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(出版者 / Publisher)
法政大学工学部
(雑誌名 / Journal or Publication Title)
法政大学工学部研究集報 / 法政大学工学部研究集報
(巻 / Volume)
35
(開始ページ / Start Page)
15
(終了ページ / End Page)
29
(発行年 / Year)
1999-03
(URL)
https://doi.org/10.15002/00003792

A simple vehicle dynamic model for real-time computer simulation

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Abstract

This paper describes how to derive a simple but accurate model of four-wheeled vehicle via an object-oriented technique for the purpose to analyze and design the vehicle control systems. The model is derived by the Newton-Euler method and given by a set of non-linear differential equations. The model has the 16-degree of freedom including three rotational, transitional dynamics around the vehicle center of mass, four wheel rotations, suspension deflection as well as x-y rotation of the front wheels. In the object-oriented approach, a well-known standard modeling software is used. An example of straightforward analysis and a design of vehicle dynamic system are shown. The model and the approach examined, show the capability of virtual real-time simulation and the validity of the model.

Keyword: vehicle dynamics, simulation, object oriented design, GUI programming

1.Introduction

The commercial vehicle market has increasingly required and/or demanded the innovative and advanced vehicle control for the active safety purpose. Many vendors have developed the advanced technologies such as ABS[1], TCS, 4WD, 4WS, air-bag etc., which are already available in the commercial market. The successful implementation of such the advanced technologies have to take into account the dynamics of vehicle-body with rigid and elastic substructures. Their behavior is inherently non-linear over their operational range. It is commonly recognized that effective design of such the vehicle and their control strategies relies on computer analysis for composing and screening alternative design concepts before constructing expensive prototypes [2]. Thus, they have developed considerable computational tools to support the analysis and the embedded control. Many venders have developed the vehicle dynamics models and dynamics analysis software for the vehicle. For example, the vehicle dynamics model software called ADAMS and the vehicle model derived by Applied dynamics[3] are typical and well known. These models can be customized, but the details are not open because of company proprietary. In addition, in order to increase accuracy in simulations, the knowledge of numerical algorithm characteristics in the software is prerequisite. The DADS can analyze the multi-body dynamics and can handle general model, in turn, it required the long calculation time, which is not suitable for developing the real time simulation and/or control scheme.

In control community, many researchers use the simplified two-wheel vehicle dynamics model which neglects the effect of chassis rolling effects[4]. However, for the more advanced technologies such as Vehicle Stability Control (VSC), de-coupling control of vehicle suspension, are necessary to take into account the four wheeled vehicle dynamics in order to get the rolling effects [5].

In this paper, we develop the four wheeled vehicle dynamics model. The modeling is based on the object oriented technique by using the SIMLUINK block diagram like GUI based representation.

2. General scheme, definition and assumptions for vehicle modeling

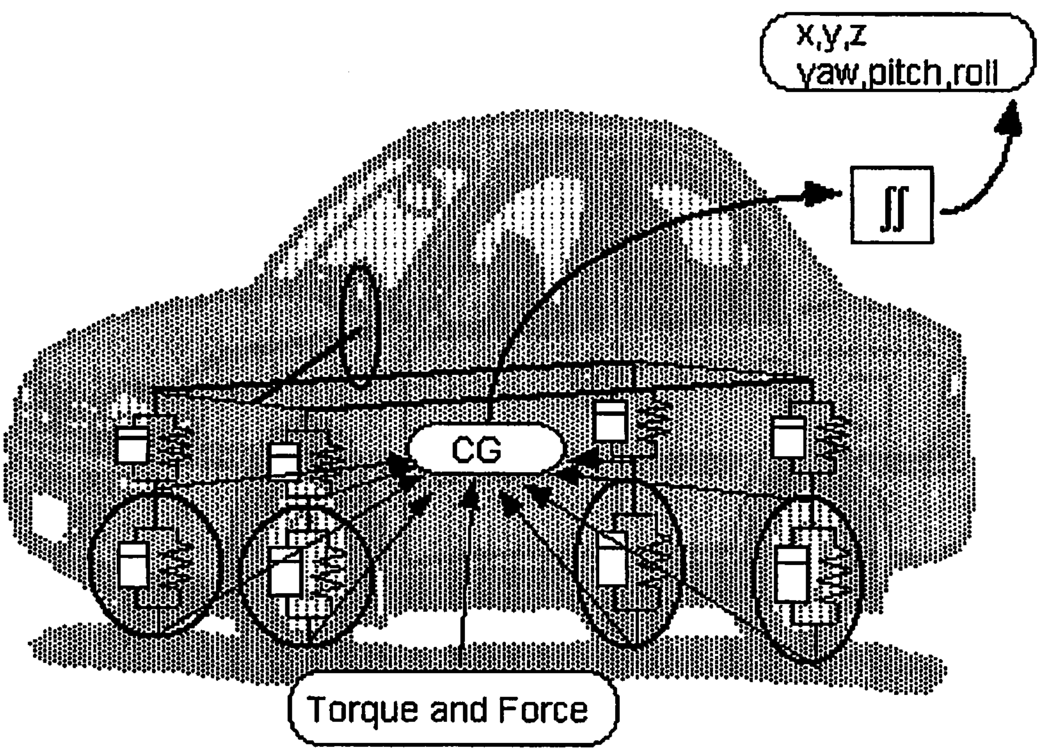

Figure 1 Basic scheme of the simple vehicle dynamics

In normal driving situation, we can neglect the effect of distortion of chassis. To take into account of the speed of calculation as well as the easiness to implement the vehicle dynamics, we assume the lumped mass vehicle model in center of gravity. Due to the D'Alembert principle, it will act the effect the translation as well as rotation acts to the center of mass. Figure 1 shows the overview of proposed vehicle dynamics model. Force and torque are produced by each part of chassis and tire and suspension. From the summation of each part of the force and the torque, we can simulate the behavior around the center of vehicle by calculating the double integral of the summed force and torque.

2.1 Definition of frame for the proposed vehicle dynamics model

Figure 2 shows the coordinate and frame for proposed vehicle model, whose scheme is shown in Figure 1. In order to describe the vehicle dynamics model, we employ the three different coordinate frames. I.e.,A frame which describes the origin point as a vehicle center of a mass, N frame which describes the absolute coordinate, and W frame whose coordinate is determined by tire direction each. To describe the direction of vector and frame, we use following notation; ${}^{Frame}Vector_{to}^{from}$, "from,to" describes the direction of vector which connects each links. To transform the coordinate, frame transformation matrix R with the following notation is used; ${}^{to(Frame)}R_{from(Frame)}$. To define the vehicle kinematics, six reference frames are employed: an inertial reference frame N, a body fixed reference frame A and four-wheel fixed reference frames W. The inertial coordinate is on an absolute coordinate such as ground surface. The body fixed coordinate has its origin at the vehicle center of gravity. The x-axis points forward and forms the roll axis of the vehicle, the y-axis points to the right and forms the pitch axis and the z direction is straight upwards and forms the yaw axis of the vehicle body.

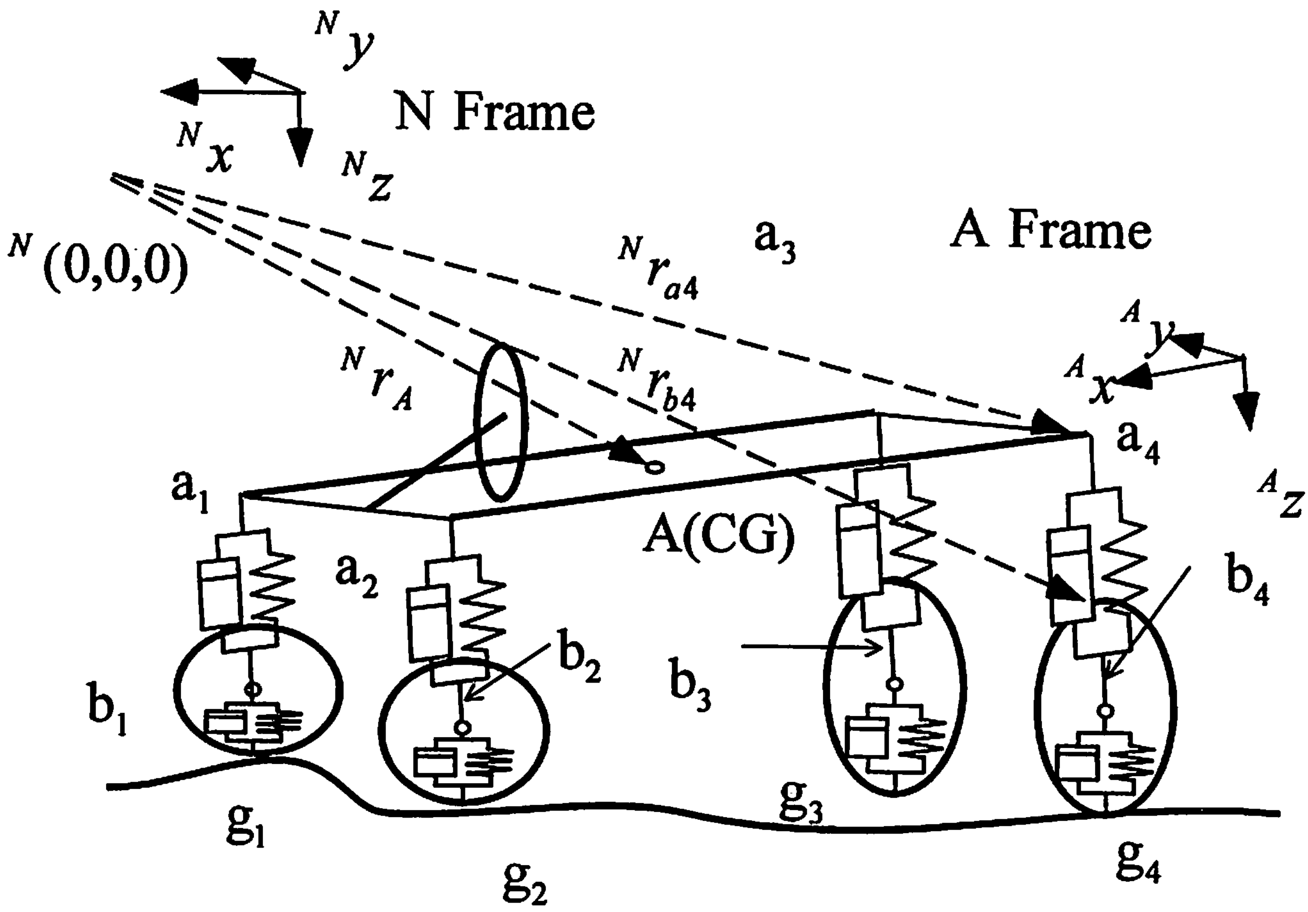

Figure 2 Coordinates for the vehicle dynamics model

[A frame : body coordinate system and definition of angular velocity]

[${}^{A}x$] point to the forward to be positive.

[${}^{A}y$] point to the right to be positive.

[${}^{A}z$] point to upwards to be positive

[${}^{A}\omega_x$] roll angular velocity around ${}^{A}x$ axis

[${}^{A}\omega_y$] pitch angular velocity around ${}^{A}y$ axis

[${}^{A}\omega_z$] yaw angular velocity around ${}^{A}z$ axis

[N frame : inertial coordinate and the corresponding absolute angle]

[ϕ] absolute angle around ${}^{N}x$ axis

[θ] absolute angle around ${}^{N}y$ axis

[φ] absolute angle around ${}^{N}z$ axis

[W frame : tire fixed coordinate]

W frame is defined on each wheel independently. The z-axis is same as N frame coordinate.

First, we will explain the coordinate system in Figure 2 for the rear left side wheel. Figure 2 shows the geometry relationship of left rear wheel b_4 and it's suspension position a_4 and ground position g_4 as well as the relationship between the inertia frame N and body frame A. The vector from the origin of N frame to the rear left side of suspension a_4 can represent the distance vector as ${}^{N}r_{a4}$. By using transformation matrix ${}^{N}R_{A}$, the distance vector ${}^{A}r_{a4}^{b4}$ in A frame can be translated to the distance vector ${}^{N}r_{a4}^{b4}$ in N frame as follows:

$$ {}^{N}r_{a4}^{b4} = {}^{N}r_{A} + {}^{N}R_{A}\,{}^{A}r_{a4}^{b4} \qquad (1) $$

2.2 Definition of variables and constants for vehicle model

We define following variables and constants for the proposed vehicle dynamics model. In order to distinguish the front and rear as well as left and right of position of the vehicle parts, we define suffix as i = 1 to 4. i.e., 1(front right),2(front left), 3(rear right), 4(rear left)

[General]

[s] Laplace transform operator , [g] acceleration of gravity,[×] cross product of matrix

[Constant and variables for vehicle body dynamics]

[I] inertial matrix around the center of gravity of vehicle body

[M_{tot}] mass matrix for the vehicle

[B_{air}] air-drag coefficient matrix for high speed driving.

[${}^{N}x$] the vehicle body position in inertial frame.

[${}^{N}\dot{x}$] the vehicle body speed in inertial frame.

[${}^{N}R_{A}$ ${}^{A}R_{N}$ ${}^{W}R_{A}$] frame transformation matrix

[${}^{N}r_{A}$] vector from the vehicle center of mass in inertial frame origin.

[Suspension model]

[r_{ti}]tire radius

[${}^{N}U$] unit vector in frame A is transformed into inertial frame N

[${}^{N}U(z)$] ${}^{N}U$ z axis component of U vector

[${}^{N}r_{gi}$] ground contact point coordinate of each tire

[B_{sus}] suspension dumping coefficient.

[$f_{sus}()$] force effect to suspension.

[R_{sr} , R_{sf}]anti-rolling coefficient.

[$r_{sus,i} = \left| {}^{A}r_{ai}^{bi} \right|$] the nominal suspension length

[${}^{A}f_{roll}$] anti-roll force

[Tire model]

[B_{wi}] tire dumping coefficient

[$f_{tire}()$] force that acts to tire.

[I_{wi}] inertial moment of each tires

[M_W] tire mass

[Tire ground interaction due to friction model]

[μ_x , μ_y] friction coefficient

[${}^{A}\omega(z)$] yaw angular velocity

[Tqd , Tqb] driving/braking torque

[${}^{W}x_{xi}$] tire directional speed in tire frame.

[sgn()] sign function

[$\alpha_{slip,i}$, $r_{slip,i}$] slip angle/ slip ratio

2.3 Assumption for derivation for vehicle modeling.

To derive a simple vehicle dynamics model, we cite following assumption for tire and suspension.

(A1) The suspension length changes only body z-axis in frame A.

(A2) Tires always contact to the ground.

(A3) Tire radius changes only on z-axis in frame N.

(A4) The coefficient of ground and tire friction is always constant.

(A5) The tire-ground contact position does not change due to the mass of vehicle body.

By assumption(A1), we mean suspension is of ideal double-wishbone. Assumption(A2) is required to simplify the calculation of the tire radius in the vehicle dynamics model. By assumption(A3), we mean the chamber angle of wheel is zero. Assumption(A4) is just to simplify the model. If we can know the friction coefficient for each tire/ground position, this assumption is not required. In usual driving situation, assumption(A5) is reasonable.

3. Vehicle dynamics modeling by using object oriented approach

3.1 Suspension dynamics

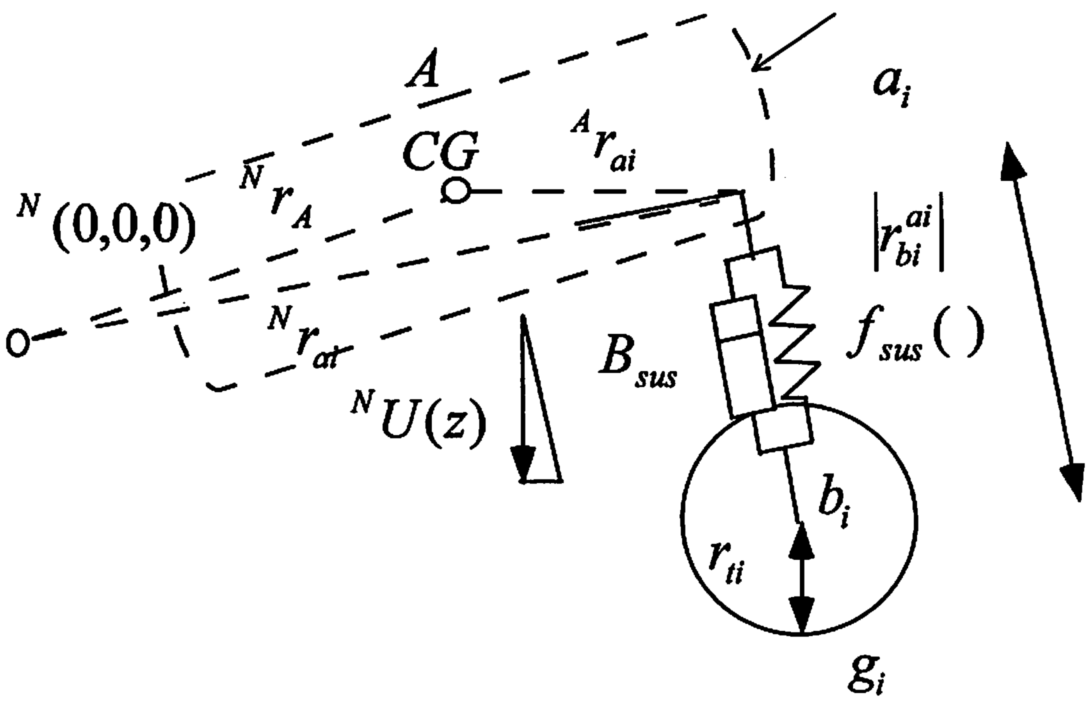

Figure 3 Schematics of suspension dynamics

Figure 3 shows relationship about the vector which describes about the suspension. The link position a_i connects to the suspension and the vehicle body is described by as follows:

$$ {}^{N}r_{ai} = {}^{N}r_A + {}^{N}R_A \, {}^{A}r_{ai} \tag{2} $$

From assumption (A1), the deflection of the suspension influences only to z-axis in A frame. To calculate the suspension position in N frame, the ratio is calculated by using the unit vector in A frame to transform in inertial coordinate in N frame.

$$ {}^{N}U = {}^{N}R_A [0 \quad 0 \quad 1]^T \tag{3} $$

From assumption(A2), the length of suspension is given by

$$ \left| {}^{A}r_{ai}^{bi} \right| = \frac{{}^{N}r_{ai}(z) - {}^{N}r_{gi}(z) + r_{ti}}{{}^{N}U(z)} \tag{4} $$

From the suspension length $\left|{}^{A}r_{ai}^{bi}\right|$, we can define the force $f_{sus}()$ assuming general spring-mass-dumped model.

$$
{}^{A}f_{ai}^{bi}=\frac{{}^{A}r_{ai}^{bi}}{\left|{}^{A}r_{ai}^{bi}\right|}\left(f_{sus}\left(\left|{}^{A}r_{ai}^{bi}\right|\right)-B_{sus}\frac{d\left|{}^{A}r_{ai}^{bi}\right|}{dt}\right) \tag{5}
$$

3.1.1 Anti-roll mechanism of vehicle body

When a vehicle drives the curve, we can feel side acceleration which is enhanced by vehicle body rolling. The rolling make driver scared as well as it is bad as the behavior of vehicle dynamics. Therefore, the vehicle inherently implements the anti-rolling mechanism to suppress the rolling effect. For modeling the anti-rolling mechanism, first we define following variables and constants. This mechanism is determined by body width and roll hardness coefficient. We calculate the anti-roll force ${}^{A}f_{roll}$ as follow;

$$
{}^{A}f_{roll}=R_{sf}\left(r_{sus1}-r_{sus2}\right)+R_{sr}\left(r_{sus3}-r_{sus4}\right) \tag{6}
$$

In addition, calculate the force ${}^{N}f_{sus}$ and the torque ${}^{A}t_{ai}$ given by suspension movement or deflection as follows.

$$
{}^{N}f_{ai}^{bi}={}^{N}R_{A}\left({}^{A}f_{ai}^{bi}+{}^{A}f_{roll}\right) \tag{7}
$$

$$
{}^{A}t_{ai}={}^{A}r_{ai}\times\left({}^{A}f_{ai}^{bi}+{}^{A}f_{roll}\right) \tag{8}
$$

3.2 Interaction between tire and ground dynamics

3.2.1 Dynamics of tire

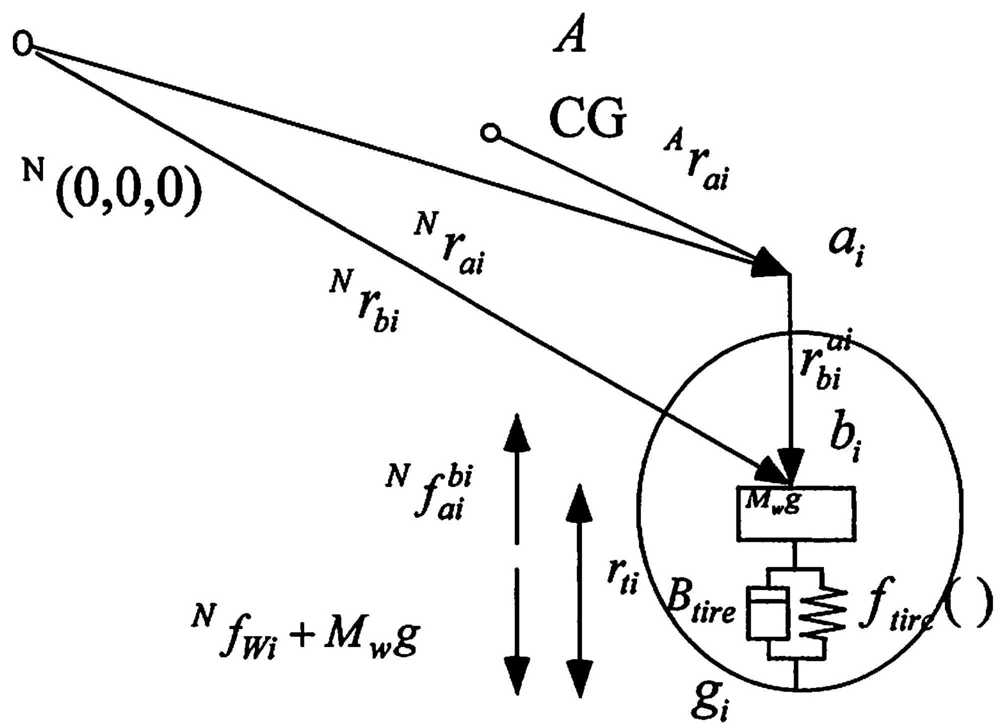

Figure 4 Schematics of tire dynamics

Figure 4 shows the geometry relationship between the tire and suspension. The link point between tire and suspension b_i is represented in N frame as follows;

$$^{N}r_{bi}={}^{N}R_{A}\left[{}^{A}r_{ai}+\begin{bmatrix}0\\0\\\left|{}^{A}r_{ai}^{bi}\right|\end{bmatrix}\right]+{}^{N}r_{A} \tag{9}$$

From assumption (A3), the tire radius of r_{ti} can be obtained by double integral of force around z-axis in inertial frame N.

$$
{}^{N}f_{wi}=\begin{bmatrix}0\\0\\f_{tire}\left(\left|{}^{N}r_{bi}^{gi}\right|\right)-B_{tire}\dfrac{d\left|{}^{N}r_{bi}^{ai}\right|}{dt}\end{bmatrix} \tag{10}
$$

$$
r_{ti}=\frac{-1}{s^{2}M_{W}}\begin{bmatrix}0 & 0 & 1\end{bmatrix}\left[{}^{N}f_{Wi}+\begin{bmatrix}0\\0\\M_{W}g\end{bmatrix}-{}^{N}f_{ai}^{bi}\right] \tag{11}
$$

3.2.2 Tire and ground friction model

It is difficult to analyze the interaction between the tire and ground, since the effect is caused by fuzzy friction. Many researchers developed the model for the tire/ground interaction, such as Fiala, Pacejka[6] and Dugoff[7]. In this paper, we adopt the Dugoff model, which explains by using the lateral slip. Dugoff model provides the friction coefficient by the slip angle and slip ratio in tire and ground contact point. The slip angle and slip ratio is calculated from absolute speed in N frame at tire/ground contact point and the tire rotational speed. By using the slip angle and slip ration, the friction force is calculated by using Dugoff tire model.

(a) Absolute velocity at tire/ground contact point.

The velocity at the tire ground contact point in W frame can be approximated by the following equation.

$$
{}^{W}v_{gi}={}^{W}R_{A}\left({}^{A}R_{N}\,{}^{N}\dot{x}+\begin{bmatrix}0\\0\\{}^{A}\omega(z)\end{bmatrix}\times{}^{A}r_{ai}\right) \tag{12}
$$

When i = 3,4 ,i.e., rear wheels form eq.(12), rear wheel cannot rotate neither to left nor right side which leads to the transformation matrix unity.

(b) Tire revolution speed

The tire revolution speed is determined from several turquoises, i.e., engine torque, brake torque, inertial moment of tire itself, and reflection force caused by friction from the ground. The tire revolution speed ${}^{W}v_{xi}$ is obtained by multiplying the tire rotational speed and the tire radius.

$$
{}^{W}v_{xi}=r_{ti}\frac{Tqd-Tqb\,\mathrm{sgn}\left({}^{W}v_{xi}\right)-{}^{W}f_{xi}r_{ti}}{sI_{wi}+B_{wi}} \tag{13}
$$

3.2.3 Estimation of slip ratio $r_{slip,i}$ and slip angle $\alpha_{slip,i}$

Tire slip angle caused by tire slip, is always zero in tire fixed frame. Thus, all we must care is the body velocity on x-y axis plane at W frame.

$$\alpha_{slip,i} = \arctan 2\left({}^{W}v_{gi}(y), {}^{W}v_{gi}(x)\right) \tag{14}$$

$$r_{slip,i} = \frac{{}^{W}v_{xi}(x) - \left|{}^{W}v_{gi}\right|}{\max({}^{W}v_{xi}, \left|{}^{W}v_{gi}\right|)} \tag{15}$$

3.2.4 Friction coefficient μ by Dugoff friction model

In the Dugoff tire model, the friction coefficient is obtained from the slip ratio and slip angle. A ground and tire friction coefficient are constant from assumption (A4), we use look up table for μ calculation. Each friction coefficient μ_x, μ_y has different characteristics given by look up table[8].

3.2.5 Force and torque calculation at tire contact point

From the obtained friction coefficient μ and each tire force f_{Wi}, we can calculate the torque and the force around the center of the mass of the vehicle. From assumption (A5), the ground position will not change due to the weight of vehicle. Therefore the calculated slip forces in z-axis in W frame(which is same as z-axis in N frame) and the velocity can be set to zero. In eq.(16), the slip force ${}^{W}f_{gi}$ is effect only for x and y axis in W frame.

$${}^{W}f_{gi} = \begin{bmatrix} \operatorname{sgn}\left({}^{W}v_{xi} - {}^{W}v_i(x)\right)\mu_x\left(\left|r_{slip,i}\right|, \left|\alpha_{slip,i}\right|\right) \\ -\operatorname{sgn}\left({}^{W}v_i(y)\right)\mu_y\left(\left|r_{slip,i}\right|, \left|\alpha_{slip,i}\right|\right) \\ 0 \end{bmatrix} {}^{N}f_{Wi} \tag{16}$$

The tire/ground contact point ${}^{A}r_{gi}$ can be expressed by following equation.

$${}^{A}r_{gi} = {}^{A}r_{ai} + \begin{bmatrix} 0 \\ 0 \\ r_{ti} + \left|{}^{A}r_{ai}^{bi}\right| \end{bmatrix} \tag{17}$$

The tire/ground interaction force and torque can be calculated as follows:

$${}^{N}f_{gi} = {}^{N}R_A\left({}^{A}R_W\, {}^{W}f_{gi}\right) \tag{18}$$

$$ {}^{A}t_{gi} = {}^{A}r_{gi} \times \left({}^{A}R_{W} \, {}^{W}f_{gi} \right) \tag{19} $$

3.3 The equation of motion of the center of gravity

The total torque and force are obtained from eq(8) and (9) that use the suspension eqs. (19),(18). The suspension equations obtained from tire/ground interaction. The total torque and force act to the center of mass of the vehicle. From the D'Alembert principle, the double integral of translation and rotation eqs.(20),(21) yields the position and rotation angle of the vehicle around the center of the mass.

$$ M_{tot} \, {}^{N}\ddot{x} + B_{air} \, {}^{N}\dot{x}^{2} = \sum_{i=1}^{4} \left({}^{N}f_{ai}^{bi} + {}^{N}f_{gi} \right) \tag{20} $$

$$ I \, {}^{A}\dot{\omega} + {}^{A}\omega \times I \, {}^{A}\omega = \sum_{i=1}^{4} \left({}^{A}t_{ai} + {}^{A}t_{gi} \right) \tag{21} $$

(a) From eq.(20), the double integral of force which acts to the center of mass yields the position. When the vehicle moves relatively fast, the effect of air drag cannot be neglected. In order to take the effect of air drag into account, we add the air-drag block whose resistive characteristic is proportional to the square of velocity of the vehicle.

(b) The calculated instantaneous angular velocity ${}^{A}\omega$ is different from the derivatives of Euler angle in N frame.

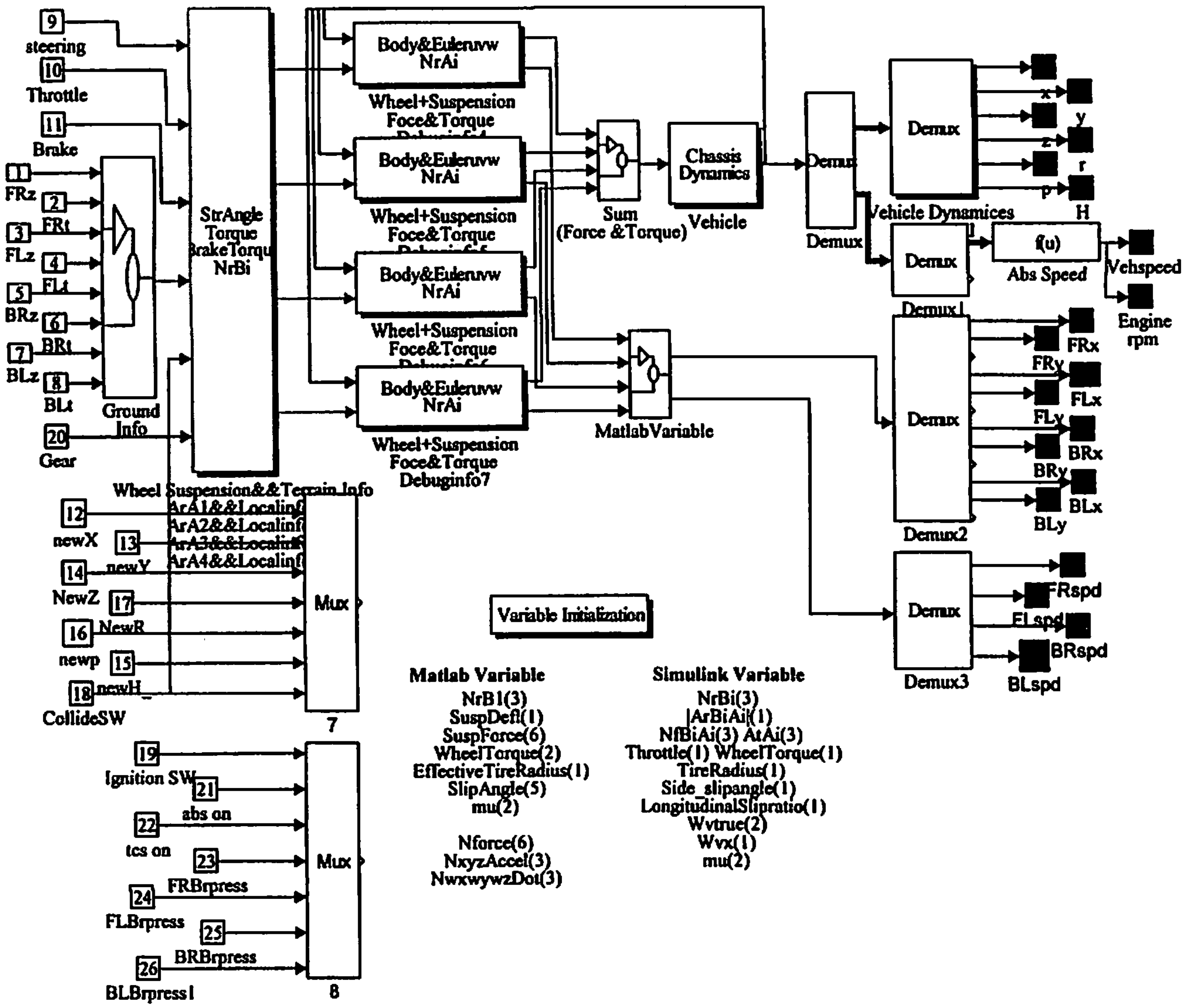

Figure 5 Developed whole vehicle dynamics model

In Figure 5, the SIMULINK blocks of the whole vehicle dynamics are shown. The four suspensions and tires object blocks are connected to the chassis dynamics block, which produce the dynamical vehicle behavior.

4 Simulation

4.1 Example of 3 dimensional vehicle dynamics simulation

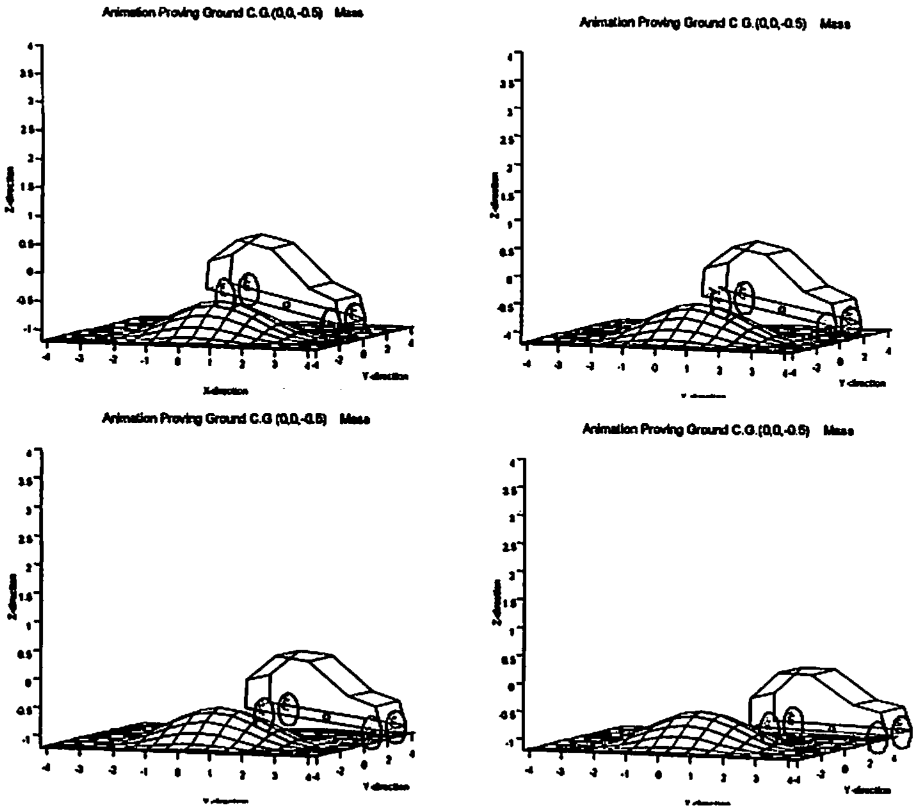

Figure 6 Simulation when bumping on the hill

Figure 6 shows the dynamical behavior when bumping on the hill. The sampling interval is set to 0.01 sec and snapshot interval is 0.2 see. We simulated total 0.8 seconds snapshot. From the three dimensional model, we can simulate any kind of ground situations.

5. Conclusion

In this paper, the vehicle dynamics modeling by using object-oriented approach is shown. The applied model is based on the Newton-Euler method. In order to validate the vehicle dynamics model, we implement vehicle model by using the SIMULINK, a general-purpose control simulation software which have the programming capability of the GUI. The proposed object oriented modeling approach and GUI programming have advantage to design and change the model as well as to make implement action of non-linearity easy. Further to enhance of modeling, we also to make used the software named real time workshop which translates to SIMLINK diagram to C source code. The generated code from real time workshop can simulate 25 % faster than real time (at sampling time is set to 0.01 sec, workstation SGI Max Impact) . In this vehicle dynamics model, we did not take the effect of engine dynamics as well as effect of torque distribution gear into account. These are our further task.

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