

# Numerical Analysis of Asymmetrical Spiral Antenna

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TABLE I  
COMPARISON OF NUMERICAL AND ASYMPTOTIC RESONANCES FOR  
ELLIPTIC CYLINDER;  $b/a = 1.5$

Pole #	Pole Location		Magnitude error(%)
	Numerical	Asymptotic	
1	$-0.561 + j 0.376$	$-0.543 + j 0.478$	6.6
2	$-0.674 + j 1.123$	$-0.685 + j 1.189$	4.6
3	$-0.750 + j 1.886$	$-0.784 + j 1.924$	2.3
4	$-0.800 + j 2.634$	$-0.863 + j 2.670$	1.9
5	$-0.838 + j 3.397$	$-0.929 + j 3.424$	1.4
6	$-0.810 + j 4.125$	$-0.988 + j 4.182$	2.0

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## Numerical Analysis of Asymmetrical Spiral Antenna

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**Abstract**—The radiation characteristics of a two-wire spiral antenna in which the one arm is truncated are evaluated on the basis of the analyzed current distribution. It is numerically demonstrated that either sense of circular polarization is generated by changing the phase relation of feed points. The variation in the axial ratio is examined as a function of truncation amount of the one arm.

## INTRODUCTION

In order to generate either sense of circular polarization from a single two-wire spiral antenna, Kaiser proposed a spiral antenna whose arm lengths differ by a quarter wavelength [1]. He experimentally showed that changing the phase relation of feed points allows generation of either sense of circular polarization. Although arm asymmetry, such as a Kaiser's model mentioned above, introduces attractive features, no rigorous studies on an asymmetrical spiral antenna have not appeared to date.

The purpose of this communication is to reveal theoretically

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the radiation characteristics of an asymmetrical spiral antenna. We use a spiral antenna with two off-center sources, which is close to Kaiser's experimental model. The radiation characteristics are calculated from the current distribution determined numerically using an integral equation [2]. The behavior of the axial ratio is investigated as a function of the truncation amount of the one arm.

## CONFIGURATION

Fig. 1 shows the antenna configuration and the excitation modes. The antenna arm is characterized by Archimedean spiral function  $r = a\phi_s$  ( $a$  = spiral constant,  $\phi_s$  = winding angle). In the present spiral, the outer circumference is taken to be  $1.5 \lambda$  ( $\lambda$  = wavelength of operating frequency), and the spiral arm length is  $4.73 \lambda$ .

The spiral is excited at two symmetrical points with respect to the origin [3], as in the case of a dipole antenna with two sources [4]. These two points are located near the origin and the distance between the two points is  $0.125 \lambda$ . The excitation modes shown in Figs. 1(b) and 1(c) are called "antiphase excitation" and "in-phase excitation," respectively.

## NUMERICAL AND EXPERIMENTAL RESULTS

The current distributions for the truncation amount of  $l = \lambda/4$  are shown in Fig. 2. The analysis is carried out using a simplified integral equation [2]. The applied voltage of each source is 1 V. It is found for the antiphase excitation that the currents gradually decay due to radiation. The current distribution has fashion similar to that of a conventional center-fed spiral antenna with symmetrical arms [5]. Since the sense of the spiral as viewed from the  $-z$  side is right-handed as shown in Fig. 1(a), the traveling currents toward the arm ends radiate a circularly polarized wave of right-hand sense on the  $+z$  axis.

The theoretical current distribution for the in-phase excitation shows standing wave fashion, as shown in Fig. 2(b). It should be noted that the standing-wave ratio of the current is reduced toward unity as the point of observation is moved away from the arm ends toward the origin. This behavior can be explained in terms of the interference of two currents. One is a traveling current from the source toward the arm end with little attenuation and the other is a reflected current from the arm end toward the source, decaying due to radiation. The reflected current radiates a circularly polarized wave of left-hand sense on the  $+z$  axis, which is the opposite of the sense obtained for the antiphase excitation. Thus, selection of either sense of circular polarization is realized by changing the excitation mode.

Fig. 3 shows the theoretical and experimental radiation patterns. It is seen that the broad axial beam is radiated for each excitation mode. The axial ratios for the antiphase excitation and in-phase excitation are 1.3 dB (measured value = 0.9 dB) and 1.8 dB (measured value = 3.4 dB), respectively. It is worth mentioning that both senses of a circularly polarized wave are obtained with the radiation pattern being nearly unchanged.

So far, the study has been restricted to the case where the truncation amount is  $\lambda/4$ . It is significant to examine how the changes in the truncation amount  $l$  affect the radiation characteristics.

Fig. 4 shows the axial ratio on the  $z$  axis as a function of truncation amount. The axial ratio for the antiphase excita-

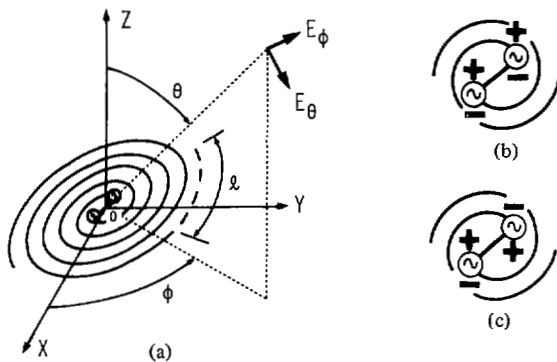


Fig. 1. Antenna configuration and excitation modes. (a) Antenna configuration. (b) Antiphase excitation. (c) In-phase excitation. Spiral constant  $a = 5.83 \times 10^{-3}\lambda$ ; wire radius  $= 2.33 \times 10^{-3}\lambda$ ; maximum winding angle  $\phi_s = 41.4$  rad.

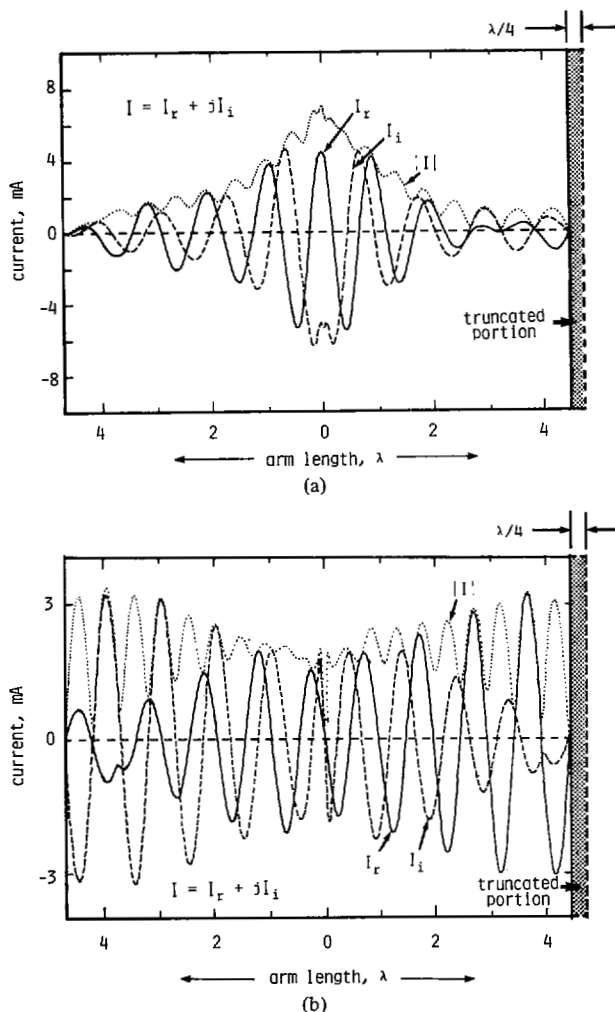


Fig. 2. Current distributions when the one arm is truncated by  $\lambda/4$ . (a) Antiphase excitation. (b) In-phase excitation.

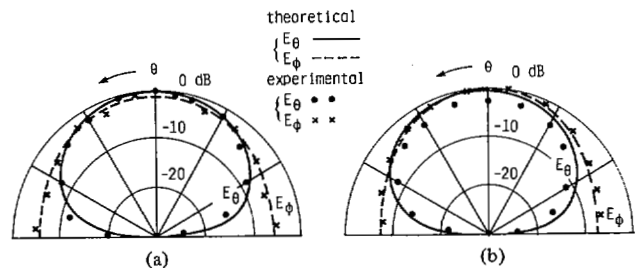


Fig. 3. Radiation patterns when the one arm is truncated by  $\lambda/4$ . (a) Antiphase excitation. (b) In-phase excitation.

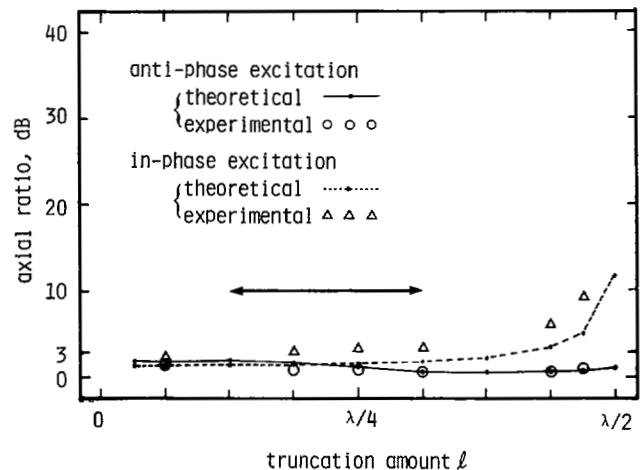


Fig. 4. Axial ratio as a function of truncation amount of the one arm.  $\longleftrightarrow$ : a region where the deviation angle of the maximum radiation from the  $z$  axis is within  $6^\circ$ .

tion is almost independent of the truncation amount of the one arm. Axial ratios of less than 2.0 dB are obtained in the direction of the  $z$  axis. The direction of the maximum radiation remains on the  $z$  axis even when the truncation amount is  $\lambda/2$ .

For the in-phase excitation, there exists a truncation range in which a circularly polarized wave is radiated. Although axial ratios of less than 3.0 dB are obtained over a relatively wide truncation range, the direction of the maximum radiation tends to deviate from the  $z$  axis as the truncation amount is increased or decreased from  $\lambda/4$ . The sense selection for the tolerable deviation angle within  $6^\circ$  is possible in a truncation range of  $2\lambda/16$  to  $5\lambda/16$ , as shown in Fig. 4.

## CONCLUSION

The radiation characteristics of an asymmetrical spiral antenna have been investigated on the basis of the analyzed current distribution. It is quantitatively confirmed that changing the excitation mode allows selection of either sense of circular polarization. If the tolerable deviation of the maximum radiation from the  $z$  axis is within  $6^\circ$ , the sense selection can be made in a truncation range of  $2\lambda/16$  to  $5\lambda/16$ .

## ACKNOWLEDGMENT

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## Simple Expressions for a Function Occurring in Diffraction Theory

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**Abstract**—The Maliuzhinets (integral) function arises in connection with diffraction by a half-plane. Two simple expressions are derived which, when used in conjunction with known identities, serve to approximate the function to a high degree of accuracy throughout the entire complex plane.

### I. INTRODUCTION

The function

$$\psi_{\pi}(z) = \exp \left\{ -\frac{1}{8\pi} \int_0^z \frac{\pi \sin u - 2\sqrt{2}\pi \sin(u/2) + 2u}{\cos u} du \right\} \quad (1)$$

with  $z = x + jy$  was first introduced by Maliuzhinets [1] and is commonly referred to as the Maliuzhinets half-plane function. It arises in connection with the electromagnetic problem of diffraction by a resistive or impedance half-plane [2], and in the latter application  $z = \pi - \phi \pm \chi$  with  $\cos \chi = \eta$  (or  $1/\eta$ ), where  $\eta$  is a normalized surface impedance and  $\phi$  is the angle of incidence or diffraction. Thus,  $z$  is complex if  $\eta$  is complex or if  $\eta$  is real and  $\eta > 1$  (or  $< 1$ ).

With the growing interest in how the material properties of a body affect its scattering, it has become important to employ the diffraction coefficients [2], [3] for nonperfectly conducting edges, and this requires a knowledge of the Maliuzhinets function. Bucci [4] has tabulated  $\psi_{\pi}(z)$  and Maliuzhinets [5] mentions another tabulation but gives no reference. For routine applications, however, the apparent complication of the expression (1) is a major deterrent to its use, and if  $\psi_{\pi}(z)$  is to be incorporated into a scattering code it is desirable (if not essential) to be able to compute the function in a simple manner.

Two simple analytical approximations whose accuracy is adequate for most purposes are presented here. The expressions correspond to the small argument and large imaginary argument approximations of  $\psi_{\pi}(z)$ . These, when used in conjunction with the identities given below, cover the entire complex  $z$  plane with a maximum amplitude error of 0.27 percent.

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### II. SOME PROPERTIES OF THE FUNCTION

The Maliuzhinets half-plane function is an even meromorphic function of  $z$  and some of its properties have been developed by Bowman [6]. An alternative expression is

$$\psi_{\pi}(z) = \left[ \frac{\sqrt{2} \cos z/2 + 1}{\sqrt{2} + 1} \right]^{1/2} (\cos z)^{-1/8} \cdot \exp \left\{ -\frac{1}{4\pi} \int_0^z \frac{u du}{\cos u} \right\}, \quad (2)$$

and one of the recurrence relations satisfied by  $\psi_{\pi}(z)$  is

$$\psi_{\pi}(z) = \{ \psi_{\pi}(\pi/2) \}^2 \frac{\cos \left( \frac{z}{4} - \frac{\pi}{8} \right)}{\psi_{\pi}(z - \pi)} \quad (3)$$

with

$$\psi_{\pi}(\pi/2) = 0.96562.$$

In addition,

$$\psi_{\pi}(-z) = \psi_{\pi}(z) \quad (4)$$

$$\psi_{\pi}(z^*) = \psi_{\pi}^*(z) \quad (5)$$

where the asterisk denotes the complex conjugate, and by using (3)-(5)  $\psi_{\pi}(z)$  can be determined throughout the entire complex plane from a knowledge of its behavior in the strip  $0 \leq x \leq \pi/2$ ,  $y \geq 0$ . It is therefore sufficient to confine our attention to the strip shown shaded in Fig. 1.

### III. ANALYTICAL APPROXIMATIONS

The Maliuzhinets half-plane function is real when  $x = 0$  or  $y = 0$ , and to illustrate the general behavior of the function, Fig. 2 shows the amplitude and phase of  $\psi_{\pi}(z)$  as functions of  $y$  on the boundaries  $x = 0$ ,  $x = \pi/2$  and center  $x = \pi/4$  of the strip. The variation with  $y$  (as well as with  $x$ ) is approximately that of a quadratic function over at least the range of  $y$  covered in Fig. 2, and this suggests that a simple analytical expression could provide an adequate approximation to  $\psi_{\pi}(z)$  for all except the largest values of  $y$ .

The Taylor series expansion of  $\psi_{\pi}(z)$  about  $z = 0$  is [4]

$$\psi_{\pi}(z) = 1 - az^2 + 0(z^4)$$

where

$$a = \frac{1}{16} \left( 1 - \sqrt{2} + \frac{2}{\pi} \right) = 0.01390,$$

and a small argument approximation to  $\psi_{\pi}(z)$  is therefore

$$\psi_{\pi}(z) = 1 - 0.01390 z^2. \quad (6)$$

If  $y \gg 0$  an alternative approximation can be obtained from (2) by replacing, for example,  $\cos z$  by  $(1/2)e^{-jz}$  and is

$$\psi_{\pi}(z) = b \left\{ \cos \frac{1}{4} (z - j\gamma) \right\}^{1/2} \exp \left\{ \frac{jz}{2\pi} e^{jz} \right\}$$

with

$$\gamma = \ln 2 = 0.69315$$