多相誘導電動機の計算機シュミュレーション

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(出版者 / Publisher)
法政大学工学部
(雑誌名 / Journal or Publication Title)
法政大学工学部研究集報 / 法政大学工学部研究集報
(巻 / Volume)
12
(開始ページ / Start Page)
67
(終了ページ / End Page)
80
(発行年 / Year)
1976-01
(URL)
https://doi.org/10.15002/00004199

Digital Simulation of Polyphase Induction Motors

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Abstract

The fundamental equations of polyphase induction motors (linear simultaneous differential equations with periodic coefficients) are directly solved by finite difference methods for balanced and for unbalanced conditions.

Principal symbols

$V[t]$	$= \{v_a, v_b, v_c, v_d, v_e, v_f\}$, voltage (column) matrix.
$I[t]$	$= \{i_a, i_b, i_c, i_d, i_e, i_f\}$, current (column) matrix.
$Z[t, \omega_m]$	$= R+\omega_m G[t, \omega_m]+L[t, \omega_m](\mathrm{d}/\mathrm{d}t)$, impedance matrix.
R	$= \lceil r_a, r_b, r_c, r_d, r_e, r_f \rfloor$, resistance (diagonal) matrix.
$G[t, \omega_m]$	$= (1/\omega_m)(\mathrm{d}/\mathrm{d}t)L[t, \omega_m]$, torque matrix.
$L[t, \omega_m]$	$= l+L'+M[t, \omega_m]$, inductance matrix.
l	$= \lceil l_a, l_b, l_c, l_d, l_e, l_f \rfloor$, leakage inductance (diagonal) matrix.
L'	= self-inductance matrix.
$M[t, \omega_m]$	= mutual inductance matrix.
$S[t, \omega_m]$	$= L^{-1}[t, \omega_m](R+\omega_m G[t, \omega_m])$, coefficient matrix of the differential state equation.
C	= current connection matrix.
$I'[t]$	$= \{i_1, i_2, i_3, i_4\}$, new coordinate current (column) matrix.
s	$= (\omega-\omega_m)/\omega$, slip.
$L_{ij}\cos(\frac{i-j}{3}2\pi)$	denotes the i-th row and j-th column element in self-inductance matrix.
$M_{ij}\cos(\omega_m t+\frac{i-j}{3}2\pi)$	denotes the i-th row and j-th column element in mutual inductance matrix.
B^t	denotes the transpose of matrix B.
B^{-1}	denotes the inverse of matrix B.
$/n$	denotes the unit matrix of order n.
ω	denotes the impressed voltage source angular velocity (rad/sec.)
ω_m	denotes the mechanical angular velocity transformed into the electrical angular velocity (rad/sec).
p	denotes the number of pole pairs.
T	denotes the torque $(N-m)$.
t	denotes the time (sec).
Δt	denotes the stepwidth (sec).
A	denotes the parameter of approximate exponential function.

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Subscript

a,b,c,d,e and *f* refer to the *a,b,c,d,e* and *f*-phase (branch) quantities respectively.
i and *j* refer to the row and column in inductance matrix respectively.

1. Introduction

Among all types of alternating current motors, the one of induction type is by far the most popular and is used very widely, which is quite often designed for use on polyphase circuit (usually three phase) over whole horse-power range. A polyphase induction motor has many excellent characteristics such as the simpleness of its structure, inherent self-starting character and high reliability in its behavior. This machine is equipped with both a primary winding (usually stator) and secondary winding (usually rotor) as shown in Fig. 1, in normal use an energy source is connected to one winding alone, the primary winding.

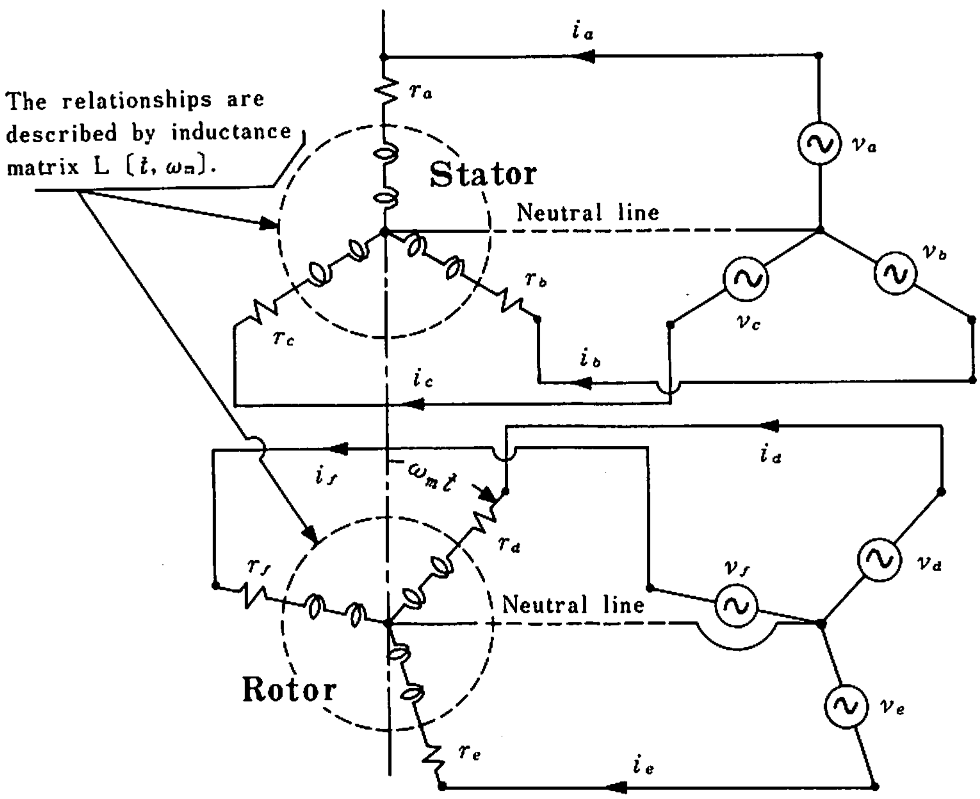

Fig. 1. Circuit diagram of the three phase induction motor.

Currents are made to flow in the secondary winding by induction, thereby creating an ampere-conductor distribution that interacts with the primary magnetic field distribution and produces a net unidirectional torque.

Many theoretical studies has been performed for the transient and steady state in polyphase induction motors for a long time [1 to 9].

In these studies, very complex and tedious tensor transformations have conventionally performed.

At starting or controlled by semiconductor elements, we choose some combinations of stator voltages, stator resistances and rotor resistances. By these choices, two cases are classified "Balanced conditions" and "Unbalanced conditions", which are explained in detail in Appendix.

In this paper, the fundamental equations of the three phase induction motor, for balanced and for unbalanced conditions, are directly solved by numerical methods instead of performing tedious tensor transformations mentioned above. The results are compared with those of conventional tensor transformation methods and also with experimental results.

2. Fundamental equations

The circuit diagram of the three phase induction motor is shown in Fig. 1 and is composed of six main branches, three of stator and three of rotor.

The set of Kirchhoff's equations for the circuit system consists of simultaneous six differential equations and is preferably expressed as a matrix equation between a voltage matrix (vector) $V[t]$ and a current matrix (vector) $I[t]$. The first three components of voltage matrix, v_a, v_b, v_c, are respectively impressed voltages of stator branches and the remaining three, v_d, v_e, v_f, are those of rotor branches and each component of current matrix has a corresponding meaning.

Denoting impedance matrix by $Z[t, \omega_m]$, the fundamental equation is given as

$$V[t]=Z[t, \omega_m]I[t] \quad \cdots\cdots (1)$$

where the quantity ω_m is a mechanical angular velocity.

The torque T of the motor is given by the number of pole pairs p, current matrix $I[t]$ and torque matrix $G[t, \omega_m]$ whose meaning will be given later, as

$$T=(p/2)I^t[t]G[t, \omega_m]I[t] \quad \cdots\cdots (2)$$

This equation is called torque equation.

Impedance matrix $Z[t, \omega_m]$ is given by the terms of three matrices, resistance matrix R, torque matrix $G[t, \omega_m]$ and inductance matrix $L[t, \omega_m]$ as follows:

$$Z[t, \omega_m]=R+\omega_m G[t, \omega_m]+L[t, \omega_m](d/dt) \quad \cdots\cdots (3)$$

The resistance matrix R has only diagonal elements, of which the first three, r_a, r_b, r_c, are respectively resistances of three stator branches and the remaining three, r_d, r_e, r_f, are rotor ones. Inductance matrix $L[t, \omega_m]$ is composed of leakage inductance matrix l, self-inductance matrix L' and mutual inductance matrix $M[t, \omega_m]$, and this is as follows:

$L[t, \omega_m]=l+L'+M[t, \omega_m]$

l_a+L_{11}	$L_{12}\cos(2\pi/3)$	$L_{13}\cos(4\pi/3)$	$M_{14}\cos(\omega_m t)$	$M_{15}\cos(\omega_m t+2\pi/3)$	$M_{16}\cos(\omega_m t+4\pi/3)$
	l_b+L_{22}	$L_{23}\cos(2\pi/3)$	$M_{24}\cos(\omega_m t-2\pi/3)$	$M_{25}\cos(\omega_m t)$	$M_{26}\cos(\omega_m t+2\pi/3)$
		l_c+L_{33}	$M_{34}\cos(\omega_m t-4\pi/3)$	$M_{35}\cos(\omega_m t-2\pi/3)$	$M_{36}\cos(\omega_m t)$
			l_d+L_{44}	$L_{45}\cos(2\pi/3)$	$L_{46}\cos(4\pi/3)$
SYMMET-			-RICAL	l_e+L_{55}	$L_{56}\cos(2\pi/3)$
					l_f+L_{66}

. . . . (4)

where L_{ij} is (ordinary) self-inductance coefficient and M_{ij} is (ordinary) mutual inductance coefficient.

Torque matrix $G[t, \omega_m]$ is obtained by differentiating inductance matrix $L[t, \omega_m]$ with time t and by dividing by mechanical angular velocity ω_m. That is

$$G[t, \omega_m]=(1/\omega_m)(d/dt)L[t, \omega_m] \quad \cdots\cdots (5)$$

In order to solve the eq. (1), it is neccesary to take the following two facts into consideration. The third through sixth components of the voltage matrix $V[t]$ are always zero, e.g. $v_d=v_e=v_f=0$ and three other components, stator voltages v_a, v_b, v_c satisfy following relation for balanced conditions, as shown in Appendix, $v_a+v_b+v_c=0$.

When balanced conditions are satisfied or neutral line is disconnected, there exist two respective relations among three stator components, i_a, i_b, i_c, and among three rotor components, i_d, i_e, i_f, of the current matrix $I[t]$, $i_a+i_b+i_c=0$ and $i_d+i_e+i_f=0$

As the mechanical angular velocity ω_m is smaller than, or at most equal to, the angular velocity ω of stator impressed voltage in actual operation of the three phase induction motor without space harmonics [10]. the time variations of matrix elements in inductance matrix $L[t, \omega_m]$ are slow at most equal to the time variations of stator impressed voltages.

Therefore, we assumed that the elements in inductance matrix $L[t, \omega_m]$ take constant values at time interval from t to $t+\Delta t$. Namely, the fundamental equations of three phase induction motor reduce to linear simultaneous differential equations with constant coefficients during interval from t to $t+\Delta t$.

Now that eq. (1) can be taken as linear simultaneous differential equations with constant coefficients, the difference method is applicable to numerically solving it.

By considering eq. (3), eq. (1) can be rewritten as

$$(\mathrm{d}/\mathrm{d}t)I[t] = -S[t, \omega_m]I[t] + L^{-1}[t, \omega_m]V[t] \quad \cdots\cdots (6)$$

where the matrix $S[t, \omega_m]$ has following meanings.

$$S[t, \omega_m] = L^{-1}[t, \omega_m](R + \omega_m G[t, \omega_m]) \quad \cdots\cdots (7)$$

Then we formally get a following equation of difference methods.

$$I[t+\Delta t] = I[t]\exp(-\Delta t S[t+\Delta t, \omega_m]) + \{/_\sigma - \exp(-\Delta t S[t+\Delta t, \omega_m])\}(R + \omega_m G[t+\Delta t, \omega_m])^{-1} V[t+\Delta t] \cdot \quad \cdots\cdots (8)$$

where the unit matrix of order 6 is denoted by $/_\sigma$.

A further approximation is applied to the exponential function $\exp(-\Delta t S[t+\Delta t, \omega_m])$, as in the following, by introducing a new parameter A.

$$\exp(-\Delta t S[t+\Delta t, \omega_m]) = (/_\sigma + A \ \Delta t S[t+\Delta t, \omega_m])^{-1}(/_\sigma - \langle 1-A \rangle \Delta t S[t+\Delta t, \omega_m]) \cdot \quad \cdots\cdots (9)$$

According to the value of the parameter A, the approximation is classified into the well known centered difference method (A = 0.5), forward difference method (A = 0) and backward difference method (A = 1) [11]. In this paper, practical computations are carried out with A = 0.5, and compared with results computed by other values of A as discussed in section 3.

Eq. (1) is directly solved by the numerical method for balanced conditions, because no currents flow in the neutral line if the neutral is connected. However, six relationships implied in eq. (1) are excessive for the majority of situations and it is desirable to reduce the number of independent variables. Since we are dealing with star-connected machines with no neutral line as shown in Fig. 1, the currents i_c and i_f can be eliminated by use of the following relationships which are explained in Appendix.

$$\begin{pmatrix} i_a \\ i_b \\ i_c \\ i_d \\ i_e \\ i_f \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} \quad \cdots\cdots (10)$$

Hereafter we denote the new current vector in the right hand side of eq. (10) by $I'[t]$, and define the current connection matrix C as

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 \end{pmatrix} \quad \cdots\cdots (11)$$

Then we obtain the transformed equation of eq. (1) and torque equation in terms of the transformed current vector $I'[t]$, and they are described by eq. (12).

$$\begin{cases} I[t]=CI'[t] \\ C^tV[t]=C^tZ[t,\omega_m]CI'[t] \\ T=(p/2)I'^t[t]C^tG[t,\omega_m]CI'[t] \end{cases} \quad \cdots\cdots (12)$$

The transient and steady state characteristics of the three phase induction motor for balanced and unbalanced conditions are computed by eq. (12), using the procedures of eq. (8) and eq. (9).

3. Numerical solutions

The various constants of a motor used in calculation are listed in Table 1 for balanced conditions, in Table 2 for unbalanced conditions, and the constants of the actual experimented motor are listed in Table 3.

Table 1. *Various constants of the calculated motor for balanced conditions.*

Voltages	$\omega=100\pi$	(rad/sec)
	$v_a=\sqrt{2/3}\,200\ \sin(\omega t)$	(v)
	$v_b=\sqrt{2/3}\,200\ \sin(\omega t-2\pi/3)$	(v)
	$v_c=\sqrt{2/3}\,200\ \sin(\omega t-4\pi/3)$	(v)
	$v_d=v_e=v_f=0$	(v)
Initial currents are all zero.		
Stepwidth $\Delta t=0.00005$ (sec)		
Number of pole pairs $p=2$		
Resistances	$r_a=r_b=r_c=1.13$	(Ω)
	$r_d=r_e=r_f=1.25$	(Ω)
Inductances	$L_{11}=L_{22}=L_{33}=L_{12}=L_{13}=L_{23}=0.11466$	(H)
	$L_{44}=L_{55}=L_{66}=L_{45}=L_{46}=L_{56}=0.11466$	(H)
	$M_{14}=M_{15}=M_{16}=M_{24}=M_{25}=0.109$	(H)
	$M_{26}=M_{34}=M_{35}=M_{36}=0.109$	(H)
	$l_a=l_b=l_c=l_d=l_e=l_f=0.00533$	(H)

Table 2. *Various constants of the calculated motor for unbalanced conditions.*

(a) Unbalanced stator impressed voltage	$v_a=\sqrt{2/3}\,200\ \sin(\omega t),\ v_b=v_c=0$	(v)
	The other constants are same in Table 1.	
(b) Unbalanced stator resistance.	$r_a=10.0$	(Ω)
	The other constants are same in Table 1.	
(c) Unbalanced rotor resistance.	$r_d=10.0$	(Ω)
	The other constants are same in Table 1.	

Table 3. *Various constants of the experimented motor for balanced and unbalanced conditions.*

Voltages	$\omega=100\pi$	(rad/sec)
	$v_a=\sqrt{2/3}\,200\ \sin(\omega t)$	(v)
	$v_b=\sqrt{2/3}\,200\ \sin(\omega t-2\pi/3)$	(v)
	$v_c=\sqrt{2/3}\,200\ \sin(\omega t-4\pi/3)$	(v)
	$v_d=v_e=v_f=0$	(v)
Stepwidth	$\Delta t=0.00005$	(sec)
Number of pole pairs $p=2$		
Resistances	$r_a=r_b=r_c=10.835$	(Ω)
	$r_d=r_e=r_f=1.0$	(Ω)
Inductances	$L_{11}=L_{22}=L_{33}=L_{12}=L_{13}=L_{23}=0.245$	(H)
	$L_{44}=L_{55}=L_{66}=L_{45}=L_{46}=L_{56}=0.0369$	(H)
	$M_{14}=M_{15}=M_{16}=M_{24}=M_{25}=0.0952$	(H)
	$M_{26}=M_{34}=M_{35}=M_{36}=0.0952$	(H)
	$l_a=l_b=l_c=0.02119$	(H)
	$l_d=l_e=l_f=0.003194$	(H)

Unbalanced stator resistance.	$r_a=30.835$	(Ω)
	The other constants are same in above table.	

For the comparison, we solve numerically the simultaneous differential equations linearized by the conventional tensor transformation methods [4 to 9] by using the backward difference method, centered difference method and forward difference method. Among the results obtained by each method, there are only small differences. These results also agree fairly well with our numerical solutions of the eq. (1) or eq. (12) for balanced conditions computed by the centered difference method. Rigorous analytical solutions are obtained for balanced conditions by the conventional revolving field theory [4 to 9]. Numerical solutions by our method reproduce these rigorous solutions within discrepancies of a few percents.

The numerical solutions of eq. (1) or eq. (12) for balanced conditions computed by the backward difference method were somewhat small compared with the results obtained by the centered difference method. On the contrary, the forward difference method yielded larger results. Therefore, we adopted the centered difference method for the digital simulation of polyphase induction motor.

Some examples of numerical solutions of eq. (12) for balanced conditions computed by the centered difference method are shown in Fig. 2. The steady state numerical solutions of eq. (12) for balanced conditions computed by the centered difference method are shown together with the above mentioned rigorous solutions in Fig. 3. Some examples of numerical solutions of eq. (12) for unbalanced conditions computed by the centered difference method are shown in Fig. 4, and the comparisons of the steady state experimental and computational results are shown in Fig. 5.

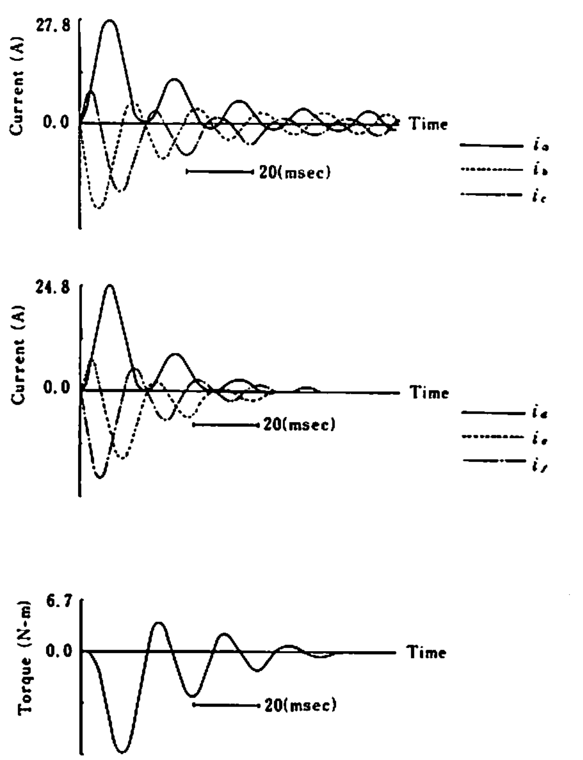

Fig. 2-a

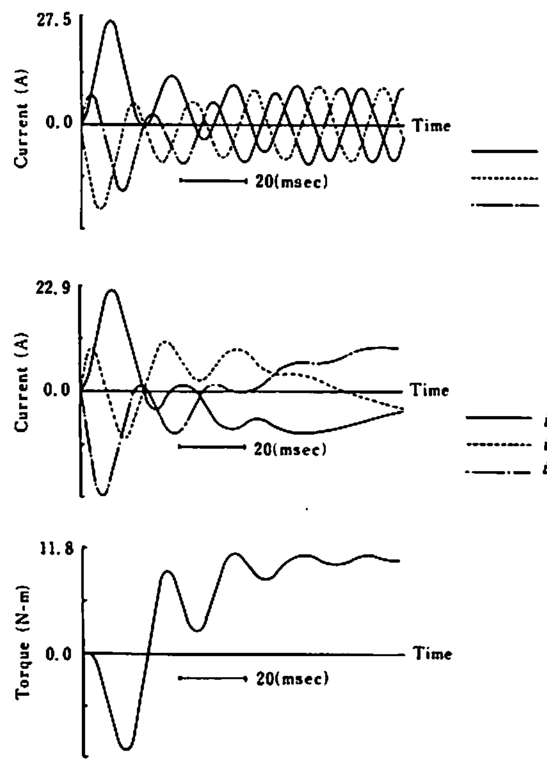

Fig. 2-b

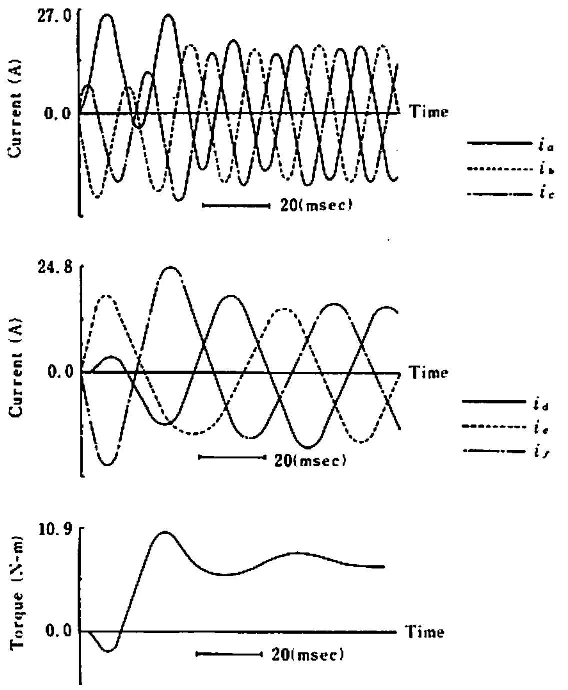

Fig. 2-c

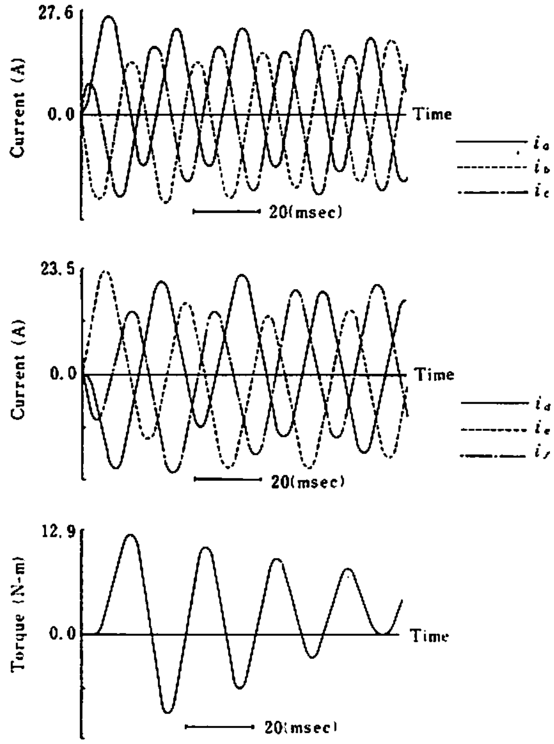

Fig. 2-d

Fig. 2. Numerical examples for balanced conditions.

(a) s = 0.0　　(c) s = 0.4

(b) s = 0.1　　(d) s = 0.8

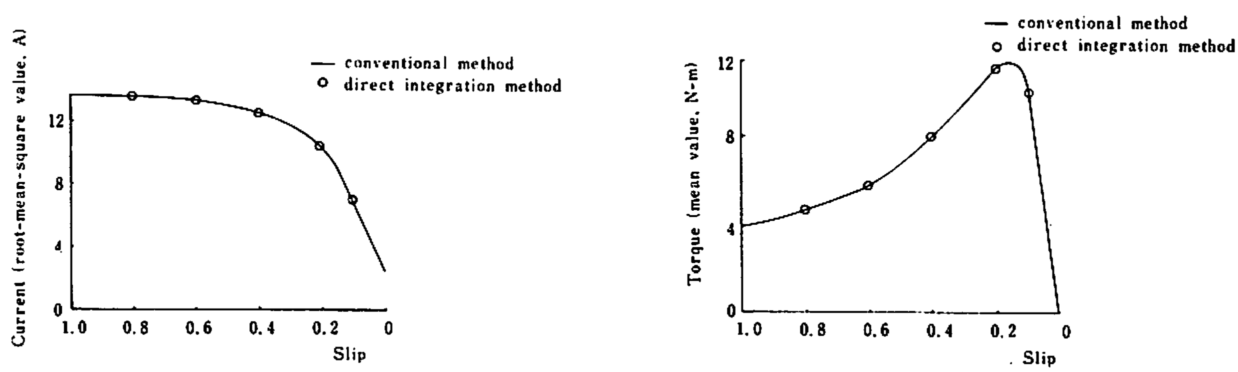

Fig. 3. Numerical examples of the steady state characteristics compared with the values that are computed by the conventional revolving field theory (each stator current i_a, i_b, i_c and rotor i_d, i_e, i_f represented by the root mean square value are the same values respectively).

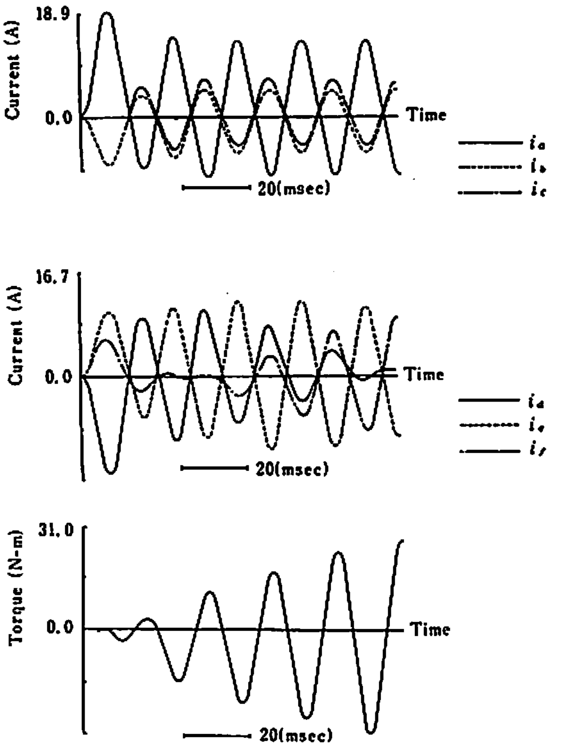

Fig. 4-a

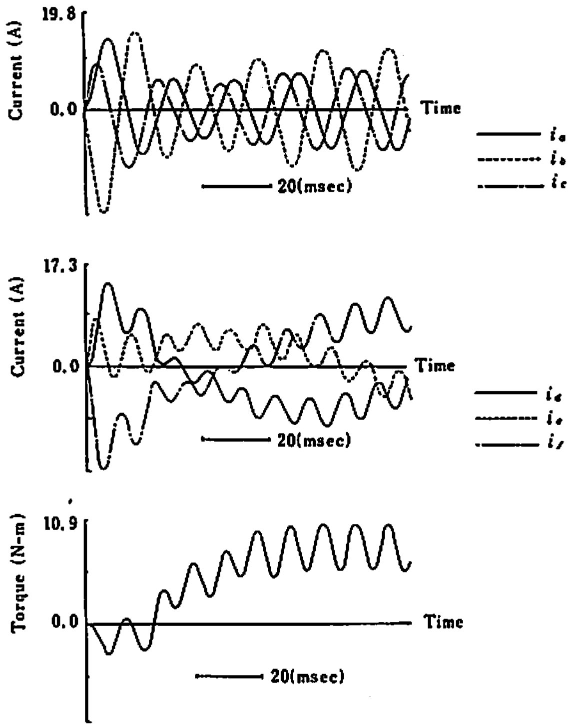

Fig. 4-b

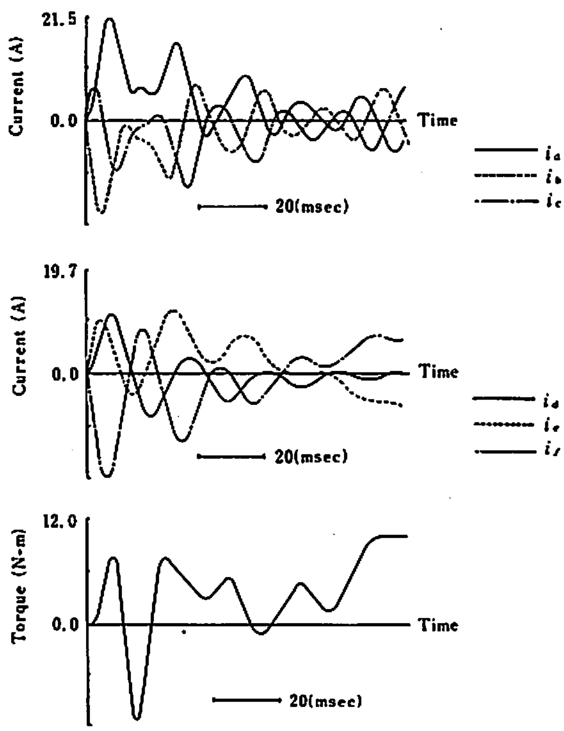

Fig. 4-c

Fig. 4. Numerical examples for unbalanced conditions.

(a) Unbalanced stator impressed voltage (s = 0.95).

(b) Unbalanced stator resistance (s = 0.1).

(c) Unbalanced rotor resistance (s = 0.1).

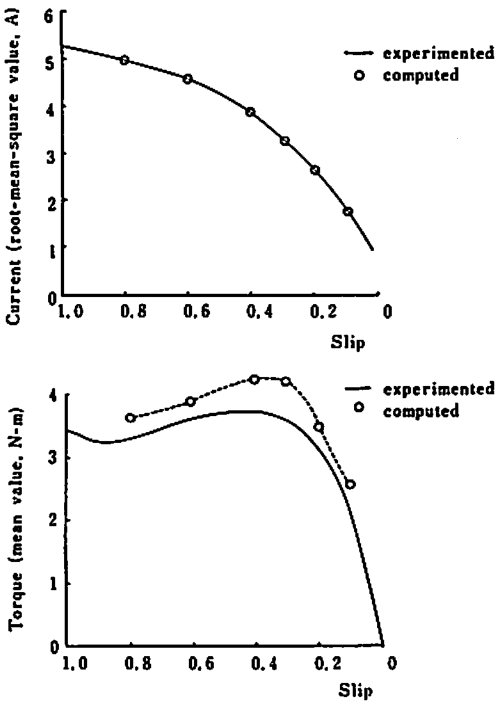

Fig. 5-a

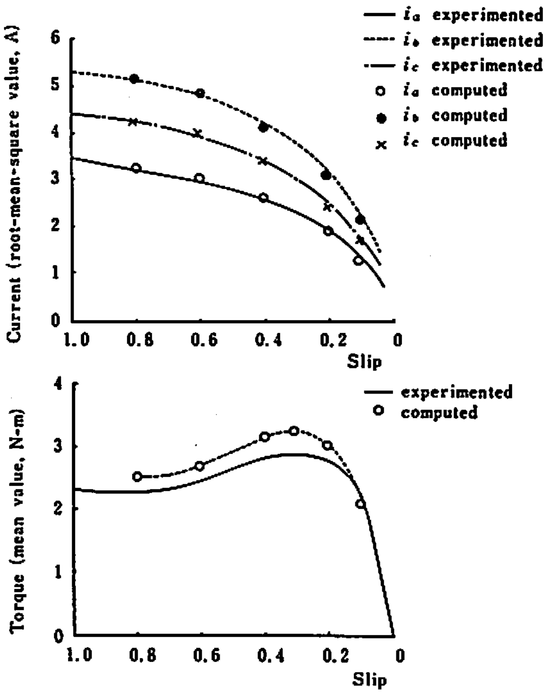

Fig. 5. Comparisons of the steady state experimental and computational results.

(a) Balanced conditions (each stator current i_a, i_b, i_c and rotor current i_d, i_e, i_f represented by the root mean square value are the same values repectively).

(b) Unbalanced stator resistance.

4. Conclusion

One of the merits of our direct integral method is that mathematical treatments are common both for balanced conditions and for any unbalanced conditions, although the conventional tensor transformations are confronted with serious difficulties in obtaining a linearized model for unbalanced conditions. So the method proposed here is not only useful for the anlysis of the polyphase induction motor, but may be applicable to any other alternating current machines.

The method is so simple in algorithm that computer program of our direct integration method can be easily written down directly from the fundamental differential equations without any other manual works, and can be used for any unbalanced conditions, therefore, programming effort is the least but obtained program has the most generality.

If the relevant stepwidth Δt is chosen, then the numerical solutions obtained by our direct integration method have enough accuracy for engineering problems; in this paper, sufficiently accurate solutions are obtained by the selection of stepwidth Δt which is enough small to obtain the correct wave forms of the stator impressed voltages.

Finally, we have recently proposed quite effective digital simulation method of polyphase induction motors. We wish to apply the direct integration method to the problems of polyphase induction motor supplied with the nonsinusoidal waves as the stator impressed voltage, and one of the authors (Saito) now intends to apply the method to the polyphase induction motor with space harmonics [10].

Acknowledgements

The authors are grateful to professor I. Fujita and professor T. Yamamura for their helpful interest and stimulating advice, and to professor T. Sebe and Mr. M. Kadoi for the facilities given by them at the computer center of Hosei University.

The practical computations given in this paper are carried out by using computers FACOM 230-45S of Hosei University and HITAC 8800 of Tokyo University.

Appendix: Balaced and Unbalanced conditions

If the conditions that the stator impressed voltages (in this paper, sinusoidal waves) v_a, v_b and v_c have the same amplitude, same angular velocity and relative phase difference $2\pi/3$ (in the case of three phase voltage) are satisfied, then such a case is called "Balanced stator impressed voltage", and the case in which one or more conditions are not satisfied is called "Unbalanced stator impressed voltage".

If the conditions that the stator resistances r_a, r_b and r_c, rotor resistances r_d, r_e and r_f stator leakage inductances l_a, l_b and l_c, rotor leakage inductances l_d, l_e and l_f, stator self-inductances L_{11}, L_{12}, .. and L_{33}, rotor self-inductances L_{44}, L_{45}, .. and L_{66} and mutual inductances M_{14}, M_{15}, .. and M_{36} are same values respectively, are satisfied, then such a case is called "Balanced impedance" and the case in which one or more conditions are not satisfied is called "Unbalanced impedance". Especially, if the stator or rotor resistances are not same values, then such a case is called "Unbalanced stator or rotor resistance" respectively.

In general, if the conditions of balanced stator impressed voltage and balanced impedance are satisfied, then such a case is called "Balanced conditions" and the case in which one or more conditions of balanced stator impressed voltage and balanced impedance are not satisfied, then such a case is called "Unbalanced conditions". ·

In balanced conditions, from the conditions of balanced stator impressed voltage, following relationship is established.

$$v_a + v_b + v_c = 0 \tag{A-1}$$

The examples of eq. (A-1) are shown in Table 1 and Table 3, and from the conditions of balanced impedance, as example is shown in Table 1, the following relationships are established.

$$i_a + i_b + i_c = 0 \tag{A-2}$$

$$i_d + i_e + i_f = 0 \tag{A-3}$$

The examples which satisfy the eq. (A-2) and eq. (A-3) are as follows:

$$i_a = I_s \sin(\omega t - h)$$
$$i_b = I_s \sin(\omega t - h - 2\pi/3)$$
$$i_c = I_s \sin(\omega t - h - 4\pi/3)$$
$$i_d = I_r \sin(s\omega t - k)$$
$$i_e = I_r \sin(s\omega t - k - 2\pi/3)$$
$$i_f = I_r \sin(s\omega t - k - 4\pi/3)$$

Where I_s, I_r, h, h and k are constants respectively.

Therefore, in balanced conditions, if the neutral line is connected or disconnected, the same numerical solutions can be obtained from eq. (1) (which is written as it were equipped with the neutral line) in section 2.

In unbalanced conditions, especially in unbalanced stator impressed voltage, the relationship (A-1) is not established, and if the neutral line is disconnected (star-connected machine as shown in Fig. 1), then the previously described relationships (A-2) and (A-3) must be established.

Then, in unbalanced conditions, the relationships (A-2) and (A-3) must be introduced to the fundamental equations as described by eq. (1) in section 2. Therefore, arbitary one independent variable of eq. (A-2) and eq. (A-3) are represented by the remaining terms of eq. (A-2) and eq. (A-3) respectively, and their relationships are described by the current connection matrix C [in this paper, current i_c in eq. (A-2) and current i_f in eq. (A-3) are represented by the remaining currents in eq. (A-2) and eq. (A-3) respectively, as described by eq. (10) and eq. (11) in section 2.], which is introduced to the fundamental equations and torque equation as described by the procedures of eq. (12).

However, if the neutral line is connected, then the relationships (A-2) and (A-3) are not satisfied, in this case, numerical solutions are obtained by the eq. (1) in section 2 without any transformations [eq. (12) in section 2].

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