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Iwamura, Hideyuki

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Institute of Comparative Economic Studies, Hosei University / 法政大学比較経済研究所

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Welfare Analysis of Currency Union

Hideyuki Iwamura
Nagoya University of Commerce and Business

Abstract

This paper aims to focus on one of the *non-exclusive* benefits of a currency union and to analyze its welfare impacts on the regions inside and outside. Presenting a three-region model with optimizing consumers, the paper demonstrates that the non-exclusive benefits depend on *the fixed costs of cross-currency transactions* and *the elasticity of input substitution*. Elasticities close enough to one ensure the non-exclusive benefits to exceed non-exclusive costs. The implications are that the external effects of a currency union can cause the union to be too small in size, and that the elasticity is a critical criterion for optimum currency areas.

JEL classification: F33, F36
Key words: Currency Union, Optimum Currency Areas

1 Introduction

Usually one currency circulates within one nation. Is a nation, however, an optimum currency area? Since Mundell (1961)'s discussion on how the degree of labor mobility determines the impacts of monetary unification, numerous researchers have been developing the theory of optimum currency areas (OCA) and proposing its criterion.

The theory has yet to develop in at least two directions. Firstly, researchers have been more concerned with which regions would have less cost in a currency union than with which regions would benefit more. As a result, there's a strong tendency toward negatively evaluating any currency union. Among the classical and most famous OCA criteria, degree of labor mobility (Mundell (1961)), openness of an economy (McKinnon (1963)), and diversity of production (Kennen (1969)), all determine the costs of a union. In addition, asymmetry of shocks, the most popular criterion in empirical researches on the EMU (Bayoumi and Eichengreen (1993), Bini Smaghi and Vori (1994)), is also a determinant of the costs. Secondly, discussions on the OCA criteria have usually been lacking in microeconomic foundations. The empirical researches have been employing the criteria, the validity of which has rarely been argued.[1]

This study focuses on the benefits of a currency union in contrast to previous literatures that placed focus on the cost aspects. This study evaluates a currency union favorably. The objective requires the model to include optimizing consumers, in order to investigate *welfare impacts* of monetary integration. The model should also involve at least three regions, in order to examine the impacts outside the union as well as inside. Finally, it should accommodate the

[1] Mongelli (2002) provides a detailed review of OCA theory and evidence, including its recent development.

non-exclusive benefits of a currency union, to support a widely accepted intuition that *a decrease in regional currencies benefits all regions non-exclusively*. Bayoumi (1994) has proposed a model with optimizing consumers and *n* regions. In order to represent the non-exclusive benefits of a currency union, this paper has modified his model in two ways. First, we introduce fixed costs of cross-currency transactions, in addition to variable costs, the source of *exclusive* benefits in the original model.[2] Second, the CES technology with elasticities higher than one replaces the Cobb-Douglas technology.

The analysis shows that 1) the non-exclusive benefits increase with fixed costs but decrease with the elasticity of input substitution, and that 2) especially when the elasticity is close enough to unity, the non-exclusive benefits exceed *non-exclusive costs* of a union, even if a fixed cost is small. It also examines how traditional OCA criteria work. Finally, the paper concludes that it is likely that a currency union has a net positive external effect that causes a currency union to be too small, and it also emphasizes that the elasticity of input substitution is crucial to welfare impacts of a currency union.

2 The model

2.1 Utility and consumption

Cobb-Douglas utility function

We have three regions in the world: region 1, region 2, and region 3. Each region specializes in producing one good and each good is consumed. A consumer in region i has a Cobb-Douglas utility function.

$$U_i = \beta_{i1}\log(C_{i1}) + \beta_{i2}\log(C_{i2}) + \beta_{i3}\log(C_{i3}) + \psi_i, \qquad (1)$$
$$\beta_{i1} + \beta_{i2} + \beta_{i3} = 1,$$

where people in region i consume a C_{ij} amount of good j . We let p_i be the relative price of good i to good 1. ψ_i is a constant, which is equal to

$$\psi_i = -\sum_j \beta_{ij}\log(\beta_{ij}) - \sum_j \beta_{ji}\log(\Omega_j / \Omega_i),$$

where

$$\Omega_1 \equiv (1-\beta_{22})\beta_{31} + \beta_{21}\beta_{32}$$
$$\Omega_2 \equiv (1-\beta_{33})\beta_{12} + \beta_{13}\beta_{32}$$
$$\Omega_3 \equiv (1-\beta_{22})\beta_{13} + \beta_{12}\beta_{23}.$$

Therefore, region i 's demand for good j is

$$D_{ij} = \beta_{ij} X_i \frac{p_i}{p_j}, \qquad (2)$$

where X_i is the amount of good i produced.

Variable costs of cross-currency transactions

Because we assume that variable costs of cross-currency transactions take the iceberg form, people can consume only a $T(\leq 1)$ portion of one unit of import. We assume that the

[2] Baldwin (2006) focuses on the fixed costs, and assuming that it falls for *all* exporters to the Eurozone, not just for firms located inside the Eurozone, suggests that it gives rise to the non-exclusive benefits.

variable costs are the same across regions. Combining the variable costs with (2) yields the actual consumption C_{ij} :

$$C_{ij} = \begin{cases} \beta_{ij} X_i & (i = j) \\ T\beta_{ij} X_i (p_i / p_j) & (i \neq j). \end{cases} \tag{3}$$

Taking the logarithm, we have

$$c_{ij} = \begin{cases} \log(\beta_{ij}) + x_i & (i = j) \\ \log(\beta_{ij}) + x_i + \log(p_i) - \log(p_j) - \tau & (i \neq j), \end{cases} \tag{4}$$

where $\tau \equiv -\log(T)$.

Substituting the actual consumption (3) into the utility function (1), we obtain an indirect utility function:

$$U_i(p_2, p_3, x_i) = x_i - \left(\sum_{j \neq i} \beta_{ij} \right) \tau + \sum_{j \neq i} \beta_{ij} \left[\log(p_i) - \log(p_j) \right] + \sum_j \beta_{ij} \log(\beta_{ij}) + \psi_i \tag{5}$$

The use of the market equilibrium relative prices eliminates p_2 and p_3 :

$$U_i(x_1, x_2, x_3) = \sum_j \beta_{ij} x_j - \left(\sum_{j \neq i} \beta_{ij} \right) \tau. \tag{6}$$

It shows that the utility at the equilibrium depends on the total output of all goods, the size of a variable transaction cost, and the share of each good in the total expenditure.

2.2 Production and labor

Labor supply and variety of labor services

Technology is the CES, and the only input is labor. Labor is continuously differentiated in the range of [0, n].

$$X_i = \left[\int_0^n L_i(z)^{\frac{\rho-1}{\rho}} e^{\varepsilon_i} dz \right]^{\frac{\rho}{\rho-1}}, \tag{7}$$

where ε_i is a stochastic disturbance term specific to region i and is normally distributed with a mean of zero and a variance of σ_i^2. We assume that ε_1=0. As in Matsuyama (1994), ρ>1, which implies that the range of labor services varies.

Firms want to employ a wider range of labor services because it improves the average productivity of an unit labor. Workers also want to differentiate their labor further. We assume that the marginal cost of differentiation is zero. The assumption, however, results in an incentive for a worker to divide his labor into many kinds of small parts and as a result, raise the average product of an unit labor infinitely.

In order to avoid the indeterminancy of the range, we assume that each differentiated labor must supply at least one unit of service. We assume, in addition, that each specialized labor service is inelastically supplied. Therefore, the resource constraint,

$\int_0^n 1dz = \overline{L},$ determines the range at $[0, \overline{L}]$.

Fixed costs of cross-currency transactions

We introduce fixed costs of cross-currency transactions in order to represent the non-exclusive benefits of a currency union. Cross-currency transactions cost consists of two parts:

a variable part, which depends on the volume of cross-currency trade, and a fixed part, which depends on the *number* of regional currencies required in trade.[3] Take the currency union between region 2 and 3 for example. For region 1, since the volume of cross-currency trade remains the same before and after the currency union, the total variable cost remains the same. On the other hand, the number of currencies which region 1 needs for trade does decrease. As a result, its total fixed cost decreases. A currency union, hence, cuts down part of the fixed costs, giving benefits to the region outside the union as well.[4]

In detail, a region is required to spend a certain fixed amount of labor on each cross-currency trade, regardless of the volume of each trade. The size of the fixed cost depends on the trading partner. Any region trading with region i spends an F_i amount of labor. We, however, assume that fixed costs are the same across regions, which means that $F_i=F$. Then, the currency union between region 2 and 3 decreases the regional currencies by one, saving *every* region, both inside and outside, its total fixed cost by an amount of F.

If we normalize labor endowment in each region to 1, then each region has a $(1–2F)$ amount of labor available for production. With the currency union between region 2 and 3, the amount of labor available in each region increases to $(1–F)$, whether the region is inside or outside the union. As a result, outputs increase as resources available for production increase. At the same time, the increases in endowments also expand the potential ranges of differentiated labor inputs, from $[0, 1–2F]$ to $[0, 1–F]$, raising the average product of each unit input.

Nominal wage rigidity and exchange rate

Firms are price-takers in both goods and labor markets. The demand for labor z , $L_i^d(z)$, is

$$L_i^d(z) = P_i^{\rho} X_i W_i(z)^{-\rho} \exp(\rho\varepsilon_i), \tag{8}$$

where P_i and $W_i(z)$ represent the price of good i and the wage of labor z in region i. Both are measured in the same *local* currency.

Nominal wages in a local currency are fixed at the *normal level* $\omega_i(z)$,

$$\omega_i(z) = P_i E_i X_i^{\frac{1}{\rho}}, \tag{9}$$

where E_i is the price of region i's currency denominated by region 1's currency. At the wage rate, when $\varepsilon_i=0$ and $E_i=1$, labor demand is equal to supply. Substituting this wage rate into the labor demand (8) gives

$$L_i^d(z) = \exp(\rho(\varepsilon_i - e_i)), \tag{10}$$

where $e_i=\log(E_i)$. The relative size of ε_i and e_i determines the labor demand.

As in Bayoumi (1994), the exchange rate is a shock absorber in labor markets. When region i lets its currency float, the exchange rate nullifies any region-specific shock and ensures its full employment. Hence, the full employment condition $L_i^d(z)=1$ determines the exchange rate at

$$e_i = \varepsilon_i. \tag{11}$$

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[3] A couple of examples of fixed costs of cross-currency transactions are provided. If two firms, each located in areas with different currencies, trade their goods, both firms usually have to have a certain amount of checking accounts, which usually yield almost no interests, in the partner's local currency. Another example is the reserve assets held in another currencies by the government or the central bank. The amount of assets held as reserves contains a fixed part which usually bear relatively lower interests than other available assets would.

[4] Micco, Stein, and Ordonez (2003) provides a suggestive result that the Europan Monetary Union has reduced trade costs between the Eurozone and the rest of the world.

3 Equilibrium

3.1 Equilibrium with no currency union

As mentioned in the last section, if a region lets its currency float, full employment is ensured. Hence, $L_i(z)=1$ for all i and z . The output is

$$X_i = (1-2F)^{\frac{\rho}{\rho-1}} \exp\left(\frac{\rho}{\rho-1}\varepsilon_i\right). \tag{12}$$

Taking the logarithm of X_i and approximating it linearly, we obtain

$$x_i = -\left(\frac{\rho}{\rho-1}\right)2F + \left(\frac{\rho}{\rho-1}\right)\varepsilon_i. \tag{13}$$

Substituting it into the utility function (6) gives

$$U_i = -\left(\frac{\rho}{\rho-1}\right)2F + \left(\frac{\rho}{\rho-1}\right)\sum_j \beta_{ij}\varepsilon_j - \left(\sum_{j\neq i}\beta_{ij}\right)\tau \tag{14}$$

3.2 Equilibrium with a currency union

We consider the currency union between region 2 and 3, and let E_{23} (or e_{23} in the logarithm) be the common exchange rate.

Exchange rate of the common currency and employment in a currency union

We assume that workers cannot move between region 2 and 3 even when the regions form a currency union. Therefore, for full employment in both regions, the common exchange rate must satisfy these conditions simultaneously: $e_{23}=\varepsilon_2$ and $e_{23}=\varepsilon_3$. Evidently, e_{23} cannot ensure full employment inside a union, which burdens them with costs, except when $\varepsilon_2=\varepsilon_3$.

Unlike the no-union equilibrium, where the full employment conditions determine the exchange rates, we need another assumption about determination of the common exchange rate e_{23}. The common exchange rate is determined by a geometric average of *shadow exchange rates*. A shadow exchange rate, $\hat{E}_i$ or $\hat{e}_i$, is the exchange rate that results if a region does not join a union. Therefore, $\hat{e}_i=\varepsilon_i$. Thus, $e_{23}=\alpha\varepsilon_2+(1-\alpha)\varepsilon_3$, where $0<\alpha<1$.

The common exchange rate absorbs any productivity shock, *but only partially*, inside the union. The demands for region 2's and region 3's labor are

$$\log(L_2^d(z)) = \rho(1-\alpha)(\varepsilon_2-\varepsilon_3) \tag{15}$$

$$\log(L_3^d(z)) = \rho\alpha(\varepsilon_3-\varepsilon_2). \tag{16}$$

Other than when $\varepsilon_2=\varepsilon_3$, one region would have excess demand, while the other would have excess supply and unemployment. This causes losses in output and costs to the union.

The parameter α determines more of which disturbance in region 2 and 3 the common exchange rate will mitigate. The equations (15) and (16) clearly shows that, as α gets larger, the exchange rate absorbs more of ε_2 and less of ε_3. Therefore, we can interpret the parameter α as the relative size of economy 2 to 3; the price of a common currency reflects the disturbances in the dominant region more than those in the dominated one.

Welfare impacts of a currency union

Inspecting equation (6) yields

$$\Delta U_1 = \sum_j \beta_{1j} \Delta x_j \tag{17}$$

$$\Delta U_2 = \sum_j \beta_{2j} \Delta x_j + \beta_{23}\tau \tag{18}$$

$$\Delta U_3 = \sum_j \beta_{3j} \Delta x_j + \beta_{32}\tau, \tag{19}$$

where ΔU_i and Δx_i represent the changes by the formation of a currency union. The equations allow us to concentrate only on the changes in outputs.

By combining both the negative effects of possible unemployment and the positive effects of expansion of labor endowment, we obtain the outputs within a union.

$$x_2 = \begin{cases} -\left(\frac{\rho}{\rho-1}\right)2F + \left(\frac{\rho}{\rho-1}\right)\varepsilon_2 & \text{if } \varepsilon_2 \geq \varepsilon_3 \\ -\left(\frac{\rho}{\rho-1}\right)F + \left(\frac{\rho}{\rho-1}\right)\varepsilon_2 - \rho(1-\alpha)(\varepsilon_3 - \varepsilon_2) & \text{if } \varepsilon_2 < \varepsilon_3 \end{cases} \tag{20}$$

$$x_3 = \begin{cases} -\left(\frac{\rho}{\rho-1}\right)2F + \left(\frac{\rho}{\rho-1}\right)\varepsilon_3 & \text{if } \varepsilon_3 \geq \varepsilon_2 \\ -\left(\frac{\rho}{\rho-1}\right)F + \left(\frac{\rho}{\rho-1}\right)\varepsilon_3 - \rho\alpha(\varepsilon_2 - \varepsilon_3) & \text{if } \varepsilon_2 > \varepsilon_3. \end{cases} \tag{21}$$

Taking the differences between the equilibrium outputs with and without a currency union gives

$$\Delta x_2 = \begin{cases} \left(\frac{\rho}{\rho-1}\right)F & \text{if } \varepsilon_2 \geq \varepsilon_3 \\ \left(\frac{\rho}{\rho-1}\right)F - \rho(1-\alpha)(\varepsilon_3 - \varepsilon_2) & \text{if } \varepsilon_2 < \varepsilon_3 \end{cases} \tag{22}$$

$$\Delta x_3 = \begin{cases} \left(\frac{\rho}{\rho-1}\right)F & \text{if } \varepsilon_3 \geq \varepsilon_2 \\ \left(\frac{\rho}{\rho-1}\right)F - \rho\alpha(\varepsilon_2 - \varepsilon_3) & \text{if } \varepsilon_3 < \varepsilon_2. \end{cases} \tag{23}$$

Because region 1 stays outside and labor is always fully employed, only a decrease in the total fixed transaction cost can affect x_1.

$$\Delta x_1 = \left(\frac{\rho}{\rho-1}\right)F \tag{24}$$

Taking the expected values yields

$$E(\Delta x_1) = \left(\frac{\rho}{\rho-1}\right)F \tag{25}$$

$$E(\Delta x_2) = \left(\frac{\rho}{\rho - 1}\right)F - 2\rho(1-\alpha)\varphi(0)\sqrt{\sigma_2^2 + \sigma_3^2 - 2\sigma_{23}} \tag{26}$$

$$E(\Delta x_3) = \left(\frac{\rho}{\rho - 1}\right)F - 2\rho\alpha\varphi(0)\sqrt{\sigma_2^2 + \sigma_3^2 - 2\sigma_{23}} \tag{27}$$

where σ_{23} represents the covariance between ε_2 and ε_3, while $\varphi(\cdot)$ denotes the probability density function of a standard normal variate.[5]

Then, we calculate the impacts of the output changes on welfare. We take the expected output changes, (25), (26), and (27), and substitute them into the expected values of the utility changes, (17), (18), and (19), to arrive at the following equations.

$$E(\Delta U_1) = \left(\frac{\rho}{\rho - 1}\right)F - [\beta_{12}(1-\alpha) + \beta_{13}\alpha]\rho\varphi(0)\sqrt{\sigma_2^2 + \sigma_3^2 - 2\sigma_{23}} \tag{28}$$

$$E(\Delta U_2) = \left(\frac{\rho}{\rho - 1}\right)F + \beta_{23}\tau - [\beta_{22}(1-\alpha) + \beta_{23}\alpha]\rho\varphi(0)\sqrt{\sigma_2^2 + \sigma_3^2 - 2\sigma_{23}} \tag{29}$$

$$E(\Delta U_3) = \left(\frac{\rho}{\rho - 1}\right)F + \beta_{32}\tau - [\beta_{32}(1-\alpha) + \beta_{33}\alpha]\rho\varphi(0)\sqrt{\sigma_2^2 + \sigma_3^2 - 2\sigma_{23}} \tag{30}$$

These equations show the parameters that affect the welfare impacts of a currency union and how they do it. It is also important to note that, as is implied by (28), even the region outside is affected.

4 Criteria for optimum currency areas

4.1 Variable transaction costs, opneness of an economy, and asymmetry of disturbances

Variable transaction costs

As shown by the second terms in (29) and (30), the larger the variable transaction cost, the larger the welfare improvement by a currency union.

$$\frac{\partial E(\Delta U_2)}{\partial \tau} = \beta_{23} > 0 \tag{31}$$

$$\frac{\partial E(\Delta U_3)}{\partial \tau} = \beta_{32} > 0 \tag{32}$$

Since region 2 does not need to convert currencies in a trade with region 3, it can consume a larger portion of *unit* import from 3 than before. Otherwise, the increase in consumption is larger as the cost of a cross-currency trade takes away a larger part of unit import, that is, T is smaller or τ is larger.

Furthermore, the effect increases with β_{23} and β_{32}. A large value of β_{23} means a large amount of import from region 3, which in turn means that the people in region 2 give up a large portion of the total import for variable transaction costs. The monetary unification with region 3 saves the costs, increasing the actual consumption of good 3 by the same amount. The increase is larger as the total import is larger, *i.e.*, β_{23} is larger.

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[5] These calculations are explained in detail in the appendix.

Openness of an economy

The openness of a region is defined as the share of imported goods in its total expenditure.[6]

$$\lambda_i = \sum_{j \neq i} \beta_{ij} = 1 - \beta_{ii}. \tag{33}$$

A relatively open region, *i.e.*, a region with smaller β_{ii}, prefers the other goods to its own good. The impact of openness, however, is ambiguous with the definition. For example, a large value of β_{23} (region 2's further openness to region 3) has both positive and negative impacts on region 2. On the one hand, large β_{23} strengthens the effect of a decrease in the total variable transactions cost. On the other hand, it also strengthens a negative effect because unemployment in region 3 forces region 2 to consume less of good 3, which region 2 prefers.

Nevertheless, we have one particular case of interest where openness can be an unambiguous criterion. That is where an economy is more open to the other member economy and is relatively small at the same time. Then, an increase in openness reinforces the benefits of a currency union, and besides, decreases the costs. Since a common exchange rate stabilizes the output of a larger economy (region 3) better, region 2 often experiences unemployment, while region 3 rarely does. But if region 2 prefers good 3 to its own, *i.e.*, more open to region 3, it suffers less from its own unemployment, while benefitting more from the savings in variable transaction costs.

We can check the result by inspecting (29).

$$E(\Delta U_2) = \underbrace{\left(\frac{\rho}{\rho - 1}\right) F + \beta_{23} \tau}_{\text{benefits of a union}}$$

$$\underbrace{-[\beta_{22}(1-\alpha) + \beta_{23}\alpha]\rho\varphi(0)\sqrt{\sigma_2^2 + \sigma_3^2 - 2\sigma_{23}}}_{\text{costs of a union}}.$$

Suppose that we increase β_{23}, with β_{21} fixed and $\alpha \leq 1/2$, then the second term increases and the absolute value of the third term either decreases or stays unchanged. The analysis implies that a relatively small and open economy can enjoy larger benefit and pay smaller cost when forming a currency union with a large economy.[7]

Asymmetry of disturbances

As explained in Section 3, with no currency union, each exchange rate completely absorbs each region-specific shock. Once two currencies are united into one, however, the common exchange rate absorbs the shocks inside the union only partially. Therefore, one region has unemployment, reducing its output and consumption of the good. As the shocks are more asymmetric, or negatively correlated, the two regions suffer larger costs. Even with the negative correlation, however, if both of the variances were small, the shocks would be often similar in size and the common exchange rate would work better. The third terms in the

[6] McKinnon (1963) defines openness as the share of tradable goods to income. In our model, since every good is tradable, his definition cannot be applied.

[7] This statement is similar to McKinnon (1963). A small open economy should abandon its own currency and adopt the currency of a large partner. Though his suggestion and ours are alike, the rationales behind them are somewhat different. His discussion is as follows. Openness of an economy limits the effect of the exchange rate variability on adjustment of trade balances, because if imported goods from a certain region have a large share in consumption, a change in the exchange rate with the region will immediately transform into a change in the *general* price level and can not affect the terms of trade between the two regions. For such an economy, variability in nominal exchange rate is not effective as an adjustment tool, and the loss of variability is not costly. Rather, exchange rate variability is even harmful, destabilizing internal prices.

expected utility changes, (29) and (30), shows that the costs are increasing in σ_2^2 and σ_3^2 and decreasing in σ_{23}.

Furthermore, we should note that a monetary union has negative external effects. As is shown in the second term in (28), the region outside suffers as well. This is because when the asymmetric disturbances decrease the outputs inside the union, region 1 is also forced to consume less of the goods.

4.2 Fixed transaction costs and elasticity of substitution

Fixed transaction costs

If the fixed transactions cost is larger, the welfare improvement of the regions inside is larger. Differentiating (29) and (30) with respect to *F* gives the result:

$$\frac{\partial E(\Delta U_i)}{\partial F} = \frac{\rho}{\rho - 1} > 0. \tag{34}$$

The inequality is ensured by the assumption of $\rho > 1$.

As previously explained, fixed costs affect welfare impacts of a currency union in two ways. Decreases in the total fixed transaction cost expand the amount of labor available for production and directly increase the potential outputs. At the same time, the increases **in the availability of labor** also expand the potential range of differentiated inputs, raising the average productivity of each unit labor.

We can show the effect of variety expansion in the same way as Romer (1987) shows. We let $\bar{L}$ denote the total amount of labor input, which is fixed, and suppose the labor input is continuously differentiated in the range of [0, *M*]. If we let the input of labor *z* be $L(z) = \hat{L}$, the resource constraint, $\int_0^M \hat{L} dz = \bar{L}$, determines $\hat{L}$ as the function of M: $\hat{L}(M) = \bar{L}/M$.

Substituting it into the production function, $X = [\int_0^M L(z)^{(\rho-1)/\rho} dz]^{\rho/(\rho-1)}$, we obtain

$$X = \left[\int_0^M \left(\frac{\bar{L}}{M} \right)^{\frac{\rho-1}{\rho}} dz \right]^{\frac{\rho}{\rho-1}} = \bar{L} \cdot M^{\frac{1}{\rho-1}}.$$

Dividing it by the total input $\bar{L}$ gives the average productivity of unit input, $M^{1/(\rho-1)}$. This is an increasing function of M, which is the variety of labor.

More importantly, as shown in the first term in (28), the two effects are not exclusive to the union members. The benefits spill over to the region that does not participate. The positive *external* effect offsets, or in some cases, exceeds the negative external effect of asymmetric shocks within a union. The result has two implications. First, the region *outside* the union, depending on the parameters, enjoys *net* benefit. Second, closely related to the first implication, because of its positive externalities, the actual size of a currency union can be smaller than the socially optimal one. This is in contrast with Bayoumi (1994), where a currency union has only negative external effects and tends to be too large.

There, however, can naturally arise a question about the size of a fixed cost. The concern is that it is so small that the welfare impact can be neglected. One possible answer is given in the following discussion.

Elasticity of substitution

The last parameter to be examined is elasticity of substitution between specialized labor inputs. Differentiating (29) with respect to ρ shows straightforwardly its impact.

$$\frac{\partial E(\Delta U_2)}{\partial \rho} = -\frac{1}{(\rho-1)^2} F - [\beta_{22}(1-\alpha)+\beta_{23}\alpha]\varphi(0)\sqrt{\sigma_2^2+\sigma_3^2-2\sigma_{23}} < 0. \tag{35}$$

As ρ decreases and approaches to one, a currency union gains a greater positive impact on welfare.

The elasticity affects welfare by controlling the impact of fixed costs. Differentiating (34) with respect to ρ shows that low elasticities reinforce the *marginal* impact of fixed costs.

$$\frac{d}{d\rho}\left(\frac{\partial E(\Delta U_2)}{\partial F}\right) = -\frac{1}{(\rho-1)^2} < 0. \tag{36}$$

Especially, when an elasticity is close enough to one, the benefits exceed the costs, provided that all variances of the shocks are finite. Taking the limit of the first term of (29) shows

$$\lim_{\rho\to+1}\left(\frac{\rho}{\rho-1}\right)F = \infty. \tag{37}$$

The first term goes to infinity as ρ approaches to unity, and the absolute value of the second term is increasing in ρ. So the net benefit (29) is decreasing in ρ, unless the square root in the second term is infinite. Then, for any level of F, there always exists $\rho_2^*(F)>1$, at which the net benefit is exactly equal to zero. When $\rho < \rho_2^*(F)$, the benefits exceed the costs for region 2. Similarly, we can also find $\rho_3^*(F)$ and $\rho_1^*(F)$. The smallest of them, $\rho^*(F)$, is the critical value of elasticity, at which the net benefit of a currency union is at least zero for every region inside or outside. Because we can find $\rho^*(F)$ for any small F, when an elasticity is sufficiently close to one, the size of a fixed cost is almost irrelevant.

On the other hand, when ρ is far larger than one, the effect is no greater than that of a mere increase in labor.[8]

$$\lim_{\rho\to\infty}\left(\frac{\rho}{\rho-1}\right)F = F \tag{38}$$

With high elasticity, the size of a fixed cost is crucial to welfare impacts of a currency union.

These results imply that, depending on elasticity, a currency union is a public good and its size tends to be too small, though the previous study often suggests that the EMU should be smaller. The results also assert that the elasticity of substitution is an crucial criterion for OCA.

5 Concluding remarks

This paper focuses on the non-exclusive benefits of a currency union with the intention to balance the views on the cost and benefit aspects, and to evaluate a monetary union favorably,

[8] When an elasticity of substitution is infinite, the CES production function degenerates to a linear production function, giving no incentive for further specialization. Then, the effect of expansion in endowment is only to increase the potential output by exactly the same proportion. On the other hand, when the elasticity equals unity, we have the Cobb-Douglas production technology, with no possibility of change in varieties of inputs, and the effects of currency union crucially depend upon the size of a fixed cost.

assuming that the non-exclusive benefits arise due to the nature of the fixed costs in cross-currency transactions and input variety in production. In addition, it is assumed that a currency union improves the welfares of the regions inside and outside. An implication of the model is that because of its external effect, the actual size of a currency union can be smaller than the socially optimal one, despite the many discussions on Europe that focus mainly on the cost aspects and suggest that the EMU should be smaller. Elasticities close to one ensure that the effect depends little on the size of a fixed cost. On the other hand, higher elasticities diminish the non-exclusive benefit and make a currency union more likely to lower the welfare of the region outside, and in some cases, even the welfares of the union members. Therefore, the model claims the elasticity to be a crucial criterion for OCA.

Finally, further research should focus on other benefits that this paper does not address. They include benefits from high degree of capital mobility and from stable relative price between tradables and non-tradables. Capital is usually allowed to move freely inside a currency union to enhance the chance for the member regions to smooth out their consumptions overtime. On the other hand, when nominal prices are not completely flexible, nominal exchange rate fluctuations caused by nominal shocks disturb the relative price, causing reallocation of resources. Therefore, a currency union eliminates any nominal exchange rate fluctuation inside, and thus prevents nominal shocks from affecting the real side of the economy through the nominal exchange rates channel.

The underlying shocks, other than productivity shocks, could be taken into account. The welfare impacts of a common currency depend on the nature of underlying disturbances, and many empirical researches have attempted to indentify separate shocks; Bayoumi and Eichengreen (1993) identifies supply and demand shocks, while Chamie, DeSerres and Lalonde (1994) further decomposes demand shocks into real and nominal shocks.

The welfare impact of a currency union among developed and underdeveloped regions should also be investigated. By allowing fixed costs to be different across regions, our model will also give insights on why the eastern European countries are eager to join the EMU, not from a political perspective, but from an economic one.

Appendix

This appendix demonstrates how to take the expected value of Δx_2 in Section 3.2.

First, we derive a formula for calculating expected values. Suppose that y is a stochastic variable and takes any value in $S \subseteq R$. If S is divided into the two subsets S_1 and S_2 so that $S_1 \cap S_2 = \varnothing$, the expected value of y can be computed in the following way.

$$E(y) = E(y \mid y \in S_1) Prob(y \in S_1) + E(y \mid y \in S_2) Prob(y \mid y \in S_2) \qquad (39)$$

Next, we calculate the expected value of Δx_2. Defining $y = \varepsilon_2 - \varepsilon_3$, we have,

$$\varepsilon_2 \geq \varepsilon_3 \leftrightarrow y \geq 0$$
$$\varepsilon_2 < \varepsilon_3 \leftrightarrow y < 0.$$

With the earlier described assumption on ε_2 and ε_3, y is also a normal variate with a mean zero and a variance of $\sigma_2^2 + \sigma_3^2 - 2\sigma_{23}$. If we represent Δx_2 in terms of y ,

$$\Delta x_2 = \begin{cases} \left(\dfrac{\rho}{\rho-1}\right)F & if \quad y \geq 0 \\ \left(\dfrac{\rho}{\rho-1}\right)F + \rho(1-\alpha)y & if \quad y < 0 \end{cases}$$

Here, setting $S_1 = [0,\infty)$ and $S_2 = (-\infty,0)$ and applying (39) to Δx_2, we obtain

$$\begin{aligned} E(\Delta x_2) &= E(\Delta x_2 \mid y \geq 0)Prob(y \geq 0) + E(\Delta x_2 \mid y < 0)Prob(y < 0) \\ &= \frac{1}{2}E\left[\left(\frac{\rho}{\rho-1}\right)F \mid y \geq 0\right] + \frac{1}{2}E\left[\left(\frac{\rho}{\rho-1}\right)F + \rho(1-\alpha)y \mid y < 0\right] \qquad (40) \\ &= \left(\frac{\rho}{\rho-1}\right)F + \frac{1}{2}\rho(1-\alpha)E(y \mid y < 0) \end{aligned}$$

We apply the following formula from Bayoumi (1994) on the expected value of a normal variate with a mean of zero to the second term on the last line:

$$E(y \mid y < Y) = -\sqrt{Var(y)}\,\frac{\varphi\left(\dfrac{Y}{Var(y)}\right)}{\Phi\left(\dfrac{Y}{Var(y)}\right)}, \qquad (41)$$

where $\varphi(\cdot)$ and $\Phi(\cdot)$ denote the p.d.f. and the c.d.f. of a *standard* normal variate.

Finally, substituting it into (40), we obtain

$$E(\Delta x_2) = \left(\frac{\rho}{\rho-1}\right)F - \rho(1-\alpha)\varphi(0)\sqrt{\sigma_2^2 + \sigma_3^2 - 2\sigma_{23}}.$$

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