The Old Chinese Mathematics : On Chiuchang-Suanshu(九章算術)

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The Old Chinese Mathematics

——On Chiuchang-Suanshu (九章算術)——

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Abstract

In ancient China, Suanching-Shihshu (算経十書) or the Ten Books on Arithmetic came into being between the 2nd century B.C. and the 6th century A.D., from the Chao (秦) to the Han (漢) eras. The most famous among them is Chinchang-Suanshu (九章算術) or the Book of Arithmetic, which contains nine chapters. It treats the elementary mathematical knowledge of daily life. This was introduced into Japan in the Heian (平安) era, but the Wasan was not directly influenced by this.

We first mention the abstruct of Chinchang-Suanshu (九章算術) and then we mainly deal the problems as to the erea and the volum in this paper and examine the algorithms in modern mathematical sense.

§1. Introduction

In ancient China, the most remarkable era as to science and technology is the East Chou (東周) era (4th century B.C.). The foundation of many scientific works was established in the Han (漢) era (the 2nd century B.C.). In the East Chou era, the trading business developed and the people who was working at the trading business needed the mathematical knowledge. But the government administration needed the more advanced mathematics for demanding a tax and engineering work.

In the Chou era, the mathematics was thought as one of the Six Arts (六芸).

Suanching-Shihshu (算経十書), or the Ten Books on Arithmetic—namely Choupi-Suanching (周髀算経), Chiuchang-Suanshu (九章算術), Haitao-Suanching (海島算経) [edited by Liu Hui (劉徽撰)], Suntzu-Suanching (孫子算経), Wutsao-Suanching (五曹算経), Hsiahouyang-Suanching (夏侯陽算経), Changchiu-Suanching (張邱建算経), Wuching-Suanshu (五経算術), Chiku-Suanching (緝古算経) [edited by Wang Hsiao-Tong (唐王考通撰)], and Shushu-Chiyi (数術記遺) [edited by Hsu Yue (徐岳撰)]—came into being between the 2nd century B.C. and the 6th century A.D., from the Chao (秦) to the Han (漢) eras, with the exception of the Chiku-Suanching compiled in the Tang (床) era (960 A.D.).

The most important among them is Chiuchang-Suanshu (九章算術), or the Book of Arithmetic, which contains nine chapters. It treats the elementary mathematical knowledge of daily life.

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§2. Chiuchang-Suanshu (九章算術)

2.1. Abstruct of Chiuchang-Suanshu

This book contains nine chapters.

1) Chapter 1 Houden (方田)

This chapter contains 38 problems. We show the way to calculate the area of a field. Houden means a field, whose shape is a square or a rectangle. The shapes of the fields are, for example, a right-angled triangular, right-angled trapezoid, a circular, a half circle, an arc and a daughnut.

At the time π is 3.

2) Chapter 2 Zoku-bei (粟米)

This chapter contains 46 problems. The method of Zoku-bei was used to know the exchange rate when the people exchanged the grain.

3) Chapter 3 Sibun (衰分)

This chapter contains 20 problems. Sibun means the difference. This is the algorithm of proportional allotment.

4) Chapter 4 Shokou (商功)

This chapter contains 24 problems. By the method of Shokou we find the length of one side of a field or a solid body when the area or the volume is given.

5) Chapter 5 Shoukou (少広)

This chapter contains 28 problems. For the purpose of building a castle or digging a ditch, first we must calculate the quantity of earth and then the amount of work in terms of the man power.

6) Chapter 6 Kinyu (均輸)

This chapter contains 28 problems. We deal the problems of equality of labor, that is, we allocate the labor according to the distance from the storehouse and the number of their houses in the village. These are the problems of OR (Operations Reserch) at the present time.

7) Chapter 7 Eifusoku (盈不足)

This chapter contains 20 problems. These are the problems calculating the values of two unknown quantities from their unit total and the total of one of their attributes, such as calculating the respective number of cranes and tortoises from the totals of their heads and legs.

8) Chapter 8 Houtei (方程)

This chapter contains 18 problems. These are the problems of plural equations.

9) Chapter 9 Kouku (句股)

This chapter contains 24 problems. These are the application of the Dythagorean theorem.

What has been given above are the abstruct of Chiuchang-Suanshu (九章算術). These were inuroduced into Japan in the Heian（平安）era, but these were lost after that time. After a period of decline came another period of development from the 13th to the 17th century. During this second wave, Chines mathematical books such as Suanhsueh-Chimeng（算学啓蒙）and Suanfa taugtsung（算学統宗）were imported, along with the abacus, or soroban as it is called in Japanese, and calculating rods, sangi in Japanese, but the Wasan was not directly influenced by Chiuchang-Suanshu（九章算術）. In the next section we mainly deal the erea and the volum in Chiuchang-Suanshu.

2.2. Calculation of Area and Volum

1) Chapter of Houden

The unit of length is *Ho*（歩）which is also used for the unit of area. 1 *Ho* in length is nearly equal to 1.82 m. 1 *Ho* in area is 1 *Ho*×1 *Ho* (1 squar *Ho*), that is, nearly 3.31 m². 1 Se（畝）in area is 240 squar *Ho*, and 1 Kei（頃）in area is 100 Se.

(a) Houden（方田）

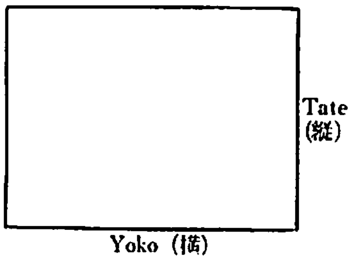

Fig. 1 Houden（方田）

The area of Houden S is calculated by

$$S = \text{Yoko} \times \text{Tate}.$$

(b) Keiden（圭田）

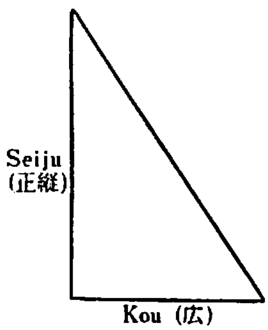

Fig. 2 Keiden（圭田）

The area of Keiden S is calculated by

$$S = (\text{Kou}/2) \times \text{Seiju}.$$

(c) Jaden（邪田）

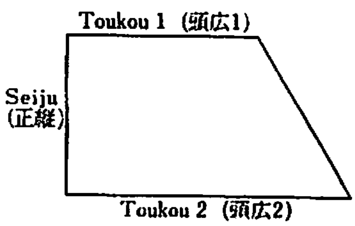

Fig. 3 Jaden（邪田）

The area of Jaden S is calculated by

$$S = (\text{Toukou 1} + \text{Toukou 2}) \times \text{Seiju} \times \frac{1}{2}.$$

(d) Kiden（箕田）

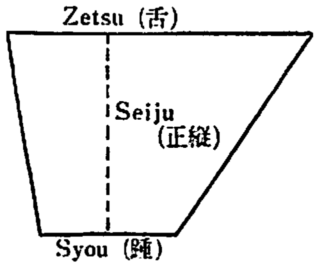

Fig. 4 Kiden (箕田)

The area of Kiden S is calculated by

$$S=(\text{Zetsu}+\text{Syou})\times\text{Seiju}\times\frac{1}{2}.$$

(e) Enden (円田)

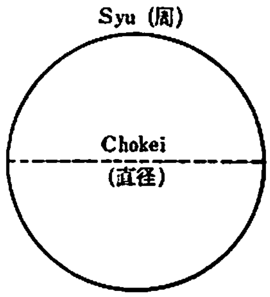

Fig. 5 Enden (円田)

The area of Enden S is calculated by three methods such as

$$S_1=\left(\text{Syu}\times\frac{1}{2}\right)\times\text{Chokei}\times\frac{1}{2},$$

$$S_2=(\text{Syu}\times\text{Chokei})\times\frac{1}{4},$$

$$S_3=(\text{Chokei})^2\times\frac{3}{4}.$$

The above three are equal if we set $\pi=3$.

(f) Eiden (宛田)

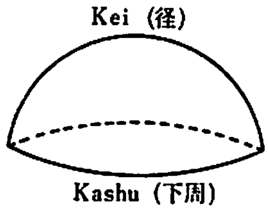

Fig. 6 Eiden (宛田)

The area of Eiden S is calculated by

$$S=(\text{Kei}\times\text{Kashu})\times\frac{1}{4}.$$

But this formula includes the error.

At the present time the area of Eiden S is given by

$S'=\pi\times(r^2+h^2)$. r: the radius of Eiden. h: the high of Eiden.

(g) Koden (弧田)

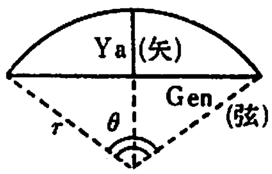

Fig. 7 Koden (弧田)

The area of Koden S is calculated by

$$S=(\text{Ya}\times\text{Gen}+\text{Ya}^2)\times\frac{1}{2}.$$

But this formula is correct if Koden is a half circular. At the present time the area of Koden S' is given by

$$S'=\frac{\theta}{360}\pi r^2-\left\{\text{Gen}\times(r-\text{ya})\times\frac{1}{2}\right\}.$$

(h) Kandan (環田)

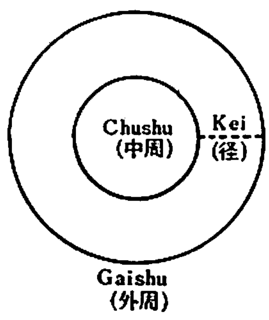

Fig. 8 Kanden (環田)

The area of Kanden S is calculated by

$$S=(\text{Chushu}+\text{Gaishu})\times\text{Kei}\times\frac{1}{2}.$$

2) Chapter of Shoukou

(a) Jou（城）

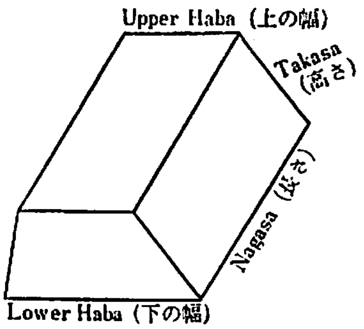

Fig. 9 Jou（城）

The volum of Jou V is calculated by

$$V=(\text{upper Haba}+\text{lower Haba})\times\text{Takasa}\times\text{Nagasa}\times\frac{1}{2}.$$

(b) Hotou（堢壔）

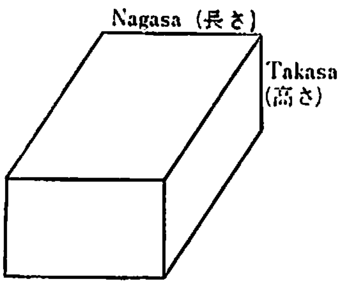

Fig. 10 Hotou（堢壔）

The volume of Hotou V is calculated by

$$V=(\text{Nagasa})^2\times\text{Takasa}.$$

(c) Houtou（方亭）（正四角錐台）

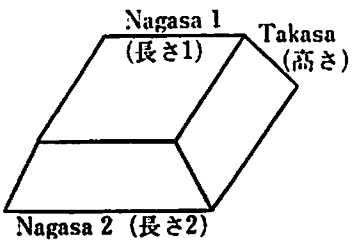

Fig. 11 Houtou（方亭）

The volum of Houtou V is calculated by

$$V=(\text{Nagasa 1}\times\text{Nagasa 2}+\text{Nagasa 1}^2+\text{Nagasa 2}^2)\times\text{Takasa}\times\frac{1}{2}.$$

(d) Housui（方錐）（正四角錐）

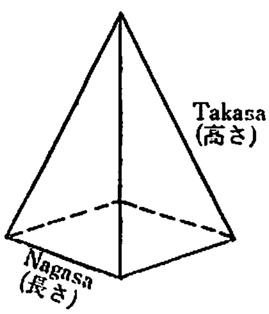

Fig. 12 Housui（方錐）

The volum of Housui V is calculated by

$$V=(\text{Nagasa})^2\times\text{Takasa}\times\frac{1}{2}.$$

(e) Senjo（羨除）

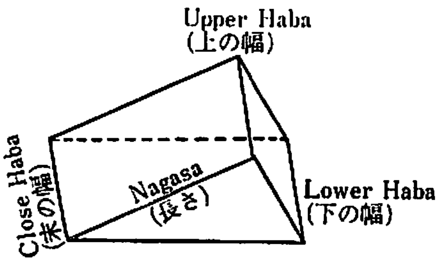

Fig. 13 Senjo（羨除）

The volum of Senjo V is calculated by

$$V=(\text{upper Haba}+\text{lower Haba}+\text{close Haba})\times \text{Fukasa}\times \text{Nagasa}\times \frac{1}{6}.$$

(f) Suibou（芻甍）

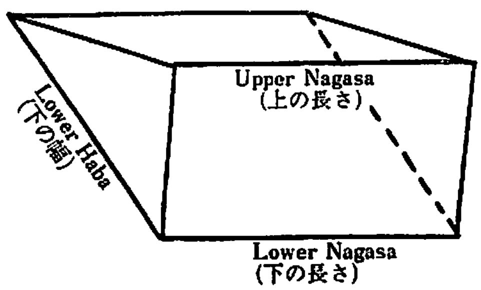

Fig. 14 Suibou（芻甍）

The volum of Suibou V is calculated by

$$V=(\text{lower Nagasa}\times 2+\text{upper Nagasa})\times \text{lower Haba}.$$

3) Chapter of Kouku

(a)

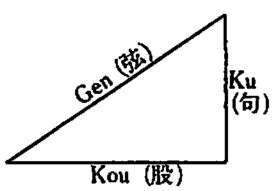

Fig. 15 Gen（弦）

$$Gen=\sqrt{(Kou)^2+(Ku)^2}.$$

(b)

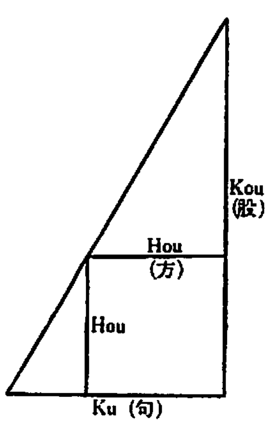

Fig. 16 Hou（方）

$$\begin{cases} Ho=Kou+Ku \\ Jitu=Kou\times Ku \end{cases}$$

$$Hou=Jitu/Ho.$$

(c)

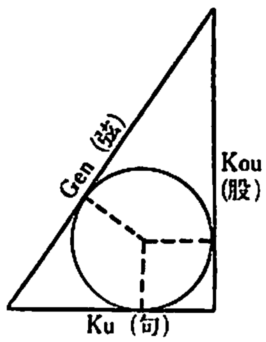

Fig. 17 Diameter of the circle

$$\begin{cases} Gen=\sqrt{(Kou)^2+(Ku)^2} \\ Ho=Gen+Kou+Ku \\ Jitu=Kou\times Ku\times 2 \end{cases}$$

Diameter of the circle=*Jitu*/*Ho*.

(d)

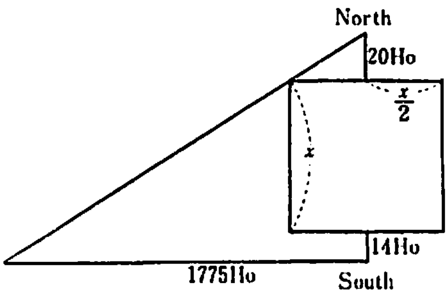

Fig. 18

There is a squar *Yu* (邑) of which we don't know how long a side. The each gate of the wall is opened. There is a tree at a distance 20 *Ho* from the north gate.

When we walk 14 *Ho* from the south gate and turn to the west and walk 1775 *Ho*. How long is a side of *Yu*? Answer: Let the length of one side of *Yu* be x. We get a quadratic equation $\frac{x^2}{2}+17x=1775\times 20$.

We can solve this equation by the method called Taiju-Kaiheihou (帯縦開平方) by which we get a solution of quadratic equation having the term of one order. If we don't have the term of one order, we have the normal squre root.

Reference

1) Chugoku no Kagaku: Chûou-Kouronsha, 1979.