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Two Sequential Screening Procedures

—Comparison between a Baysian procedure and a GUT procedure—

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Abstruct

We consider a system which has n defective items, where n is unknown and each item has common known distrihution of time to failure, $F(t)$. On the observed failure times we construct two sequential screening procedures and compare two procedures in accordauce with two criterions (a) Expectaion of duration, (b) Expectation of number of failures.

1. Introduction

Infunt mortality or decreasing failure rate in electronic systems (computers, say) is sometimes ascribed to "contamination of the population of standerd items by a small percentage of poor-quality or defective items that tend to fail soon after they are put in operation," (Barlow etal. [1]).

In this case the number of defectives n in the system is unknown. If the experimenter wants to know with reasonable certainity at what point he has observed all of the n items in the system, the given stopping rule will be appropriate.

We suppose that the standard items in the system are "immortal" (can't fail).

Consider n defective items with common known distribution of time to failure, $F(t)$ where n is unknown.

Let failures be observed at Times $T_1 \leq T_2 \leq \cdots\cdots\cdots \leq T_j$.

Let $X = -\log(1-F(T_i))$, so $X_1 \leq X_2 \leq \cdots\cdots\cdots \leq X_j$ are the order statistics from an exponential distribution with density $\exp(-y)$ $(y \geq 0)$.

We compare two stopping rules based on the datas $X_1, \cdots\cdots\cdots, X_j$ that is, ① procedure I (Bayesian procedure), ② Procedure II (GUT procedure; Giving Up Time procedure).

Comparision is made in accordance with (a) Expectation of duration, (b) Expectation of number of failures.

2. Procedure I: Bayesian procedure.

First we examine Procedure I, which is given as follows; We make the Bayes estimator n of $\hat{n}$ based on the datas $X_1, \cdots\cdots\cdots, X_j$ Sampling is stopped as soon as $\hat{n}-j \leq c$, when c is a given constant. In general, we can look for a family of prior distributions such that

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the posterion distributions also are in the same family, only the indexing qualities being changed.

Such a family is called "closed under sampling or conjugate". We construct the conjugate family of n.

First, the likelihood is

$$L(n|x_1,\cdots\cdots, x_j)=n(n-1)\cdots\cdots(n-j+1)\exp\{\sum_{i=1}^{j}x_i+(n-j)x_j\}$$

$$\propto[n!/(n-j)!]\exp(-nx_j).$$

If the prior distribution of n is taken as

$$w(n)\propto[1/n!]\exp\{\tau n\}, \tag{1}$$

then the posterior disribution of n becomes

$$w(n|x_1,\cdots\cdots, x_j)\propto[1/(n-j)!]\exp\{(\tau-x_j)n\}. \tag{2}$$

It now follows from the relations (1), (2) that the posterior distribution of n is the same form as (1) with parameters $n-j$ and $\tau-x_j$.

We then have the conjugate prior distribution of n as

$$w(n)=\frac{\exp\{\tau n\}}{\exp\{\exp(\tau)\}n!},$$

(Fig. 1), and the posterior distribution of n as

$$w(n|x_1,\cdots\cdots, x_j)=\frac{[1/(n-j)!]\exp\{(\tau-x_j)n\}}{\exp\{(\tau-x_j)j\}\exp\{\exp(\tau-x_j)\}}\quad(n=j,\ j+1,\cdots\cdots),$$

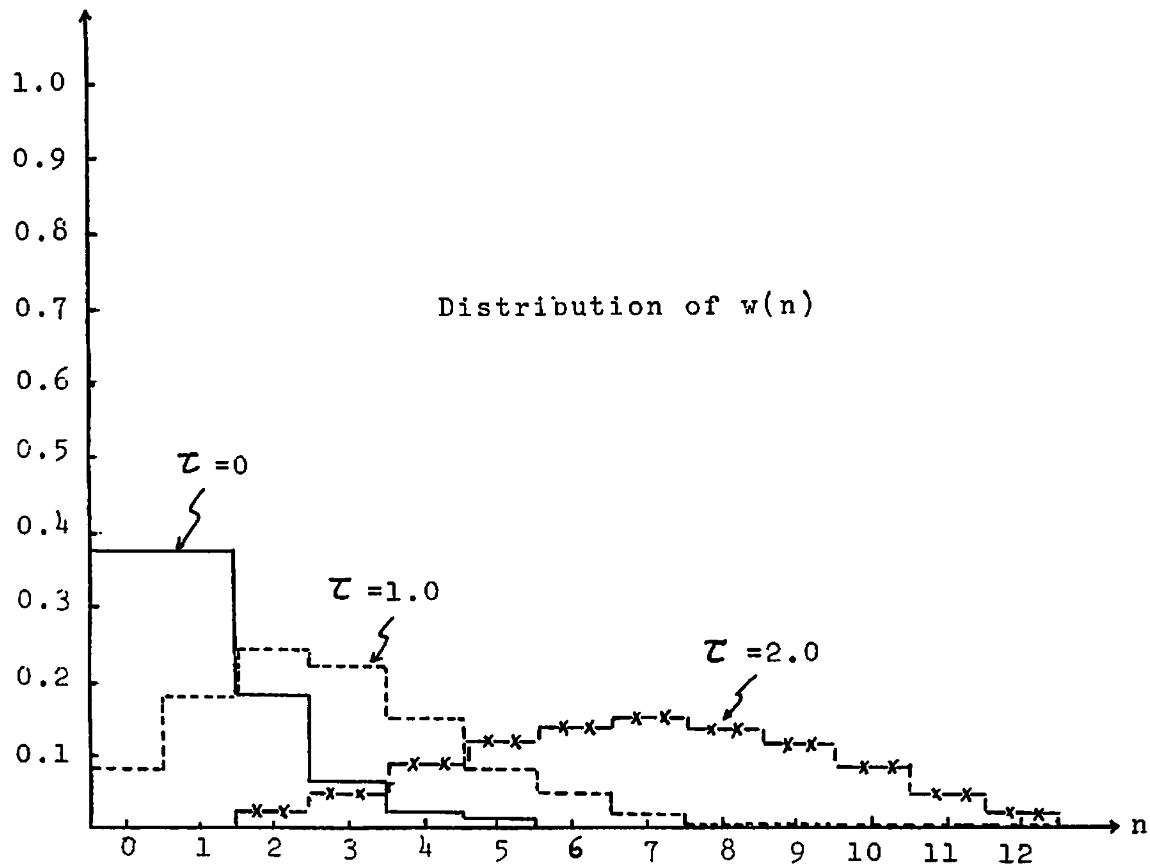

Fig. 1

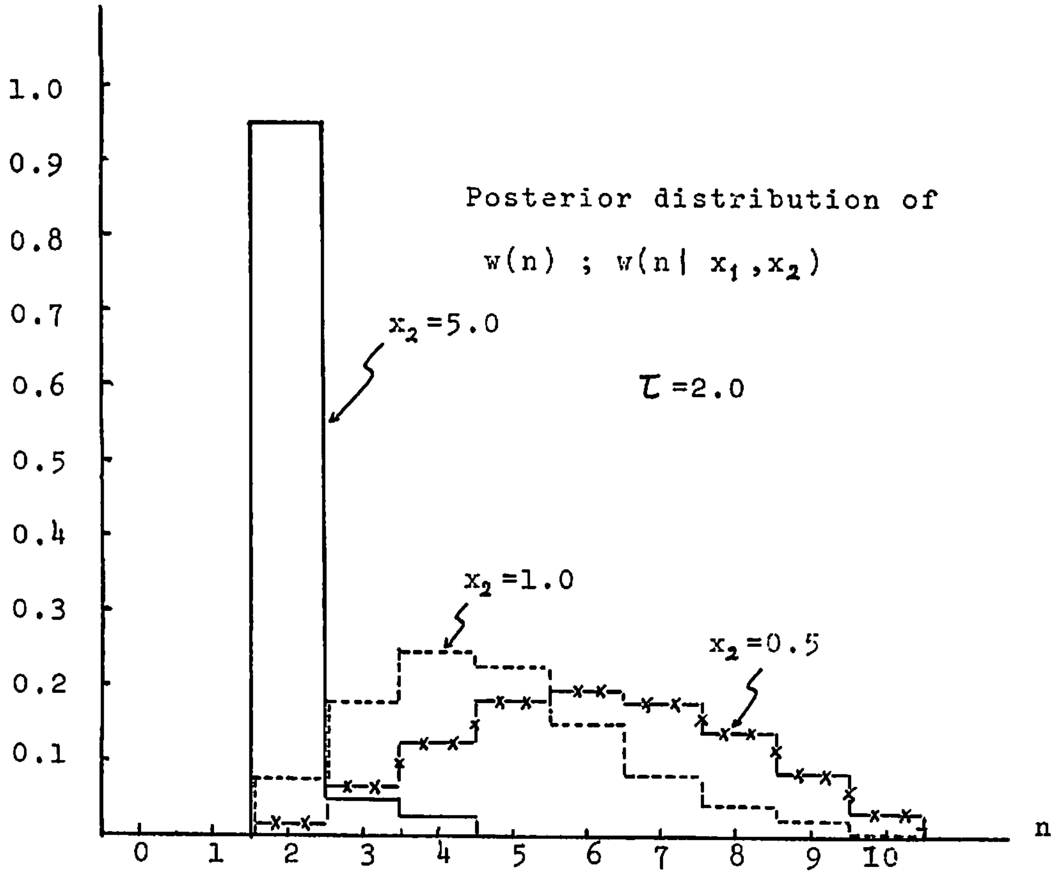

Fig. 2

(Fig. 2).

The Bayes estimator $\hat{n}$ of n is

$$\hat{n}=\sum_{n=j}^{\infty} n\ w(n|x_1,\cdots\cdots\cdots,\ x_j)=j+\exp(\tau-x_j). \qquad (3)$$

Then we conclude, from (3), that sampling is stopped at $j-1$ st failure such that $\hat{n}-j=\exp(\tau-x_j)\leq c$, that is, $x_j\geq\tau-\log c$. Procedure I becomes the constant time censoring procedure. Letting J be tha number of failure, tha mean number of failure at time T is (see Appendix A)

$$E[J]=\exp(\tau)\{1-\exp(-T)\}.$$

3. Procedure II: GUT procedure.

Second, we examine Procedure II, which is given as follows; This procedure was saggested by R. Marcus & S. Blumenthal [2]. The stopping rule of this procedure is paticulally simple, Let datas be $X_1, X_2,\cdots\cdots\cdots, X_j$ as above, so that $X_1\leq X_2\leq\cdots\cdots\cdots\leq X_j$ are the order statistics from an exponential distribution with deusity exp $(-y)$ $(y\geq 0)$.

Let $W_i=X_i-X_{i-1}$ $(X_0\equiv 0)$ be tha i-th waiting time (after transformation) between failures $(1\leq i\leq n)$. Sampling is stopped as soon as some waiting time $W_i\geq t^*$ where t^* is a given constant. Letting $(J+1)$ be the first $i\geq 1$ such that $(W_i\geq t^*)$, the number of items remaining N_r is $(n-J)$.

The distribution of the number of failures J is given by

$$P[J=k|n]=\prod_{j=n-k+1}^{n}[1-\exp(-jt^*)]\exp[-(n-k)t^*],\ (k=1,\cdots\cdots\cdots,\ n).$$

To compare above two procedure, we also suppose in Procedure II the same prior distribution of n as Procedure I, that is,

$$w(n)=\exp\{\tau n\}/\exp\{\exp(\tau)\}n!.$$

The unconditional distribution of J is given by

$$P[J=k]=\sum_{n=k}^{\infty}P[J=k|n]w(n).$$

And the expectation of J is given by

$$E[J|t^*]=\sum_{k=1}^{\infty}k\cdot P[J=k]$$

Let $E[W_j|n,\ t^*]$ be the expectation of the j th waiting time given that it does not exceed T,

$$W[W_j|n,\ t^*]=(n+1-j)^{-1}-t^*[\exp((n+1-j)t^*)-1]^{-1}$$

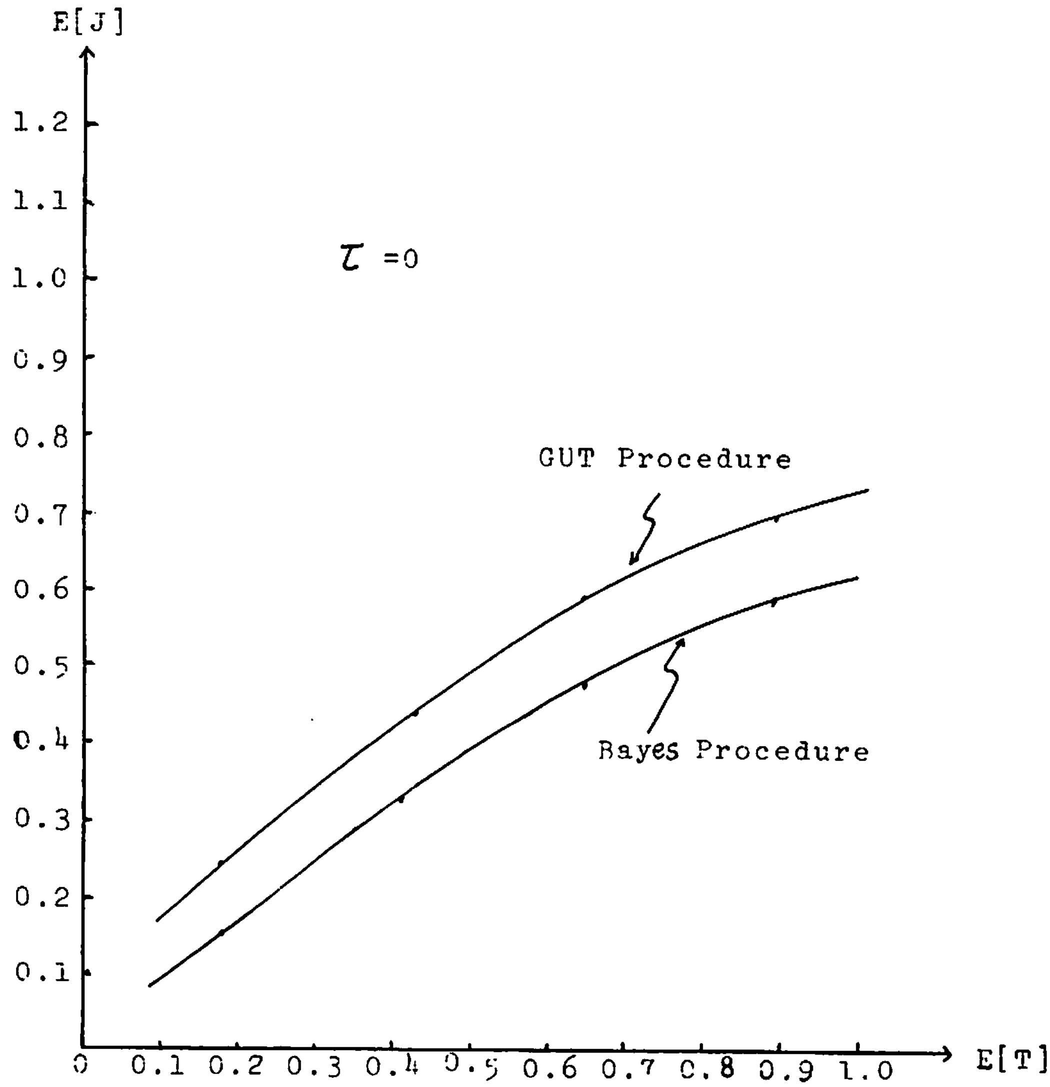

Fig. 3

Letting T represent the random duration of the sampling procedure given n and t^*,

$$E[T|n,\ t^*]=t^*+\sum_{k=0}^{n-1}p[J-n=k]\sum_{j=1}^{n-k}E[W_j|n,\ t^*].$$

The uncondition al expectation of T is given by

$$E[T|t^*]=\sum_{n=0}^{\infty}E[T|n,\ t^*]w(n)$$

4. Comparison of Procedure I & II.

I this section, we compare Procedure I & II in accordance with two criterions (a) Expectation of duration T, (b) Expectation of number of failures.

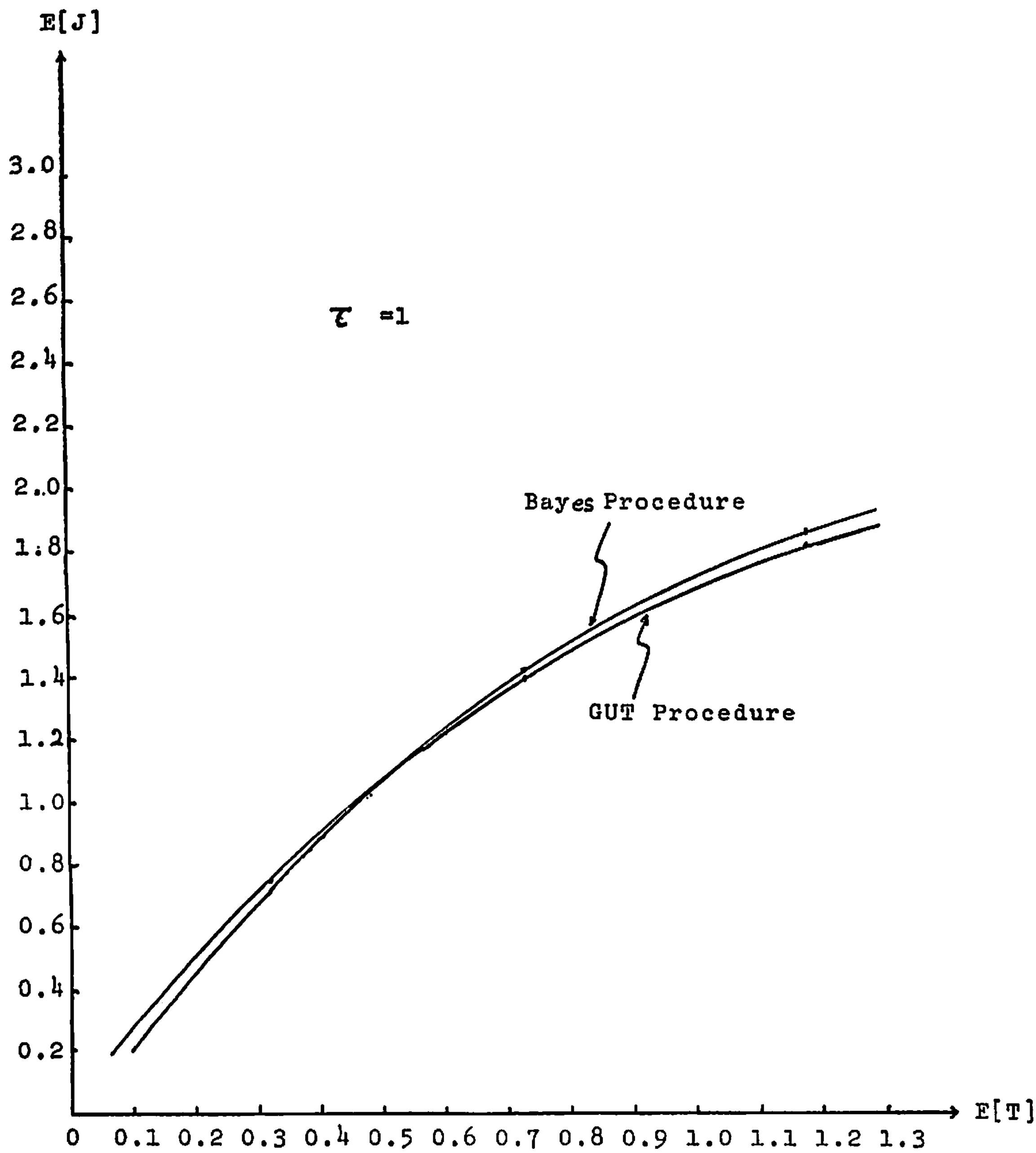

Fig. 4

The above (a), (b) criterions are represented by the ($E[T]$, $E[J]$) space. From Fig. 3. 4. 5, if τ is less than 1, Procedure I (Bayesian procedure) is preferable to Procedure II (GUT procedure), But if τ is above 1, the reverce is true.

Appendix A.

Let $X(t)$ be the number of items remaining at time t.

The distribution of $X(t)$, given $X(0)=n$, is

$$P_r\{X(t)=k|X(0)=n\}=\binom{n}{k}\exp(-kt)(1-\exp(-t))^{n-k}. \qquad (A-1)$$

Letting $Y(t)=n-X(t)$ represent the number of failures at time t, the distribution of $Y(t)$ is given by

$$P_r\{Y(t)=i|Y(0)=0\}=\binom{n}{i}(1-\exp(-t))^i\exp\{-(n-i)t\}.$$

Let $p(t)=\exp(-t)$, $q(t)=1-p(t)$.

Putting $P[Y(t)=i|n]=P[Y(t)=i|Y(0)=0]$, we have

$$P[Y(t)=i|n]=\binom{n}{i}[q(t)]^i[p(t)]^{n-i},$$

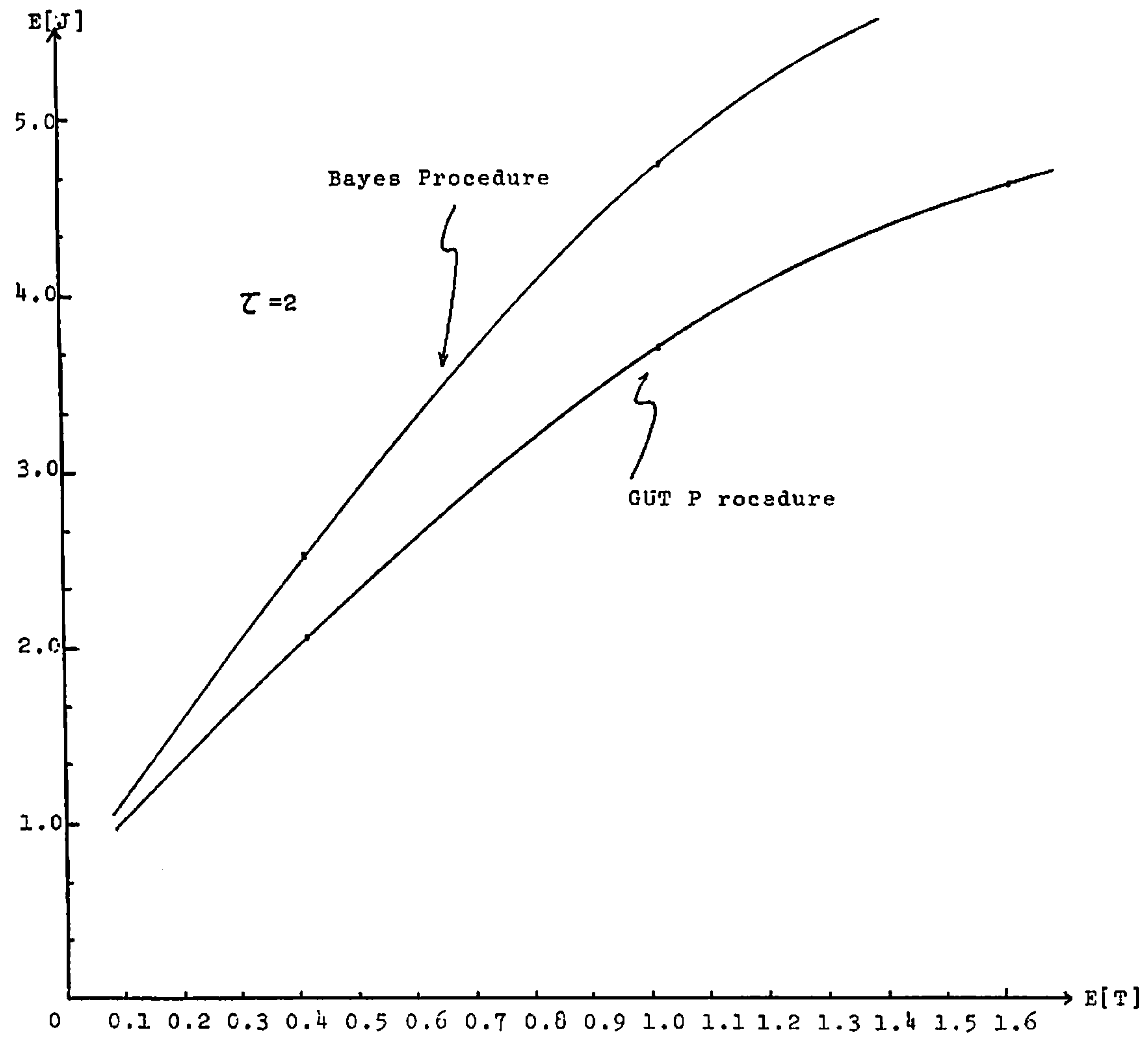

Fig. 5

Let $P[Y(t)=i]=\sum_{n=0}^{\infty}P[Y(t)=i|n]w(n)$, where $w(n)$ is the conjugate prion distribution of n.

Let $\varphi_t(u)$ be the charactristic function of $P[Y(t)=i|n]$ and $\phi(u)$ that of $w(n)$.

Thus

$$\phi(u)=\sum_{n=0}^{\infty}e^{iun}w(n)=\frac{\exp\{\exp(iu+\tau)\}}{\exp\{\exp(\tau)\}},$$

and

$$\varphi_t(u)=\sum_{n=0}^{\infty}(q(t)e^{iu}+p(t))^n w(n)$$

$$=\phi[\frac{\ell_n(q(t)e^{iu}+p(t))}{i}]$$

$$=\exp\{(q(t)e^{iu}+p(t)-1)\exp(\tau)\}.$$

The expectation of $Y(t)$ is $m_1=\frac{\varphi_t'(0)}{i}=q(t)\exp(\tau)=(1-\exp(-t))\exp(\tau)$.

References

(1) Barlow, R., Madnsky, A., Proschan, F. and Scheuer, E. (1968). "Statistical estimation procedures for the 'burn-in' process", Technomatrics 10, 51-62.

(2) Marcus, R. and Blumenthal, S. (1974). "A sequential Screening procedure", Technometrics 16, 229-234.