(A - c + \frac{1+\delta}{3+2\delta}\left(E[A]-c\right)\right)$$

is positive under

$$c \geq \frac{(1+\delta)E[A] - (3+2\delta)\inf[A]}{4+3\delta}, \qquad \text{PP[2]II}$$

The two firms' profits are

$$\pi^{LS} = \frac{(1+\delta)(2+\delta)}{2(3+2\delta)}\left(E[A]-c\right)\left(A - c - \frac{2+\delta}{3+2\delta}\left(E[A]-c\right)\right)$$

and

$$\begin{aligned}\pi^{SL} &= \frac{2+\delta}{2(3+2\delta)}\left(E[A]-c\right)\left(A - c - \frac{4+3\delta}{2(3+2\delta)}\left(E[A]-c\right)\right)\\ &\quad + \frac{\delta}{4}\left(A - c - \frac{1+\delta}{3+2\delta}\left(E[A]-c\right)\right)^2\end{aligned}$$

respectively. The welfare is

$$\begin{aligned}w^{[2]} &= \int_{q=0}^{q_i^L+q_j^I}(A-q-c)\,dq + \delta\int_{q=0}^{q_i^L+q_j^{II}[A]}(A-q-c)\,dq\\ &= (1+\delta)\frac{(A-c)^2}{2} - \frac{1}{2}\left(A - c - \frac{4+3\delta}{2(3+2\delta)}\left(E[A]-c\right)\right)^2\\ &\quad - \frac{\delta}{8}\left(A - c - \frac{1+\delta}{3+2\delta}\left(E[A]-c\right)\right)^2\end{aligned}$$

[3] If neither firm commits, then each produces an unambiguously positive quantity

$$q_i^I = \frac{E[A]-c}{3}$$

in the first marketing stage, where the price

$$p_{[3]}^I = A - 2q_i^I = A - \frac{2}{3}\left(E[A]-c\right)$$

is positive under

$$c \geq E[A] - \frac{3}{2}\inf[A]\,. \qquad \text{PP[3]}$$

In the second marketing stage, each firm produces

$$q_i^{II}[A] = \frac{A-c}{3}\,, \qquad p_{[3]}^{II} = A - 2q_i^{II}[A] = \frac{A-c}{3}$$

of which the positivity is guaranteed by

$$\inf[A] \geq c\,. \qquad \text{IR[3]}$$

This entails each firm's profit

$$\pi^{SS} = \frac{E[A]-c}{3}\left(A - c - \frac{2(E[A]-c)}{3}\right) + \delta\left(\frac{A-c}{3}\right)^2$$

and welfare

$$\begin{aligned} w^{[3]} &= \int_{q=0}^{2q_i^I}(A-q-c)\,dq + \delta\int_{q=0}^{2q_i^{II}[A]}(A-q-c)\,dq \\ &= \frac{4(1+\delta)}{9}(A-c)^2 - \frac{2}{9}(A-E[A])(2A-E[A]-c)\,. \end{aligned}$$

Hence IR[2] and IR[3] ensure the firms' participation incentives. Since the former is a stricter condition than the latter, it suffices to assume only the former. Likewise, among those conditions PP[1], PP[2]I, PP[2]II and PP[3] ensuring the positivity of market prices, PP[2]I is the most stringent.

Hereby IR[2] and PP[2]I should be imposed as assumptions throughout this paper (see section 2). Without these assumptions, the first-order conditions for the firms' maximisation problems would become non-linear, so that the closed-form solutions would be difficult to obtain without further distributional assumptions on $F(a)$ (see Hirokawa, 1997).

Under IR[2] and PP[2]I, the *expected* payoff matrix (see section 3) is derived as follows:

$$\begin{aligned} \bar{\pi}^{LL} &= E[\pi^{LL}] = (1+\delta)\left(\frac{E[A]-c}{3}\right)^2, \\ \bar{\pi}^{LS} &= E[\pi^{LS}] = \left(1+\frac{\delta}{2}\right)\left(\frac{1+\delta}{3+2\delta}(E[A]-c)\right)^2, \\ \bar{\pi}^{SL} &= E[\pi^{SL}] = (1+\delta)\left(\frac{2+\delta}{2(3+2\delta)}(E[A]-c)\right)^2 + \frac{\delta}{4}\,\mathrm{Var}[A]\,, \\ \bar{\pi}^{SS} &= E[\pi^{SS}] = (1+\delta)\left(\frac{E[A]-c}{3}\right)^2 + \frac{\delta}{9}\,\mathrm{Var}[A]\,. \end{aligned}$$

Therefore,

$$\bar{\pi}^{LL} \gtreqless \bar{\pi}^{SL} \quad \text{iff} \quad \frac{\text{Var}[A]}{(E[A]-c)^2} \lesseqgtr \frac{(1+\delta)(12+7\delta)}{9(3+2\delta)^2}$$

$$\bar{\pi}^{LS} \gtreqless \bar{\pi}^{SS} \quad \text{iff} \quad \frac{\text{Var}[A]}{(E[A]-c)^2} \lesseqgtr \frac{(1+\delta)(3+\delta)}{2(3+2\delta)^2}$$

The expected welfare becomes:

$$\begin{aligned} \bar{w}^{[1]} &= E[w^{[1]}] = \frac{4(1+\delta)}{9}(E[A]-c)^2, \\ \bar{w}^{[2]} &= E[w^{[2]}] = \frac{(1+\delta)(4+3\delta)(8+5\delta)}{8(3+2\delta)^2}(E[A]-c)^2 + \frac{3\delta}{8}\text{Var}[A], \\ \bar{w}^{[3]} &= E[w^{[3]}] = \frac{4(1+\delta)}{9}(E[A]-c)^2 + \frac{4\delta}{9}\text{Var}[A]. \end{aligned}$$

Hence

$$\bar{w}^{[1]} < \bar{w}^{[2]} \leq \bar{w}^{[3]} \quad \text{iff} \quad \frac{\text{Var}[A]}{(E[A]-c)^2} \geq \frac{(1+\delta)(12+7\delta)}{5(3+2\delta)^2},$$

$$\bar{w}^{[1]} < \bar{w}^{[3]} \leq \bar{w}^{[2]} \quad \text{iff} \quad \frac{\text{Var}[A]}{(E[A]-c)^2} \leq \frac{(1+\delta)(12+7\delta)}{5(3+2\delta)^2}$$

A.2 Numerical Examples

For a Stackelberg-like outcome to be sustainable either as a pure strategy equilbrium or as a result of a mixed strategy equilibrium, the prior demand distribution must keep the degree of uncertainty within a certain range. Observing a few algebraically simple distributions can give us some qualitative insight on what types of uncertainty are admissible for endogenising Stackelberg-like behaviour.

As a simple example, consider a uniform demand distribution. When $\delta = \frac{3}{5}$, which is the highest δ that can sustain a pure strategy Stackelberg-like equilibrium, firms' participation condition (inequality IR[2] in section 3) becomes $\inf[A] \geq c + \frac{8}{21}(E[A]-c)$. Under this proviso, the most disperse uniform distribution is $A \in \left(c + \frac{8}{21}(E[A]-c),\right.$ $\left. c + \frac{34}{21}(E[A]-c)\right]$, where $\frac{\text{Var}[A]}{(E[A]-c)^2} = \frac{169}{1323}$. This uncertainty is not sufficiently large to sustain a Stackelberg-like equilibrium.

When $\delta \downarrow 0$, firms' participation constraint IR[2] becomes $\inf[A] \geq c + \frac{1}{3}(E[A]-c)$. The most disperse uniform distribution satisfying this condition is $A \in \left(c + \frac{1}{3}(E[A]-c),\right.$

$c + \frac{5}{3}(E[A] - c)\Big]$, where $\frac{\text{Var}[A]}{(E[A]-c)^2} = \frac{4}{27}$. This is barely sufficient to entail a Stackelberg-like outcome as one of the weak equilibria.

These observations suggest that the demand uncertainty tends not to be sufficiently high to give rise to the endogenous Stackelberg-like behaviour as long as the demand distribution is symmetric around $E[A]$.

In reality, however, when a new product is introduced into the market, typically there is a small probability that the product becomes a million seller, and a larger probability that it becomes nothing more than one of less successful products. Namely, a typical prior demand distribution has a distinctively positive skewness.

One of those computationally simple distributions with a large upper-tail is an exponential distribution. To yield $\frac{\text{Var}[A]}{(E[A]-c)^2} = \frac{8}{49}$, which enables Stackelberg-like equilibria over the widest ranges of δ, the exponential demand distribution should be $a \in \left(c + (1 - \frac{2\sqrt{2}}{7})(E[A] - c), \infty\right)$, of which the c.d.f. is

$$F(a) = 1 - \exp\left[\frac{7\sqrt{2}-4}{4} - \frac{(7\sqrt{2})(a-c)}{4(E[A]-c)}\right] \qquad a \geq c + (1 - \frac{2\sqrt{2}}{7})(E[A] - c).$$

Overall, the Stackelberg-like behaviour is more likely to arise when the prior demand distribution is positively skewed, than when it is symmetric.

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