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(出版者 / Publisher)
The American Physical Society
(雑誌名 / Journal or Publication Title)
Dynamics & Design Conference / Dynamics & Design Conference
(号 / Number)
2
(開始ページ / Start Page)
024001-1
(終了ページ / End Page)
024001-5
(発行年 / Year)
2001-06
(URL)
https://doi.org/10.1103/PhysRevC.64.024001

Complete set of total cross sections for imaginary parts of *nd* forward scattering amplitudes, and three-nucleon force effects

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(Received 8 November 2000; published 29 June 2001)

In neutron-deuteron scattering, four total cross sections are shown to form a complete set for the determination of the imaginary parts of the forward amplitudes by means of the optical theorem. The amplitudes are decomposed into scalar and tensor components in spin space. Contributions of three-nucleon forces (3NF) to these amplitudes are studied by Faddeev calculations. Significant effects of the 3NF on the tensor components are predicted.

DOI: 10.1103/PhysRevC.64.024001 PACS number(s): 21.45.+v, 21.30.−x, 24.70.+s

In recent years, three-nucleon ($3N$) systems have attracted considerable attention as important sources of information on nuclear interactions, because of possible effects of three-nucleon forces (3NF) in addition to two-nucleon ones. Various versions of the two-nucleon force (2NF) have been examined as the input to $3N$ calculations, and have been found to be deficient in reproducing empirical data of specific physical observables [1]. Improvements have been achieved by introducing a 2π exchange 3NF in the calculation of the $3N$ binding energy [2] and the minimum of the proton-deuteron (pd) elastic scattering cross section between 50 and 200 MeV [3]. However, the prescription is not always effective for polarization phenomena. In fact, the 3NF cannot explain the proton vector analyzing power between 65 and 200 MeV [4] in $\vec{p}d$ scattering, and the deuteron tensor analyzing powers at 270 MeV in $\vec{d}p$ scattering [5]. Also, discrepancies in the nucleon and deuteron vector analyzing powers between calculations and measurements in low energy neutron-deuteron (nd) and pd scatterings still remain unresolved even when a 3NF is included [6–8].

These indicate that a comprehensive understanding of the role of nuclear interactions, particularly that of their spin dependence, in $3N$ observables has not yet been obtained. In the present paper, we consider nd total cross sections, which are specified by spin orientations of projectiles and targets, as a reliable scale for the criticism of the spin dependence of the nuclear interactions. Because of the optical theorem, total cross sections are linked to the imaginary parts of forward scattering amplitudes. In the following, we give a complete set of the nd total cross sections to determine unambiguously the imaginary parts of all of the nd forward amplitudes. Since the spin dependence of the nuclear interactions is reflected in the scattering observables through the spin structure of the scattering amplitudes, we decompose the nd forward amplitudes by spin space tensors, and examine the relation between these components of the amplitudes and corresponding nuclear force components. Finally, effects of 3NF on these components of the amplitudes are studied by $3N$ Faddeev calculations.

In the nd scattering, we have four nonvanishing independent forward amplitudes [9]. Designating elements of the nd scattering matrix as $\langle \nu_n', \nu_d' | \boldsymbol{M} | \nu_n, \nu_d \rangle_{\theta=0}$, where ν's are the z components of the related particles' spins

$$\begin{aligned}
M_1 &= \left\langle \frac{1}{2},1 \middle| \boldsymbol{M} \middle| \frac{1}{2},1 \right\rangle_{\theta=0}, \\
M_2 &= \left\langle -\frac{1}{2},1 \middle| \boldsymbol{M} \middle| -\frac{1}{2},1 \right\rangle_{\theta=0}, \\
M_3 &= \left\langle -\frac{1}{2},1 \middle| \boldsymbol{M} \middle| \frac{1}{2},0 \right\rangle_{\theta=0} = \left\langle \frac{1}{2},0 \middle| \boldsymbol{M} \middle| -\frac{1}{2},1 \right\rangle_{\theta=0}, \\
M_4 &= \left\langle \frac{1}{2},0 \middle| \boldsymbol{M} \middle| \frac{1}{2},0 \right\rangle_{\theta=0}.
\end{aligned} \tag{1}$$

Then, in principle, four kinds of independent total cross sections completely determine the imaginary parts of the forward scattering amplitudes by the optical theorem.

Denoting the spin density matrices of the neutrons and the deuterons [10] in the initial state by $\rho^{(n)}$ and $\rho^{(d)}$, respectively, the total cross section σ^{tot} is given as

$$\sigma^{tot} = \alpha \, \mathrm{Im}\{\mathrm{Tr}(\rho^{(n)}\rho^{(d)}\boldsymbol{M}_{\theta=0})\} \quad \text{with} \quad \alpha = \frac{4\pi}{k}, \tag{2}$$

where k is the magnitude of the nd relative momentum $\boldsymbol{k}$. One of the independent total cross sections is that for unpolarized neutrons and deuterons σ_0^{tot} [11,12], for which the spin density matrices are given by

$$\rho^{(n)} = \frac{1}{2} I^{(n)}, \quad \rho^{(d)} = \frac{1}{3} I^{(d)}, \tag{3}$$

where $I^{(n)}$ and $I^{(d)}$ are the unit matrices. We get from Eq. (2)

$$\sigma_0^{tot} = \frac{\alpha}{3} \, \mathrm{Im}(M_1 + M_2 + M_4). \tag{4}$$

Next, we choose the longitudinal ($\Delta\sigma_L$) and the transversal ($\Delta\sigma_T$) asymmetries of the total cross section for vector-polarized neutrons and deuterons, for which noticeable contributions of the 3NF have been predicted [13]. When the neutrons and the deuterons are vector polarized in the same direction along the z axis with polarizations $p_z^{(n)}$ and $p_z^{(d)}$, the corresponding cross section $\sigma_L^{tot}(p_z^{(n)},p_z^{(d)})$ is obtained by Eq. (2) with the spin density matrices

$$\rho^{(n)}=\frac{1}{2}p_z^{(n)}\sigma_z, \quad \rho^{(d)}=\frac{1}{2}p_z^{(d)}P_z, \tag{5}$$

where

$$P_z=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{6}$$

On the other hand, when the neutrons and the deuterons are vector polarized in the same direction perpendicular to the z axis with polarizations $p_y^{(n)}$ and $p_y^{(d)}$, one can choose the y axis as the polarization direction. Then the corresponding cross section $\sigma_T^{tot}(p_y^{(n)},p_y^{(d)})$ is obtained by Eq. (2) with the spin density matrices

$$\rho^{(n)}=\frac{1}{2}p_y^{(n)}\sigma_y, \quad \rho^{(d)}=\frac{1}{2}p_y^{(d)}P_y, \tag{7}$$

where

$$P_y=\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}. \tag{8}$$

The cross section asymmetries are defined as the cross section difference provided by the reversal of the deuteron spin direction:

$$\Delta\sigma_L=\sigma_L^{tot}(+1,-1)-\sigma_L^{tot}(+1,+1),$$
$$\Delta\sigma_T=\sigma_T^{tot}(+1,-1)-\sigma_T^{tot}(+1,+1), \tag{9}$$

which are equivalent to the asymmetries in Ref. [13]. Using Eqs. (2) and (9) with Eqs. (5), (6), (7), and (8), we obtain

$$\Delta\sigma_L=-\alpha\,\mathrm{Im}(M_1-M_2), \tag{10}$$

and

$$\Delta\sigma_T=-\alpha\sqrt{2}\,\mathrm{Im}(M_3). \tag{11}$$

As the last one, we consider the total cross section for the scattering of unpolarized neutrons by tensor polarized deuterons. For the unpolarized neutrons and the t_{20} tensor polarized deuterons along the z axis,

$$\rho^{(n)}=\frac{1}{2}I^{(n)}, \quad \rho^{(d)}=\frac{\sqrt{2}}{6}t_{20}P_{zz}, \tag{12}$$

where

$$P_{zz}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{13}$$

We get the total cross section σ_{20}^{tot} for $t_{20}=1$,

$$\sigma_{20}^{tot}=\frac{\alpha}{3\sqrt{2}}\,\mathrm{Im}(M_1+M_2-2M_4). \tag{14}$$

Solving Eqs. (4), (10), (11), and (14), the imaginary parts of M_1–M_4 are obtained:

$$\begin{aligned}
\alpha\,\mathrm{Im}(M_1)&=\sigma_0^{tot}+\frac{1}{\sqrt{2}}\sigma_{20}^{tot}-\frac{1}{2}\Delta\sigma_L,\\
\alpha\,\mathrm{Im}(M_2)&=\sigma_0^{tot}+\frac{1}{\sqrt{2}}\sigma_{20}^{tot}+\frac{1}{2}\Delta\sigma_L,\\
\alpha\,\mathrm{Im}(M_3)&=-\frac{1}{\sqrt{2}}\Delta\sigma_T,\\
\alpha\,\mathrm{Im}(M_4)&=\sigma_0^{tot}-\sqrt{2}\sigma_{20}^{tot}.
\end{aligned} \tag{15}$$

By these equations, one can determine the imaginary parts of the scattering amplitudes M_1–M_4 unambiguously when σ_0^{tot}, $\Delta\sigma_L$, $\Delta\sigma_T$, and σ_{20}^{tot} are measured. The present choice of the set of the independent total cross sections is not unique. However, different choices will produce the information of the scattering amplitudes equivalent to the present one, since the independent amplitudes are restricted to four ones.

In order to get deeper insights into the spin dependence of the interactions, we decompose the scattering matrix $\boldsymbol{M}$ by spin space tensors of rank K with z component κ, $\boldsymbol{S}_{K\kappa}$,

$$\boldsymbol{M}=\sum_{K\kappa}(-)^{\kappa}\boldsymbol{S}_{K,-\kappa}\boldsymbol{R}_{K\kappa}, \tag{16}$$

where $\boldsymbol{R}_{K\kappa}$ is the counterpart, a tensor in the coordinate space. The matrix element of $\boldsymbol{M}$ for a reaction $A(a,b)B$ is given in terms of invariant amplitudes [14]:

$$\begin{aligned}
&\langle \nu_b,\nu_B;\boldsymbol{k}_f|\boldsymbol{M}|\nu_a,\nu_A;\boldsymbol{k}_i\rangle\\
&\quad=\sum_{s_is_fK}(s_as_A\nu_a\nu_A|s_i\nu_i)(s_bs_B\nu_b\nu_B|s_f\nu_f)\\
&\qquad\times(s_is_f\nu_i,-\nu_f|K\kappa)(-)^{s_f-\nu_f}\\
&\qquad\times\sum_{l_i=\bar{K}-K}^{K}[C_{l_i}(\hat{k}_i)\otimes C_{l_f=\bar{K}-l_i}(\hat{k}_f)]_{\kappa}^{K}F(s_is_fKl_i),
\end{aligned} \tag{17}$$

where $\boldsymbol{k}_i(\boldsymbol{k}_f)$ is the relative momentum in the initial (final) state, s's (ν's) denote the spins (z components), and $\bar{K}=K$ for even K and $K+1$ for odd K [15]. The quantum number

$s_i(s_f)$ is the channel spin for the initial (final) state. The quantity $\hat{k}_i(\hat{k}_f)$ is the solid angle of $\boldsymbol{k}_i(\boldsymbol{k}_f)$ and $C_{lm}(\hat{k})$ is related to the spherical harmonics $Y_{lm}(\hat{k})$ as usual [16]. In Eq. (17), the geometrical parts of the matrix elements of the tensors are given by the Clebsch-Gordan coefficients and $[C_{l_i}(\hat{k}_i)\otimes C_{l_f}(\hat{k}_f)]^K_\kappa$, and their physical parts are included in $F(s_is_fKl_i)$, the invariant amplitude, which is a function of the scattering angle and the center-of-mass energy, although omitted for simplicity. The amplitude $F(s_is_fKl_i)$ describes the scattering by the tensor interaction of rank K in the spin space: for example $F(s_is_fK=0l_i)$ represents the scattering by scalar interactions, that is, central interactions in the sense of effective interactions, which include any higher order of the interactions as long as it forms a scalar in the spin space. More details are given in Refs. [14,15].

In the present case, $\boldsymbol{k}_i=\boldsymbol{k}_f=\boldsymbol{k}$ and $z\|\boldsymbol{k}$, we have two nonvanishing scalar amplitudes, U_1 and U_3, and two tensor ones, T_1 and T_3, defined as

$$U_{2s}=F(ss00),$$
$$T_{2s}=F\left(\frac{3}{2}s20\right)+\sqrt{\frac{2}{3}}F\left(\frac{3}{2}s21\right)+F\left(\frac{3}{2}s22\right), \quad (18)$$

where $s=1/2$ (the doublet state) and 3/2 (the quartet state). From Eqs. (15) and (17), we obtain

$$\begin{aligned}
\alpha\,\mathrm{Im}(U_1)&=\sqrt{2}\sigma_0^{tot}+\frac{\sqrt{2}}{3}(\Delta\sigma_L+2\Delta\sigma_T),\\
\alpha\,\mathrm{Im}(U_3)&=2\sigma_0^{tot}-\frac{1}{3}(\Delta\sigma_L+2\Delta\sigma_T),\\
\alpha\,\mathrm{Im}(T_1)&=-\sqrt{2}\sigma_{20}^{tot}-\frac{1}{3}(\Delta\sigma_L-\Delta\sigma_T),\\
\alpha\,\mathrm{Im}(T_3)&=\sqrt{2}\sigma_{20}^{tot}-\frac{2}{3}(\Delta\sigma_L-\Delta\sigma_T).
\end{aligned} \quad (19)$$

The amplitudes, U_1, U_3, T_1, and T_3, are general. To connect them with realistic interactions, we consider an explicit form $\boldsymbol{M}_{\theta=0}$, which includes two scalar amplitudes and two tensor ones,

$$\boldsymbol{M}_{\theta=0}=S_0+S_\sigma(\boldsymbol{s}_n\cdot\boldsymbol{s}_d)+W_D[\boldsymbol{s}_d\otimes\boldsymbol{s}_d]^2_0+W_T[\boldsymbol{s}_n\otimes\boldsymbol{s}_d]^2_0, \quad (20)$$

where $\boldsymbol{s}_n$ and $\boldsymbol{s}_d$ are the spin operators of the neutrons and the deuterons. Here, S_0 and S_σ are the space parts of the scalar amplitudes, and W_D and W_T are those of the tensor ones. These amplitudes are related to the invariant amplitudes, U_1, U_3, T_1, and T_3, as

$$\begin{aligned}
S_0&=\frac{1}{3}\left(U_3+\frac{1}{\sqrt{2}}U_1\right),\\
S_\sigma&=\frac{1}{3}(U_3-\sqrt{2}U_1),\\
W_D&=\frac{1}{2}(T_3-2T_1),\\
W_T&=T_3+T_1,
\end{aligned} \quad (21)$$

which lead to the following equations:

$$\begin{aligned}
\alpha\,\mathrm{Im}(S_0)&=\sigma_0^{tot},\\
\alpha\,\mathrm{Im}(S_\sigma)&=-\frac{1}{3}(\Delta\sigma_L+2\Delta\sigma_T),\\
\alpha\,\mathrm{Im}(W_D)&=\frac{3}{\sqrt{2}}\sigma_{20}^{tot},\\
\alpha\,\mathrm{Im}(W_T)&=-(\Delta\sigma_L-\Delta\sigma_T).
\end{aligned} \quad (22)$$

If we consider a folding potential between the neutron and the deuteron neglecting antisymmetrizations and other reaction mechanisms, the relation between the amplitudes, S_0, S_σ, W_D, and W_T, and nuclear force components turns out to be rather straightforward. Let us assume the nuclear force between nucleons i and j to consist of spin-independent, spin-spin central forces, and a tensor one as

$$V_{i,j}=V_0(i,j)+V_\sigma(i,j)(\sigma_i\cdot\sigma_j)+V_T(i,j)S_T(i,j). \quad (23)$$

In the first order approximation, it is easily shown that the scalar amplitudes, S_0 and S_σ, are provided by the scalar interactions, V_0 and V_σ, respectively, with the S-state component of the deuteron internal wave function, one of the tensor amplitudes W_D by the scalar interaction V_0 with the deuteron D-state component, and the other tensor amplitude W_T by the tensor interaction V_T with the S-state component. Therefore the measurements of σ_0^{tot}, $\Delta\sigma_L$, $\Delta\sigma_T$, and σ_{20}^{tot} would provide pure information on the respective interactions. From Eq. (22), one can see that σ_0^{tot} is given only by the imaginary part of the spin-independent scalar amplitude, and σ_{20}^{tot} by that of the tensor one whose origin is considered as the deuteron D state. On the other hand, the cross section asymmetries, $\Delta\sigma_L$ and $\Delta\sigma_T$, contain information of the imaginary part of the spin-dependent scalar amplitude and that of the intrinsic tensor amplitude. The quantity $(\Delta\sigma_L-\Delta\sigma_T)$ provides direct information of the nuclear nd tensor interaction.

We calculated numerically the total cross sections for the complete set at low incident energies by solving the Faddeev equation, in which the 2NF is fixed to the Argonne V_{18} model (AV18) [17], while the 3NF is the 2π exchange Brazil model (BR-3NF) [18] with the cutoff parameter adjusted so as to reproduce the empirical triton binding energy. Due to the 2π exchange mechanism, the BR-3NF is expected to

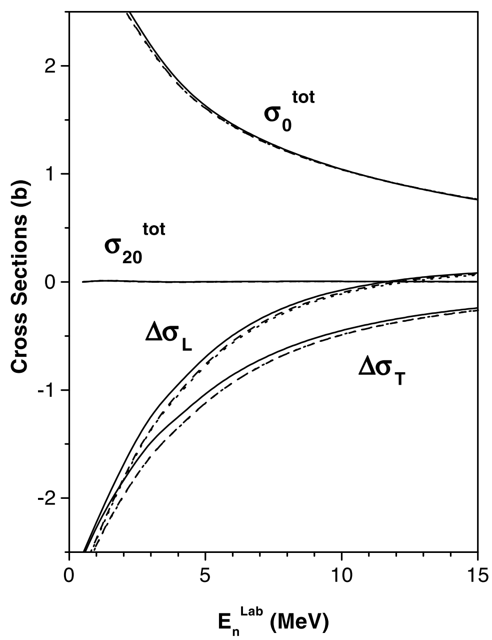

FIG. 1. The total cross sections, σ_0^{tot}, $\Delta\sigma_L$, $\Delta\sigma_T$, and σ_{20}^{tot}, of the nd scattering as functions of incident neutron energy in laboratory system for AV18 (solid lines), AV18+BR-3NF (dashed lines), and AV18+GS-3NF (dotted lines). The dashed lines and the dotted ones overlap each other.

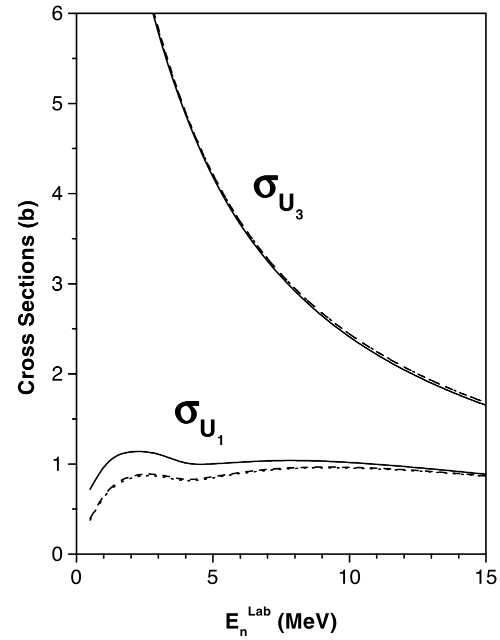

FIG. 2. The total cross sections, σ_{U_1} and σ_{U_3}, of the nd scattering as functions of incident neutron energy in laboratory system for AV18 (solid lines), AV18+BR-3NF (dashed lines), and AV18 +GS-3NF (dotted lines). The dashed lines and the dotted ones overlap each other.

contribute to not only scalar nuclear forces but also tensor ones in the spin space. To demonstrate the role of the tensor forces, we examine a fictitious spin-independent 3NF of the Gaussian form (GS-3NF)

$$V_G = V_0^G \exp\left\{ -\left(\frac{r_{21}}{r_G}\right)^2 - \left(\frac{r_{31}}{r_G}\right)^2 \right\} + (c.p.). \tag{24}$$

Values of the parameters, which are determined so as to reproduce the empirical triton binding energy, are $r_G = 1.0$ fm and $V_0^G = -45$ MeV.

The numerical calculations are performed in coordinate space [2,19,20], where $3N$ partial wave states for which 2NF and 3NF act are restricted to those with the total nucleon-nucleon angular momenta $j \leqslant 2$, and the total $3N$ angular momenta $J \leqslant 19/2$, which have been shown to be sufficient for the convergence of calculations [1]. We note that the results of σ_0^{tot}, $\Delta\sigma_L$, and $\Delta\sigma_T$ in the present calculations agree with those in Refs. [11–13] within a few percent.

In Fig. 1, the calculated cross sections, σ_0^{tot}, $\Delta\sigma_L$, $\Delta\sigma_T$, and σ_{20}^{tot}, are shown as functions of the neutron incident energy up to 15 MeV. In the figure, the 3NF contribution is very small for σ_0^{tot} but is appreciable for $\Delta\sigma_L$ and $\Delta\sigma_T$. The contribution to σ_{20}^{tot} is not clear because of the small magnitude of the cross section. More details will be discussed later in a magnified scale. From Eqs. (19) and (22), we can construct the scalar amplitudes U_1 and U_3 (or S_0 and S_σ), and the tensor amplitudes T_1 and T_3 (or W_D and W_T). From the numerical calculations, it turns out that the spin dependence of the 3NF contribution is clarified in $\mathrm{Im}(U_1)$, $\mathrm{Im}(U_3)$, $\mathrm{Im}(W_T)$, and $\mathrm{Im}(W_D)$. These amplitudes are shown in Figs. 2 and 3 by cross sections, $\sigma_A = \alpha\,\mathrm{Im}(A)$, where A is U_1, etc. In Fig. 2, the effect of 3NF on the scalar amplitude for the quartet state, σ_{U_3}, is very small, while that for the doublet state, σ_{U_1}, is remarkable, particularly at low incident ener-

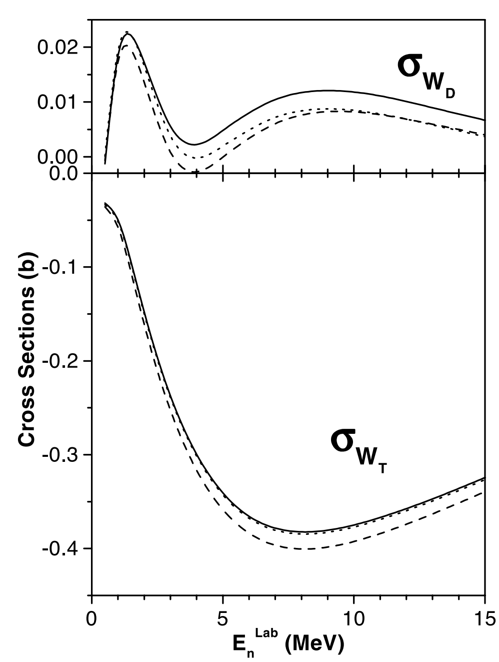

FIG. 3. The total cross sections, σ_{W_T} and σ_{W_D}, of the nd scattering as a function of incident neutron energy in laboratory system for AV18 (solid lines), AV18+BR-3NF (dashed lines), and AV18 +GS-3NF (dotted lines).

gies. In the latter, however, we cannot distinguish the effect of the BR-3NF from that of the GS-3NF. This result corresponds to a well known correlation between calculations of the triton binding energy and those of the nd doublet scattering length [21,7], which means that the doublet scattering amplitude at low energies is governed essentially by a position of the $3N$ bound state pole.

The effect of 3NF on the tensor amplitude σ_{W_T} has an interesting feature as shown in Fig. 3, where the effect of the BR-3NF on σ_{W_T} is quite appreciable at large incident energies, while that of the GS-3NF is almost negligible. This means that the BR-3NF contributes to $\mathrm{Im}(W_T)$ as a nd tensor force due to the spin dependence. In the figure, $\sigma_{W_D} = 3/\sqrt{2}\sigma_{20}^{tot}$, which is newly introduced in the present paper, shows significant 3NF effects except for very low energies with the remarkable dependence on the choice of the 3NF. The BR-3NF mostly reduces σ_{W_D} by a considerable amount and changes the sign of σ_{W_D} around 4 MeV. Although the magnitude of σ_{W_D} is small, refined measurements may identify such 3NF effects.

In summary, we have shown that the four nonvanishing independent forward amplitudes in the nd elastic scattering consist of two scalar amplitudes and two tensor amplitudes, which are related to the two central interactions and the two tensor ones, respectively, and the imaginary parts of these scattering amplitudes are given by the four total cross sections, σ_0^{tot}, $\Delta\sigma_L$, $\Delta\sigma_T$, and σ_{20}^{tot}. The Faddeev calculations are performed, by which the 3NF effects are shown to be clear in σ_{U_1}, σ_{W_T}, and σ_{W_D} for limited energy ranges, although the magnitude of the last cross section is small. It is also found that σ_{W_T} provides the information of the nd tensor interaction effect of the 3NF. These predictions will be encouraging the measurements of the total cross sections to obtain significant information of the interaction between three nucleons.

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