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The Japanese and Simple Topologies

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Abstract

In this paper, the new families of sets are examined. These families are "the Japanese Family in Edo Era", "the family of open sets", "the family of closed sets" in the general topology.

We define also "the Union Family" and "the Meet Family". With the Union and the Meet Families, we obtain the clear relations between many families.

1. The Japanese Family in Edo Era

Let X be a universe and J be a subfamily of $P=\{A; A\subseteq X\}$. We call J the Japanese Family in Edo Era, iff the following three (i), (ii), (iii), or four conditions (i), (ii), (iii), (iv), are satisfied.

(i) $X\in J$, $\phi\notin J$.

(ii) If $A, B\in J$ and $A\cap B\neq\phi$, then $A\subseteq B$ or $B\subseteq A$.

(iii) There is a unique element $\alpha\in X$ such that if $A\in J$ and $A\neq X$ then $\alpha\notin A$.

The condition (ii) means prohibition of "Nisoku-no-waraji". This is Japanese peculiar condition.

If $x\in X$, $x\neq\alpha$, then "α" of the condition (iii) is called "x's Tono" and X is called "α's Nawabari", and sometimes it is said that, "x's Amae to Tono α".

Let $\phi\neq Y\in J$, and $J'=\{A\cap Y;\ A\in J\}$

then the next conditions hold.

(1) $Y\in J'$, $\phi\notin J'$.

(2) If $M, N\in J'$ and $M\cap N\neq\phi$, then $M\subseteq N$ or $N\subseteq M$.

(iv) There is an unique element $\beta\in Y$ such that if $M\in J'$ and $M\neq Y$, then $\beta\notin M$.

The conditions (1) and (2) will be easily proved from the definition of J' and the conditions (i) and (ii).

The condition (iv) is an additional condition of (i), (ii), (iii), and we will assume (iv) holds for any $Y\in J$. If $y\in Y$, and $y\neq\beta$, then "β" of the condition (iv) is called "y's Shujin" and y is said to be "β's Miuch" and also it is said that "y's Amae to Shujin β.

For each Japanese Family J, $\bar{J}$ is defined by the following equality.

$\bar{J}=\{X\sim A;\ A\in J\}=\{A^c;\ A\in J\}$

$\bar{J}$ is said to be "Sotomono of (X, J).

Theorem 1

Let J be a Japanese Family of X, which satisfies the condition (i), (ii), (iii) and (iv) for each $Y\in J$. $\bar{J}$, J' and $\bar{J}'$ are defined by the following equalities.

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$\bar{J}=\{X\sim A;\ A\in J\}$, $J'=\{A\cap Y;\ A\in J,\ \phi\neq Y\in J\}$

$\bar{J}'=\{X\sim M;\ M\in J'\}$

then $\bar{J}$ and $\bar{J}'$ satisfy the next three conditions.

(a) $\phi\in\bar{J}$, $X\notin\bar{J}$; $\phi\in\bar{J}'$, $Y\notin\bar{J}'$.

(b) If $A, B\in\bar{J}$ and $A\cup B\neq X$, then $A\subseteqq B$ or $B\subseteqq A$; If $M, N\in\bar{J}'$ and $M\cup N\neq Y$, then $M\subseteqq N$ or $N\subseteqq M$.

(c) There is an unique element $\alpha\in X$ such that if $A\in\bar{J}$ and $A\neq\phi$, then $\alpha\in A$;

For each $Y\in J$, there is an unique element $\beta\in Y$ such that if $M\in\bar{J}'$ and $M\neq\phi$ then $\beta\in M$

($\bar{J}$ is Sotomono of J and $\bar{J}'$ is Sotomono of J'.)

Proof

(b) If $A, B\in\bar{J}$ and $A\cup B\neq X$ $\Leftrightarrow$

$X\sim A$, $X\sim B\in J$ and $(X\sim A)\cap(X\sim B)\neq\phi$ then $(X\sim A)\subseteqq(X\sim B)$ or $(X\sim B)\subseteqq(X\sim A)$ by (ii) $\Leftrightarrow$ $A\subseteqq B$ or $B\subseteqq A$.

(c) There is Tono $\alpha\in X$ such that $X\sim A\in J$ and

$X\sim A\neq X$ then $\alpha\notin X\sim A$ by (iii) $\Leftrightarrow$ $A\in\bar{J}$ and $A\neq\phi$ then $\alpha\in A$.

2. Simple Topologies

Let O be a subfamily of $P=\{A;\ A\subseteqq X\}$.

O is called an Union Family if the following two conditions are satisfied.

(α) $X\in O$.

(β) For any S such that $S\subseteqq O$;

$$\cup\{A;\ A\in S\}\in O$$

Let Q be a subfamily of P. Q is said to be a Meet Family iff the following two conditions are satisfied.

(α') $\phi\in Q$.

(β') For any T such that $T\subseteqq Q$;

$$\cap\{B;\ B\in T\}\in Q$$

Theorem 2

Let O be an Union Family which satisfies the conditions (α) and (β).

For any M such that $M\subseteqq X$; if we define M^o by the next equation

$$M^o=\cup\{A;\ A\subseteqq M,\ A\in O\}$$

then $M^o\in O$ and the next three conditions (i), (ii) and (iii) are satisfied.

(i) $X^o=X$

(ii) For any M such that $M\in O$, $M^o=M$. For any M such that $M\notin O$, $M^o\subsetneqq M$.

(iii) For each M, N such that $M\subseteqq N\subseteqq X$, $M^o\subseteqq N^o$.

But it is not always true that

(iv) $(M\cap N)^o=M^o\cap N^o$ for any $M, N\subseteqq X$.

Proof

(i) $X\in O$, $X^o=\cup\{A;\ A\subseteqq X,\ A\in O\}=X$

(ii) $M\in O$, then $M^o=\cup\{A;\ A\subseteqq O\}=M$.

$M \notin O$, then $M^o \subseteq M$, if we assume that $M^o = M$, then $M \in O$, contradicting $M \notin O$. Hence $M^o \subsetneq M$.

(iii) $x \in M^o = \cup \{A;\ A \subseteq M,\ A \in O\}$ then there is $x \in A \in O$ such that $A \subseteq M \subseteq N$

i.e. $x \in N^o$. $\therefore$ $M^o \subseteq N^o$.

(iv) Let us consider that

$O = \{\phi, \{a, b\}, \{a, c\}, X\}$; $X = \{a, b, c\}$ as Fig. 1.

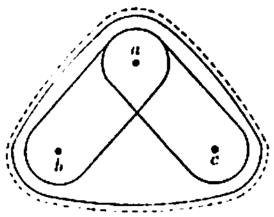

Fig. 1 The representation of $O = \{\phi, \{a, b\}, \{a, c\}, X\}$

Because O is closed by the operation of "$\cup$", O is an Union Family.

Let $M = \{a, b\} = M^o$, $N = \{a, c\} = N^o$;

then $(M \cap N)^o = \{a\}^o = \phi$, $M^o \cap N^o = \{a\}$

which shows that (iv) is not satisfied.

Theorem 3 Let $\circ$ be a function of $P = \{M;\ M \subseteq X\}$ into P, that is, $P \xrightarrow{\circ} P$; if the function $\circ$ satisfies the three conditions (i), (ii), (iii) of theorem 2, then

$O = \{M;\ M^o = M\}$

is an Union Family, *i.e.* the conditions (α) and (β) of the definition of an Union Family are satisfied.

Proof

In the case of $X = \{a, b\}$, there are four Union Families as Fig. 2.

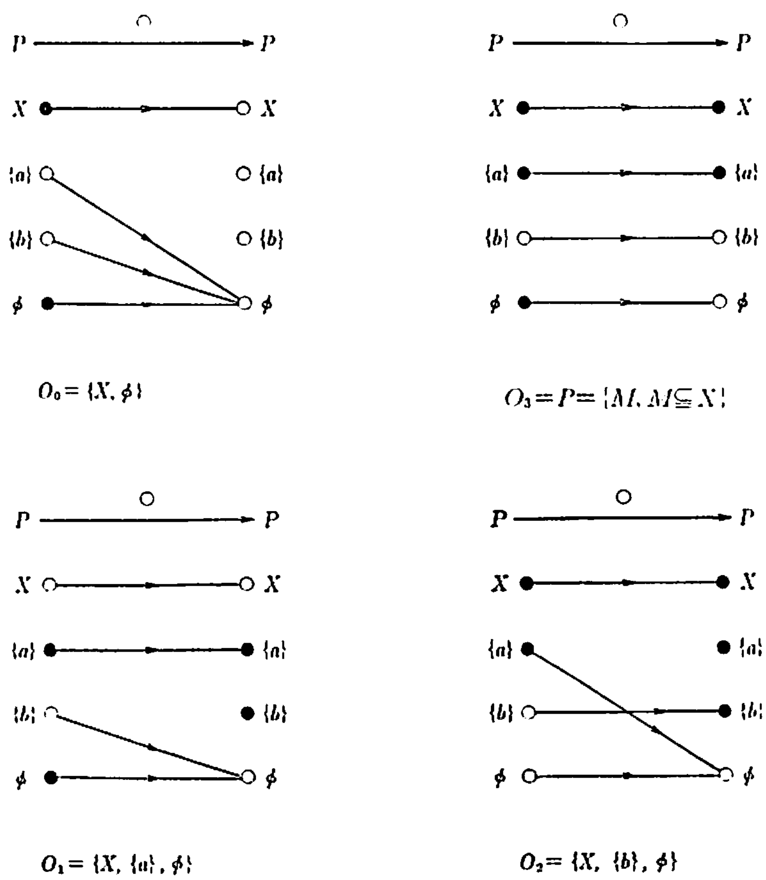

Fig. 2 The Union Families of $X = \{a, b\}$

We will prove (β) generally.

$S \subseteq \{M;\ M^o = M\} = O$.

$D = \cup \{A;\ A \in S\}$

then $A \subseteq D$ for each $A \in S$.

From (iii), $A = A^o \subseteq D^o$ for each $A \in S$.

$\cup \{A;\ A \in S\} = D \subseteq D^o$

$\therefore\ D = D^o \in O$

i.e. $\cup \{A;\ A \in S\} = (\cup \{A;\ A \in S\})^o \in O$.

(α) is proved easily.

Theorem 4

Let Q be a Meet Family satisfying the conditions (α') and (β').

For any $M \subseteq X$, if we define M^- by the next equation

$M^- = \cap \{A;\ M \subseteq A,\ A \in Q\}$

then $M^- \in Q$ and the next three conditions are satisfied

(i) $\phi^- = \phi$

(ii) For any M such that $M \in Q$, $M^- = M$.

For any M such that $M \notin Q$, $M \subsetneq M^-$.

(iii) For each M, N such that $M \subseteq N \subseteq X$, $M^- \subseteq N^-$.

But it is not always true that

(iv) $(M \cup N)^- = M^- \cup N^-$ for any $M, N \subseteq X$.

Proof

(i) $\phi \in Q$, $\phi^- = \cap \{A;\ A \subseteq \phi,\ A \in Q\} = \phi$.

(ii) $M \in Q$, then $M^- = \cap \{A;\ M \subseteq A,\ A \in Q\} = M$.

$M \notin Q$, then $M \subseteq M^-$, if we assume that $M^- = M$, then $M \in Q$, contradicting $M \notin Q$.

(iii) Let $M^- = \cap \{A;\ M \subseteq A,\ A \in Q\}$, $M \subseteq N$

and $N^- = \cap \{B;\ N \subseteq B,\ B \in Q\}$.

then $M^- \subseteq B$ for any B.

$\therefore\ M^- \subseteq N^-$.

(iv) Let us consider that

$Q = \{\phi, \{b\}, \{c\}, X\}$ and $X = \{a, b, c\}$ as Fig. 3. Because Q is closed by the operation of "$\cap$", Q is a Meet Family.

Let $M = \{b\} = M^-$, $N = \{c\} = N^-$

then $(M \cup N)^- = \{b, c\}^- = X$, and $M^- \cup N^- = \{b, c\}$. In this case, (iv) is not satisfied.

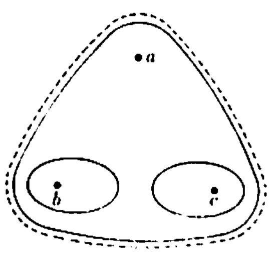

Fig. 3 The representation of $O = \{\phi, \{b\}, \{c\}, X\}$

Theorem 5

Let $-$ be a function of $P = \{M;\ M \subseteq X\}$ into P, i.e. $P \xrightarrow{-} P$; if the function $-$ satisfies the three conditions (i), (ii), (iii) of theorem 4, then

$Q = \{M;\ M^- = M\}$

is a Meet Family, that is, the conditions (α'), (β') of the definition of a Meet Family are satisfied.

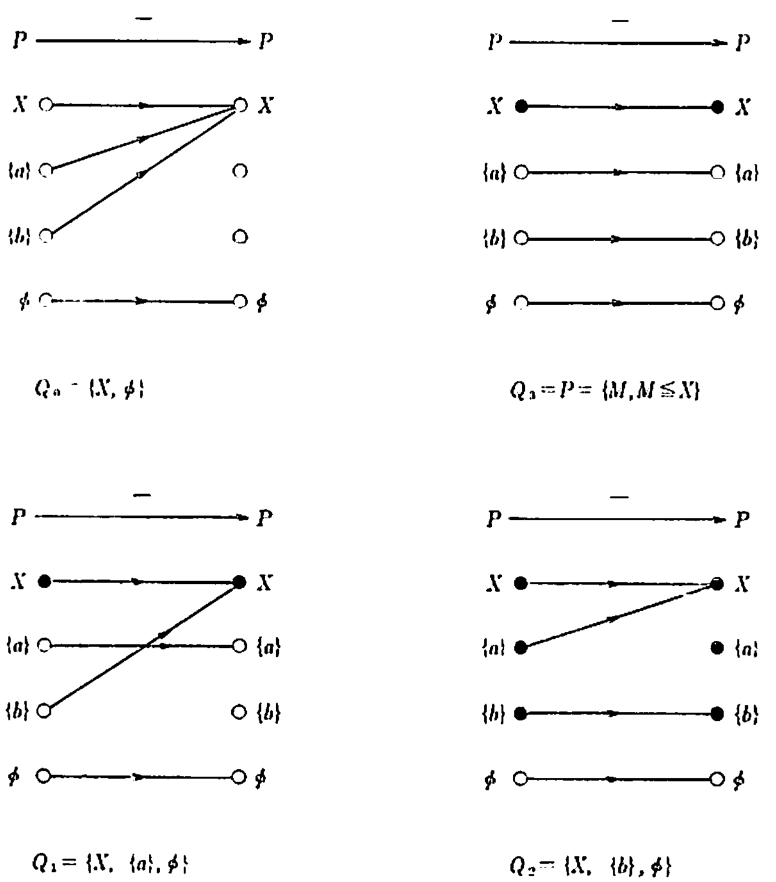

Fig. 4　The Meet Families of $X = \{a, b\}$

Proof

In the case of $X = \{a, b\}$, there are four Meet Families as Fig. 4.

We will prove (β') generally.

$$T \subseteqq \{M;\ M^- = M\} = Q.$$

$$E = \cap \{A;\ A \in T\}$$

then $E \subseteqq A$ for each $A \in T$.

From (iii) $E^- \subseteqq A^- = A$ for each $A \in T$.

$\therefore\ \ E^- \subseteqq \cap \{A;\ A \in T\} = E$

$\therefore\ \ E^- = E \in Q$

i.e. $\cap \{A;\ A \in T\} = (\cap \{A;\ A \in T\})^- \in Q$ (α') is proved easily.

3. The Relations between the Japanese Family and Topologies

The relations between the Japanese Family and Topologies are represented in Fig. 5. These are expressed in the next statements.

(1) The Japanese Family J and ϕ: $J \cup \{\phi\}$ is a Meet Family.

(2) The Sotomono $\bar{J}$ and X: $\bar{J} \cup \{X\}$ is an Union Family.

(3) A family of closed sets in the general topology is a Meet Family.

(4) A family of open sets in the general topology is an Union Family.

(5) If O is an Union Family, then $Q = \{\sim A;\ A \in O\}$ is a Meet Family.　If Q is a

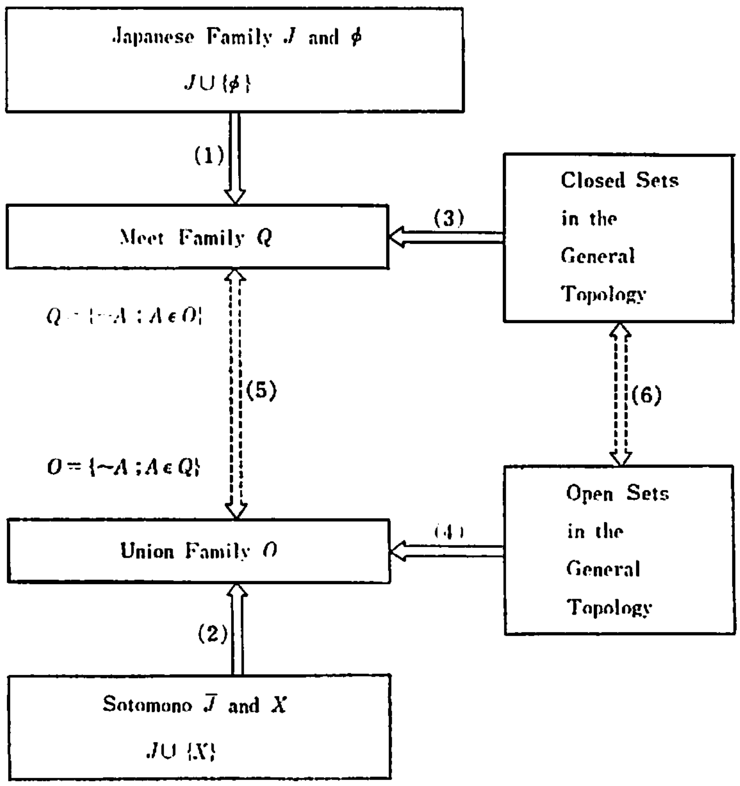

Fig. 5 The Relation between the Japanese Family and Topologies.

Meet Family, then $O=\{\sim A;\ A\in Q\}$ is an Union Family.

(6) If F is a family of open sets in the general topology, then $\bar{F}=\{\sim A;\ A\in O\}$ is a family of closed sets.

If $\bar{F}$ is a family of closed sets in the general topology, then $F=\{\sim A;\ A\in \bar{F}\}$ is a family of open sets.

Acknowledgement

Late Professor Masaru Naito in Tokyo University presented to the author the important question that the most serious problem of statistics in Japan was not existent in the numerical level but existed in the logical level. What was it?

The auther was wandering about this question for many years and this paper is an answer of this question.

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