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Improved Multistep Method for Wide-Angle Beam Propagation

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***Abstract*—To improve a wide-angle beam propagation method using a finite-difference technique, the generalized Douglas scheme for variable coefficients is applied to a multistep method. A truncation error of $O(\Delta x)^4$ is achieved in the transverse direction. The effectiveness of the present method is demonstrated in the analysis of a tilted step-index slab waveguide.**

I. Introduction

The BEAM-PROPAGATION method (BPM) has successfully been used to analyze various optical waveguides. Since the conventional paraxial approximation restricts the application, several techniques have been proposed to treat wide-angle beam propagation [1]–[8]. Of them a so-called multistep method [4] in which the Padé approximant operator is factored into a series of simpler Padé (1, 1) operators has an advantage in that it allows paraxiallike solution techniques, such as the Thomas algorithm. The formulation by Hadley is, however, based on the Crank–Nicholson (CN) scheme with a truncation error of $O(\Delta x)^2$ in the transverse direction.

Recently we developed an improved BPM based on the generalized Douglas (GD) scheme for variable coefficients [9], and studied the possibility of applying the GD scheme to a wide-angle beam propagation analysis [10]. The GD scheme ensures a truncation error of $O(\Delta x)^4$ with substantial improvement in accuracy.

The purpose of this letter is to apply the GD scheme to the multistep method for wide-angle beam propagation, and to demonstrate the effectiveness of the present method. The propagation of the TE_1 mode in a tilted step-index waveguide [11] is analyzed to assess the numerical accuracy.

II. Formulation

The method begins with the scalar propagation equation obtained using a Padé (n, n) approximation of the Helmholtz operator. Then, an Nth-order Padé propagator may be decomposed into an N-step algorithm for which the kth partial step takes the form [4]

$$E^{m+k/N} = \frac{1 + a_k P}{1 + a_k^* P} E^{m+(k-1)/N} \tag{1}$$

in which $P = \partial^2/\partial x^2 + \nu$, where $\nu = k_0^2[n^2(x,z) - n_0^2]$. k_0 is the free space wavenumber, $n(x,z)$ is the index profile, and n_0 is the reference refractive index. The superscript m indicates position along the z axis. The a's can be determined by the one-time solution of an Nth-order complex algebraic equation.

Equation (1) can straightforwardly be restored to the following differential equation

$$\frac{\partial E}{\partial z} = \frac{2j\mathrm{Im}\{a_k\}P}{(1 + \mathrm{Re}\{a_k\}P)\Delta z/N} E. \tag{2}$$

We apply the GD scheme to Eq. (2). Following the procedure described in [10], we finally obtain

$$\begin{aligned} &C_{i+1}^{m+k/N} E_{i+1}^{m+k/N} + C_i^{m+k/N} E_i^{m+k/N} + C_{i-1}^{m+k/N} E_{i-1}^{m+k/N} \\ &\quad = C_{i+1}^{m+(k-1)/N} E_{i+1}^{m+(k-1)/N} + C_i^{m+(k-1)/N} E_i^{m+(k-1)/N} \\ &\qquad + C_{i-1}^{m+(k-1)/N} E_{i-1}^{m+(k-1)/N} \end{aligned} \tag{3}$$

where

$$\begin{aligned} C_{i\pm1}^{m+(k-1)/N} &= \frac{1}{12} + \Gamma_k \left(\frac{1}{\Delta x^2} + \frac{1}{12} \nu_{i\pm1}^{m+1/2} \right), \\ C_i^{m+(k-1)/N} &= \frac{5}{6} - \Gamma_k \left(\frac{2}{\Delta x^2} - \frac{5}{6} \nu_i^{m+1/2} \right) \end{aligned}$$

in which

$$\begin{aligned} \Gamma_k &= a_k \quad \text{for } m + (k-1)/N \\ &= a_k^* \quad \text{for } m + k/N. \end{aligned}$$

Equation (3) can be solved by efficient techniques such as the Thomas algorithm, so that the computational time is almost identical to that in the conventional method based on the CN scheme. It should also be noted that a transparent boundary condition [12] can easily be incorporated.

III. Numerical Results

Fig. 1 illustrates a tilted step-index waveguide to be considered here. The core width is designated as $2D$. We treat a weakly guiding structure in which the ratio of the refractive indices of the core and cladding is $N_{\mathrm{CO}}/N_{\mathrm{CL}} = 1.002$. For easy normalization, a wavelength of $\lambda = 1$ μm is used, and the reference refractive index is chosen to be that in the cladding ($N_{\mathrm{CL}} = 1.000$). To assess the accuracy, we calculate the coupling efficiency (definition is presented in [10]) at a propagation distance of 100 μm. In the case of no calculation error, the guided-mode field continues to propagate without any distortion, so that the efficiency becomes unity. Throughout this letter, the tilted waveguide is excited with the TE_1 mode, which can be obtained analytically. The numerical parameters to be used are as follows. The propagation step

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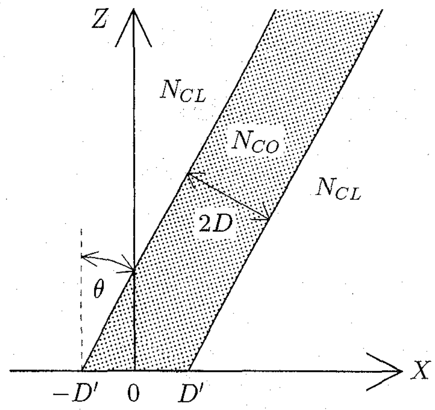

Fig. 1. Configuration of a tilted step-index slab waveguide.

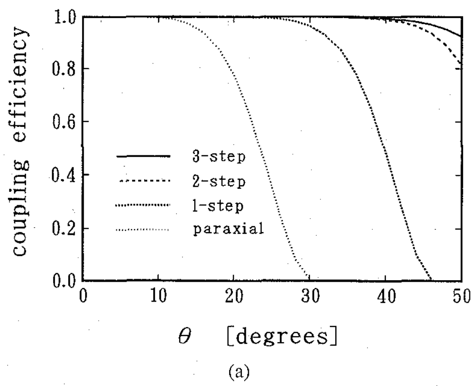

Fig. 2. Coupling efficiency for TE_1 mode as a function of tilt angle θ: (a) Generalized Douglas scheme and (b) Crank-Nicholson scheme. The results are improved as the number of steps is increased. The superiority of the generalized Douglas scheme is observed particularly at large tilt angles.

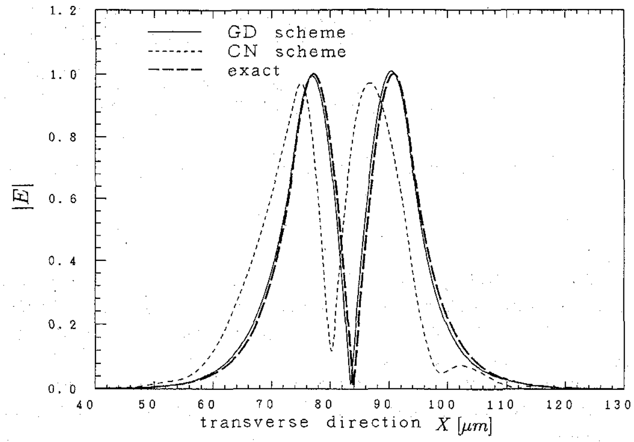

Fig. 3. Field distributions for TE_1 mode observed at a propagation distance of 100 μm. A tilt angle is 40°. The field obtained by the generalized Douglas scheme is close to the exact field.

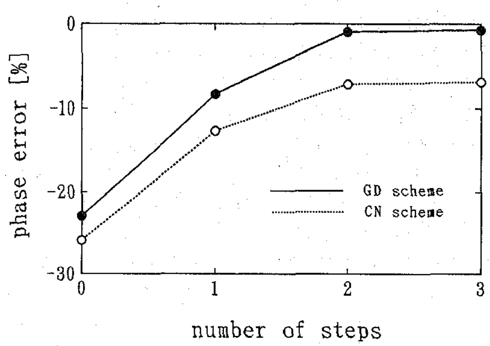

Fig. 4. Phase error for TE_1 mode observed at a propagation distance of 100 μm. A tilt angle is 40°. 0-step corresponds to the paraxial approximation.

length is $\Delta z = 0.1$ μm, the transverse sampling width is $\Delta x = D/(84 \cos\theta) = D'/84$, and the number of transverse sampling points is 1800.

Fig. 2 shows the coupling efficiency for TE_1 mode as a function of tilt angle θ. The core width is taken to be $2D = 15.092$ μm, so that the normalized frequency is $V = 3.0$. Fig. 2(a) and (b) are the cases for the GD and CN schemes, respectively. It is clearly seen that the increase in the number of steps leads to the improvement in accuracy. It is worth mentioning that the GD scheme achieves higher accuracy than the CN scheme in spite of the same discretization mesh.

To clarify the reason why the GD scheme improves the coupling efficiency, we illustrate the propagating field obtained using the three-step method in Fig. 3. The calculation is made for a tilt angle of 40° and the field is observed at a propagation distance of 100 μm. The field obtained by the GD scheme propagates with the mode profile being maintained, while that by the CN scheme not only deforms but also shifts toward the $-x$ direction.

As a further test on the accuracy, we calculate the phase error accumulated as the field propagates. The phase is evaluated using the maximum field at a propagation distance of 100 μm. The results as a function of the number of steps are shown in Fig. 4. The tilt angle is 40°. In this figure, 0-step corresponds to the paraxial approximation. We can find that the accuracy improves as the number of steps is increased, and the GD scheme achieves higher accuracy than the CN scheme. For example, the error is evaluated to be -0.7% for the three-step method with the GD scheme, while that with the CN scheme is at best -6.9%.

It is known in a wide-angle scheme that the numerical results are not sensitive to the choice of the reference refractive index n_0 [5], [6]. To confirm this fact, we also evaluate the reference-index dependence of the coupling efficiency and the phase error using the GD scheme. Calculations are

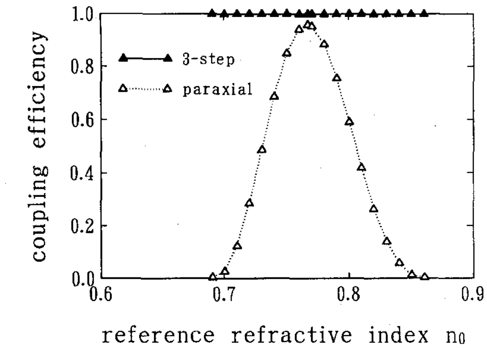

Fig. 5. Coupling efficiency as a function of reference refractive index.

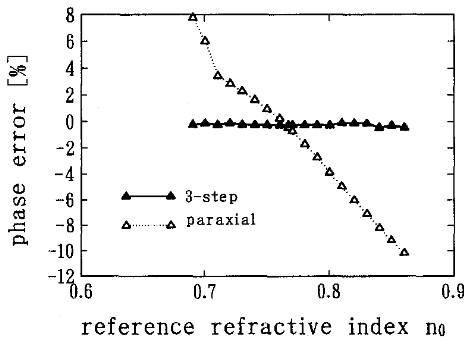

Fig. 6. Phase error as a function of reference refractive index.

made for $\theta = 40°$ at a propagation distance of 100 μm. Figs. 5 and 6 clearly show that the present results are almost independent of n_0, while the scheme on the basis of the paraxial approximation yields relatively good results only when n_0 is chosen to be $\beta \cos\theta / k_0$ ($\simeq 0.77$) [13], where β is the propagation constant of the eigenmode of the waveguide. In other words, a wide-angle scheme has an advantage in that the problem involving radiation modes can be treated more accurately than that based on the paraxial approximation [8].

So far, the discussion has been restricted to a weakly guiding structure. Final consideration is given to the propagation problem in a strongly guiding structure. We choose the waveguide discussed in [11], i.e., $N_{CO} = 3.3$, $N_{CL} = 3.17$, $2D = 8.8$ μm and $\lambda = 1.55$ μm, and consider the case where the tilted waveguide with $\theta = 40°$ is excited with the TE_1 mode. The numerical parameters are $\Delta x = D/(84 \cos\theta) = D'/84$, $\Delta z = 0.05$ μm, and the number of transverse sampling points is 1800. The reference refractive index is chosen to be that in the cladding. Using the three-step method we evaluate the coupling efficiency at $z = 100$ μm. Calculation shows that the coupling efficiency for the GD scheme is 0.97, while that for the CN scheme is 0.07. This result again demonstrates the superiority of the present method over the conventional approach.

IV. Conclusion

The generalized Douglas scheme for variable coefficients has been applied to a multistep method for wide-angle beam propagation. The coupling efficiency and the phase error in a tilted step-index waveguide are evaluated to assess the numerical accuracy. The present method offers further improvement in the analysis of wide-angle beam propagation compared with the conventional approach, while maintaining almost the same computational efficiency.

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