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ON THE PASINETTI GROWTH MODEL AND THE ANTI-PASINETTI THEORY (III)

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CONTENTS

CHAPTER 3: WELFARE ANALYSIS IN THE BALESTRA-BARANZINI MODEL

In Chapter 2, we have clarified that, if the given rate of interest, i, is lower than the equilibrium level of the capitalists' profit rate, n/s_c, the Pasinetti steady state is necessarily transformed into the anti-Pasinetti steady state, if the workers' propensity to save, s_w, rises up to the critical level, s_w^1.

However, what we have shown in Chapter 2 was simply the causal relationship from the assumption of the sufficient increase in s_w, on one hand, to the anti-Pasinetti consequence of the "euthanasia of the capitalists," on the other hand. We have made clear that the cause will bring about the effect, but we have not made clear why the initial cause has to be considered to be likely to happen.

Indeed, if the cause is unrealistic, the effect will not make sense, no matter how interesting the causal relationship from the cause to the effect may appear.

Anyway, if we can give a theoretical ground for the possibility of the initial assumption, that s_w may rise to the sufficiently large level, s_w^1, it would make the analysis in the last chapter much more meaningful and theoretically interesting.

In this chapter, we will show that, if the workers maximize their per capita consumption, the increase of s_w up to s_w^1 may be beneficial to the workers in some, not very unrealistic, cases.

If the rise of s_w up to s_w^1 is beneficial to the workers, it will follow that the workers will be motivated to increase their thrift. And thus the cause will become important and realistic, so will do the whole theory developed in Chapter 2, above.

It must be remarked in this context that the increase in s_w to s_w^1 may take either of the following two forms: the first is the quasi-stationary increase which takes a very long time, that is, a very gradual rise of s_w up to s_w^1. The second is the discrete, once-and-for-all increase which takes place instantaneously, or a jump in s_w to s_w^1.

In the first case, the steady state will continuously moves, and the economy will remain always very close to the continuously moving steady state.

In the second case, the steady state will discretely jump from that of the Pasinetti to the anti-Pasinetti case, and it will take a very long time for the economy to adjust itself toward the new anti-Pasinetti steady state. In this case, the economy will be inevitably very much disturbed and serious economic confusions will occur as the results of the sudden shift of the workers' propensity to save.

In this case, therefore, the workers will by no means become better off, but possibly rather very much worse off for a fairly long time until the new long-run equilibrium is sufficiently approached.

Since we do not intend to analyze explicitly the quantitative welfare losses of these long-term economic disturbances to be caused by the very rapid increase in s_w, we will assume away this second case, and instead, we will assume that the rise in s_w is of the first, quasi-stationary, type.

From another viewpoint, if the change in the workers' propensity to save is to be realistic, it cannot help but be a very slow change, since it is rooted deeply in the traditional culture, custom, and institutions of the nation, which cannot be drastically changed in a very short period of time.

However, even if the workers' propensity to save is difficult to change at a very high pace, it may be changed little by little if there is a welfare motivation for the workers to change it.

Though gradually, the workers will come to steadily increase s_w if it becomes apparent that the corresponding movement of the long-run equilibrium will surely bring about a more and more benefit to them, although, again, the benefit which they obtain, will be only very small relative to the total utility on the per-unit-of-time basis.

This will be the way in which we will interpret the final result of the comparative statics to be developed in this chapter.

However, before proceeding to the discussion concerning this final conclusion, we will present some very fundamental welfare propositions concerning the relationship between the propensities to save of the two classes, the capitalists and the workers.

Namely, we will first show the following two theorems: firstly, if $s_c = 1$, that is, the capitalists do not consume at all, then the workers' per capita consumption is locally maximized at $s_w = 0$; and, secondly, if $s_c < 1$, then the workers' per capita consumption is greater at a positive s_w than at $s_w = 0$.

It will be proved that these theorems hold true irrespective of what value the given rate of interest takes, provided it is less than the equilibrium capitalists' rate of profit. Particularly, they will be shown to hold good irrespective of whether the rate of interest is zero or not.

According to the first theorem, if the capitalists do not consume, then it is at least locally optimal for the workers to consume all of their incomes.

This theorem may appear to be obvious at first sight in the case where the interest rate equals zero: zero rate of interest would imply that no benefits would accrue to the workers from their starting to save, simply because there would be no interest accruing to the workers.

However, this is not really so. For, we will also know, in the form of the above second theorem, that, even if the interest rate equals zero, the workers' per capita consumption may indeed be increased by a rise in s_w from zero.

This means that the assumption of a positive interest rate is not required for the rise of s_w to be able to improve the workers' welfare.

This is the reason why the first theorem, in the particular case of $i = 0$, is not necessarily obvious, saying that, if $i = 0$ and $s_c = 0$, then $s_w = 0$ is locally optimal.

Now, according to the second theorem, if $s_c < 1$, then it will be better for the workers to save at least some of their incomes.

These two theorems will be proved under the general assumption of the well-behaved neoclassical production function.

Since we will show in this second theorem that $s_c < 1$ implies that the optimal s_w is positive, there will arise the problem as to how the optimal s_w is functionally related to the vector of the parameters, specifically $(i,\ s_c)$.

Furthermore, it will naturally be asked whether or not there exist any such vectors $(i,\ s_c)$ for which the optimal s_w will be equal to, or greater than, the critical value, s_w^1, so that there will stand the anti-Pasinetti steady state. And, if there are, what will be the set of those vectors?

These are nothing but the problems which we will consider in the final section of this chapter.

Unfortunately, however, in order to analyze these problems, it will be technically inevitable for us to restrict the general neoclassical well-behaved production functions to those of a special form. In the final section, therefore, we will specify the production function to be of the Cobb-Douglas form, and explicitly consider those above problems under this assumption.

Most importantly, in this Cobb-Douglas case, it will be proved that there does arise the possibility for the anti-Pasinetti steady state to be an optimal state for the workers. We will explicitly specify the sets of the vectors $(i,\ s_c)$ such that it will benefit the workers to determine the level of s_w so high as the anti-Pasinetti steady state is necessarily brought about. The Pasinetti and the anti-Pasinetti areas for the point $(i,\ s_c)$ will be illustrated.

The illustration will visualize that the anti-Pasinetti case cannot be dismissed as extremely exceptional: the anti-Pasinetti area will seem to be located in as realistic a position as the Pasinetti area, in the $(i,\ s_c)$ plane. This will seem to show not only a non-negligible possibility, but also a substantial reality, of the anti-Pasinetti case.

1. Assumptions

The basic model and assumptions of this chapter are the same as those made in the previous chapter. However, in this chapter, we make the following additional assumptions.

ASSUMPTION 3: The total utility of the average worker, U, is definable, and U is a monotonely increasing function of the per capita consumption of the workers.

ASSUMPTION 4: The growth rate of the working population, n_1, and the rate of technical progress, n_2, the capitalists' propensity to save, s_c, and the rate of interest, i, are determined independently of the workers' propensity to save, s_w.

ASSUMPTION 5: The workers can change their propensity to save in the long-run, provided the utility of the representative worker is not decreased by the change in their propensity to save.

As to the production function, we make the following

ASSUMPTION 6: The production function is of the well-behaved, neoclassical type, fulfilling the Inada conditions.

This Assumption 6 is made for the whole analysis of this chapter. In addition, only for Section 3, we will further make the following

ASSUMPTION 6-a: The production function is of the Cobb-Douglas type, $Y = K^a L^{1-a}$, where L denotes the quantity of effective labor and a is a constant, $1 > a > 0$.

1-1: Comparative Statics

What we will do in this chapter is the comparative statics concerning the equilibrium levels of utility of the workers' per capita consumption, corresponding to different vectors of the values of the parameters.

We will compare the utility levels of the average worker in the different steady states which will be brought about by the different vectors of the parameter values.

Specifically, we will regard s_w as a variable, and we will consider the maximization in various cases with different combinations of i and s_c. Meanwhile, n_1, n_2, (hence n), and the production function are supposed to be constant throughout this chapter.

1-2: The Objective Function to be Maximized

The workers are assumed to determine the value of their propensity to save, so as to maximize the equilibrium level of utility combined with their per capita consumption.

Since, at the beginning of this paper, we have assumed the Harrod-neutral technical progress, we may write $L(t) = A(t)L_r(t)$, where $A(t)$ denotes the coefficient of the average labor productivity which represents the labor-augmenting technical progress, and $L_r(t)$ the quantity of labor. Hence $n = \dot{L}/L = n_1 + n_2$.

Therefore, if we denote the total consumption of the workers by C_w, we have $C_w(t)/L_r(t) = A(t)\{C_w(t)/L(t)\}$. Since the rate of technical progress, n_2, is a constant, $A(t)$ is a given exponential function which is independent of s_w, i, and s_c. This implies that, given i and s_c, the workers' objective function may be written as

$$X(s_w;\ i,\ s_c) = (C_w/L)(s_w;\ i,\ s_c) \qquad (40)$$

where $(C_w/L)(s_w;\ i,\ s_c)$ denotes the equilibrium value of C_w/L as a function of s_w, given i and s_c.

Now, let us first prove the following proposition.

PROPOSITION 13: Given i and s_c, we have, for all s_w such that $s_w^1 > s_w > 0$,

$$\begin{aligned} X(s_w) &= (1-s_w)\{(W/L)+(iK_w/L)\}(s_w) \\ &= (1-s_w)\{(W/K_w)+i\}(s_w)\cdot(K_w/L)(s_w) \qquad (41) \end{aligned}$$

where

$$\{(W/K_w)+i\}(s_w) = f(k^{**}(s_w))/k^{**}(s_w) = n/s_w, \qquad (42)$$

and

$$(K_w/L)(s_w) = k_w(s_w). \qquad (43)$$

Furthermore, $(W/K_w)(s_w)$ is a decreasing function of s_w, and $(W/L)(s_w)$ and $(K_w/L)(s_w)$ are both increasing functions of s_w.

Proof. In view of $C_w = (1-s_w)(W+P_w) = (1-s_w)(W+iK_w)$, (41) is obvious.

Let us then verify (42). Since we are here considering the steady states, such that $\dot{k}_w = 0$, we have, by (10),

$$\dot{k}_w/k_w = s_w[\{f(k)-kf'(k)\}/k_w+i]-n = 0 \qquad (44)$$

so that we have, for all $s_w > 0$,

$$W/L = f(k)-kf'(k) = k_w\{(n/s_w)-i\} \qquad (45)$$

Hence, by $k_w = K_w/L$, we have

$$W/K_w = (n/s_w)-i \qquad (46)$$

which is a decreasing function of s_w.

By (46), we have $(W/K_w)+i = n/s_w$. By the definition of k^{**}, we also have $n/s_w = f(k^{**}(s_w))/k^{**}(s_w)$. Hence, we have (42).

Finally, let us show that (43) is an increasing function of s_w. As verified in Theorem 2 in Chapter 2, the capital intensity, $k_e(s_w)$, of the Pasinetti steady state corresponding to each value of s_w, is a monotonely increasing function of s_w. On the other hand, the function, $f(k)-kf'(k)$, is an increasing function of k, and so is W/L, as is seen by the first equality of (45). It follows that W/L is an increasing function of s_w.

In the equation $k_w = K_w/L = (W/L)(K_w/W)$, by (46), both of W/L and K_w/W are increasing functions of s_w. Hence, $k_w(s_w)$ is an increasing function of s_w. Q. E. D.

As seen in (41), the workers' objective function is decomposed into three factors: the workers' propensity to consume, $1-s_w$, the workers' total income per unit of the workers' capital, $(W/K_w)+i$, and the ratio of the workers' capital to the total effective labor, k_w.

The above proposition says that the second factor is a decreasing function of s_w, and the third factor is an increasing.

Furthermore, since we have seen in the above proof that W/L is also an increasing function of s_w, it will follow that the product of the second and the third factors, which equals $W/L+ik_w$, is an increasing function of s_w.

However, since the first factor, $1-s_w$ is a decreasing function of s_w, the total effect of a change in s_w on the objective function cannot be predicted without further investigations.

2. A First Approach

As a first approach to the above formulated problem, let us assume that the initial value of the workers' propensity to save equals zero. It will be proved in Appendix 6 that a steady state is established for this $s_w = 0$.

Since $s_w = 0$, the workers cannot own capital in the steady state. Because, even if they initially own some capital, their capital will not increase at all, by the lack of their savings. It will then follow that the value of k_w continues to fall toward zero.

Such a state of the economy cannot be a steady state, since it requires k_w to be constant for a steady state.

Therefore, we must have $k_w = 0$ in the initial steady state.

In such a state, what sense does the assumption that $i > 0$ make? It may be argued that, if $k_w = 0$, it will not matter anything to the workers whether the rate of interest on K_w is positive or zero. If $k_w = 0$, the workers will actually earn no interest, irrespective of $i > 0$ or $= 0$.

However, if $i > 0$, then this will mean that there is opened the opportunity for the workers to earn positive interest on their own potential capital which could be accumulated if they started to save some of their incomes.

However, even if the condition, $i = 0$, implies there is no opportunity of interest incomes for the workers, it does not necessarily follow that there is not any motivation for the workers to begin to save.

Indeed, the workers' savings will contribute to accelerate the capital deepening of the whole economy, and thereby to increase the real per capita wages. This productivity effect of the workers' savings will work even when there is not the interest-income opportunity for the workers.

Therefore, in this section, we assume i to be only non-negative, allowing for both the cases where $i = 0$ as well as $i > 0$.

Before presenting the theorem, let us make the following additional assumption.

ASSUMPTION 7: $f(k)$ has its derivative of order three.

This assumption will be needed for the proof of the following theorem.

THEOREM 3: Suppose $s_c = 1$. Then, a local maximum of the workers' objective function is attained at $s_w = 0$. Also, suppose $s_c < 1$. Then, the derivative of the objective function with respect of s_w at $s_w = 0$ is positive.

That is, in symbol,

$$X'(0;\ 1) = \text{and } X''(0;\ 1) < 0 \tag{47}$$

$$X'(0;\ s_c) > 0 \text{ for all } s_c < 1. \tag{48}$$

Proof. In Appendix 6, it will be shown that the Pasinetti steady state uniquely exists and globally stable even if $s_w = 0$.

Let $k_e(s_w)$ and $k_c^e(s_w)$ denote the equilibrium values of k and k_c in the Pasinetti steady state corresponding to each s_w, $0 \leqq s_w < s_w^1$.

The point representing the Pasinetti steady state is nothing but the intersection point between Graph of $\dot{k}$ and Graph of $\dot{k}_c$. Particularly, if $s_w = 0$, the intersection point is the point (k^*, k^*) on the 45-degree line, where $k_c = k$ and $k_w = 0$.

As s_w rises from 0, Graph of $\dot{k}$ shifts continuously to the right, and the intersection point moves to the right along Graph of $\dot{k}_c$.

Since Graph of $\dot{k}_c$ is a smooth curve in the (k, k_c) plane, it may be obvious that the movement of the intersection point is continuous.

However, it will be easy to see that, in order to verify such derivative expressions as those in (47) and (48), we have to ensure not only the continuity but also the differentiability of $k_e(s_w)$ and $k_c^e(s_w)$.

LEMMA 1: $k_e(s_w)$ and $k_c^e(s_w)$ are twice-differentiable at $s_w = 0$.

Proof of Lemma 1. Since $\dot{k} = 0$ and $\dot{k}_c = 0$ in the Pasinetti steady state, we know, by (7) and (8), that $k_e(s_w)$ and $k_c^e(s_w)$ fulfill

$$(s_c - s_w)\{(f'(k) - i)k + ik_c\} + s_w f(k) - nk = 0 \qquad (49)$$

$$s_c\{(f'(k) - i)k + ik_c\} - nk_c = 0 \qquad (50)$$

Given n, i, and s_c, the system of equations, (49) and (50), can be rewritten as

$$R_1(k, k_c, s_w) = 0 \qquad (51)$$

$$R_2(k, k_c, s_w) = 0 \qquad (52)$$

where R_1 and R_2 denote the functions on the left-hand sides of (49) and (50), respectively.

Let us denote $(k^*, k_c^*, 0)$ by P_o. Then, we have the following propositions: (i) (51) and (52) are satisfied at P_o; (ii) R_1 and R_2 are twice-differentiable with respect to (k, k_c, s_w) in the neighborhood of

P_o; and (iii) $\{D(R_1, R_2)/D(k, k_c)\}_o \neq 0$, where the left-hand side of this inequality signifies the Jacobian (functional determinant) of (51) and (52) with respect of (k, k_c), evaluated at P_o.

Let us then verify these three propositions. (i) is obvious. As to (ii), we have to use the assumption postulated just above. Indeed, the twice-differentiability of (51) and (52) requires, among other things, $\partial^2 R_i/\partial k^2$ $(i = 1, 2)$ to exist, which contain the term with the third derivative of $f(k)$. However, the existence of all the remaining derivatives of order two, including the cross-derivatives, is easy to verify.

Finally, let us consider the condition (iii).

$$\begin{aligned}
&\{D(R_1, R_2)/D(k, k_c)\}_o \\
&= \begin{vmatrix} (s_c - s_w)(f''k + f' - i) + s_w f' - n, & i(s_c - s_w) \\ s_c(f''k + f' - i), & is_c - n \end{vmatrix}_o \\
&= \begin{vmatrix} s_w(i - f''k) - n, & n - is_w \\ s_c(f''k + f' - i), & is_c - n \end{vmatrix}_o \\
&= \begin{vmatrix} -s_w f''k, & n - is_w \\ s_c(f''k + f') - n, & is_c - n \end{vmatrix}_o
\end{aligned} \tag{53}$$

By $s_w = 0$ and $f'(k^*) = n/s_c$, this equals $-ns_c f''(k^*)k^* \neq 0$. (iii) is thus proved.

According to the implicit function theorem, the conditions, (i), (ii), and (iii), together, imply there exist functions, $h_1(s_w)$ and $h_2(s_w)$, defined on the neighborhood of $s_w = 0$, such that: (1) $R_i(h_1(s_w), h_2(s_w), s_w) = 0$ $(i = 1, 2)$, for all those small positive s_w in the neighborhood; (2) $h_1(0) = h_2(0) = k^*$; and (3) $h_i(s_w)$ $(i = 1, 2)$ are twice-differentiable on the neighborhood.

Clearly, we can identify these functions, $h_i(s_w)$ $(i = 1, 2)$, with the functions, $k_e(s_w)$ and $k_c^e(s_w)$, with their domain restricted to the neighborhood of $s_w = 0$.

Therefore, $k_e(s_w)$ and $k_c^e(s_w)$ are twice-differentiable at $s_w = 0$. This concludes the proof of Lemma 1.

Now, subtracting (50) multiplied by $(s_c - s_w)/s_c$, from (49), we have

$$nk_c(s_c - s_w)/s_c + s_w f(k) - nk = 0. \tag{54}$$

For the notational convenience, let us rewrite s_w simply as s. Differentiating (50) and (54) with respect of s, we have

$$s_c\{f''(k)k + f'(k) - i\}k' + (is_c - n)k_c' = 0 \tag{55}$$

$$-nk_c/s_c + f(k) + \{sf'(k) - n\}k' + \{n(s_c - s)/s_c\}k_c' = 0 \tag{56}$$

where $k = k_e(s)$, $k_c = k_c^e(s)$, $k' = k_e'(s)$, and $k_c' = k_c^{e\prime}(s)$.

By (41), (43), and (45), we also have

$$X(s) = (1-s)\{f(k) - kf'(k) + ik_w\} \tag{57}$$

So that we have

$$\begin{aligned} X'(s) = &-\{f(k) - kf'(k) + i(k - k_c)\} \\ &+ (1-s)\{i(k' - k_c') - kf''(k)k'\} \end{aligned} \tag{58}$$

By (55),

$$-ik_c' - kf''(k)k' = \{f'(k) - i\}k' - (n/s_c)k_c' \tag{59}$$

Substituting (59) into (58),

$$\begin{aligned} X'(s) = &-\{f(k) - kf'(k) + i(k - k_c)\} \\ &+ (1-s)\{f'(k)k' - (n/s_c)k_c'\} \end{aligned} \tag{60}$$

Multiplying (56) by $(1-s)/(s_c - s)$ and rearranging, we also have

$$\begin{aligned} -\{n(1-s)/s_c\}k_c' = &\{(1-s)/(s_c - s)\}[-nk_c/s_c \\ &+ f(k) + \{sf'(k) - n\}k'] \end{aligned} \tag{61}$$

Substituting (61) into (60),

$$\begin{aligned} X'(s) = &-\{f(k) - kf'(k) + i(k - k_c)\} + (1-s)f'(k)k' \\ &+ \{(1-s)/(s_c - s)\}[-nk_c/s_c + f(k) + \{sf'(k) - n\}k'] \\ = &\ k\{f'(k) - i\} + k_c[i - \{(1-s)/(s_c - s)\}(n/s_c)] \end{aligned}$$

$$+\{(1-s_c)/(s_c-s)\}f(k)$$
$$+(1-s)k'[\{s_c/(s_c-s)\}f'(k)-\{n/(s_c-s)\}] \qquad (62)$$

Therefore, we have

$$\begin{aligned} X'(0) &= X'(0;\ s_c) \\ &= k(0)\{f'(k(0))-i\}+k_c(0)\{i-(n/s_c^2)\} \\ &\quad +\{(1-s_c)/s_c\}f(k(0)\} \\ &\quad +k'(0)\{f'(k(0))-(n/s_c)\} \end{aligned} \qquad (63)$$

On the other hand, by putting $s = 0$ in (54), we have $nk_c(0)-nk(0) = 0$, so that $k_c(0) = k(0)$ Hence, by putting $s = 0$ in (50), we have $s_c f'(k(0))k(0)-nk(0) = 0$, so that we have

$$f'(k(0)) = n/s_c \qquad (64)$$

By $s = 0$ and (50) and (64), we also have $s_c\{(n/s_c)-i\}k(0)+s_c ik_c(0)-nk_c(0) = 0$, so that $k_c(0) = k(0)$. Hence,

$$k(0) = k_c(0) = k^*. \qquad (65)$$

By (63), (64), and (65), we have

$$\begin{aligned} X'(0;\ s_c) &= k^*\{(n/s_c)-i\}+k^*\{i-(n/s_c^2)\}+\{(1-s_c)/s_c\}f(k^*) \\ &= k^*(n/s_c^2)(s_c-1)+\{(1-s_c)/s_c\}f(k^*) \\ &= \{(1-s_c)/s_c\}\{f(k^*)-k^*(n/s_c)\} \\ &= \{(1-s_c)/s_c\}\{f(k^*)-k^*f'(k^*)\} \\ &= \{(1-s_c)/s_c\}(W/L)(k^*) \end{aligned} \qquad (66)$$

where $(W/L)(k^*)$ denotes the real wage-rate at $k = k^*$. Since $f(k^*)-k^*f'(k^*) > 0$, (66) implies the equality in (47) and the proposition (48).

Let us then turn to the proof of the second-order condition in (47). We first note that, by putting $s_c = 1$ in (62), we have

$$X'(s;\ 1) = k\{f'(k)-i\}+k_c(i-n)+k'\{f'(k)-n\} \qquad (67)$$

Since $kf'(k) = (K/L)(P/K) = P/L$, the first term on the right-hand side of this equation can be rewritten as $P/L-i(K/L)$, which is equal to $(P_c/L)-i(K_c/L)$. Because, $P-iK = P_c+P_w-iK_w-iK_c = P_c-iK_c$, since $P_w = iK_w$ in the Pasinetti steady states.

On the other hand, we also have $P_c/K_c = n/s_c$ in the Pasinetti steady states (see Pasinetti, 1974, in the *References* of the Part I, Vol. 64, No. 3, p. 38, *Keizai-Shirin*). Hence, we have $P_c/L = (P_c/K_c)(K_c/L) = (n/s_c)k_c$. Therefore,

$$k\{f'(k)-i\} = \{(n/s_c)-i\}k_c \tag{68}$$

for all $s \geqq 0$, in the neighborhood of zero.

By (67) and (68), we have

$$\begin{aligned} X'(s;\ 1) &= (n-i)k_c(s)+k_c(s)(i-n)+k'(s)\{f'(k(s))-n\} \\ &= k'(s)\{f'(k(s)\}-n \end{aligned} \tag{69}$$

So that we have

$$X''(s;\ 1) = k''(s)\{f'(k(s))-n\}+\{k'(s)\}^2 f''(k(s)) \tag{70}$$

By Theorem 2 in Chapter 2, we have $k'(s) > 0$. Therefore, by (64), (65), and (70), we finally have

$$X''(0;\ 0) = \{k'(0)\}^2 f''(k^*) < 0 \tag{71}$$

This concludes the proof of Theorem 3.

Theorem 3 implies that, if the economy is in the Pasinetti steady state in which the workers do not save at all ($s_w = 0$) and the capitalists do not consume at all ($s_c = 1$) then an increase in s_w will be combined with a new steady state in which the workers' utility must be lower than initially.

This proposition holds true irrespective of whether the rate of interest is positive or zero: even in the case where there is an opportunity for the workers to earn interest at a positive interest rate, it will not raise the workers' welfare for them to begin to save some of their wages, as long as the capitalists' propensity to save is equal to unity.

Therefore, in the economy where the capitalists only save and the workers only consume, there is a strong tendency for such a perfect specialization to persist. For if s_w rises a little, the workers will suffer from a decrease in their welfare, in the resulting new steady state. Then the workers will be well-motivated to reduce s_w back to the previous level, since doing so will surely raise back their utility.

If the capitalists consume some of their incomes ($s_c < 1$), the opposite is true. The workers had better start to save, rather than they keep to consume all.

This is true, again, irrespective of whether the interst rate is zero or positive.

3. The Workers' Welfare and the Anti-Pasinetti Steady States

In the last section, we have clarified that, if $s_c < 1$, then the optimal level of s_w, which maximizes the equilibrium per capita consumption of the workers, is greater than zero.

However, we did not analyze there as to whether the optimal s_w may or may not be so high as the anti-Pasinetti steady state stands, and if it may, in what case. We will consider these problems in this section.

For the analytical purpose, we suppose the Cobb-Douglas production function, $Y = F(K, L) = K^a L^{1-a}$, where $1 > a > 0$, a is a constant.

We also suppose that s_w may or may not be lower than s_c. It is assumed that $n > is_c$.

3-1: The Critical Value for s_w Between the Pasinetti and Anti-Pasinetti Cases

THEOREM 4: A necessary and sufficient condition for an anti-Pasinetti steady state to exist, be uniquely determined, and be globally stable, is $s_w \geqq an/i$.

Proof. By Definitions 6, we have $f'(k_2) = i$. Also, by Definition 8, we have $i = f'(k^{**}(s_w^1))$. It follows that

$$k^{**}(s_w^1) = k_2 \tag{72}$$

In Chapter 2 and Appendices 3, 4, and 6, we clarified that if $s_w < s_w^1$, then the Pasinetti steady state holds. In other words, if $s_w < s_w^1$, no anti-Pasinetti steady state is possible.

Since we are assuming the Cobb-Douglas case in this section, this critical value, s_w^1, can be shown to equal an/i. Indeed, by the above definition of s_w^1, we have $f'(k^{**}(s_w^1)) = i$. It follows that $s_w^1/n = k^{**}(s_w^1)/f(k^{**}(s_w^1)) = af'(k^{**}(s_w^1))^{-1} = a/i$. Hence, $s_w^1 = an/i$.

Therefore, it follows that no anti-Pasinetti steady state is possible if $s_w < an/i$. This proves the necessity of the condition of $s_w \geqq an/i$ for an anti-Pasinetti equilibrium.

Suppose $s_w^1 < s_c$. Then, if $s_w < s_w^1$, the Pasinetti steady state exists. If $s_w \geqq s_w^1$, the anti-Pasinetti steady state exists, is uniquely determined, and is globally stable.

In other words, in the case of $s_w^1 < s_c$, $s_w \geqq s_w^1$ is necessary and sufficient for the existence, uniqueness, and global stability of the anti-Pasinetti steady state.

We will consider the case of $s_w^1 \geqq s_c$ in Appendices 3, 4, 5 and 7. There, it will be shown that the anti-Pasinetti equilibrium holds if and only if $s_w \geqq s_w^1$.

Figs. A9 and A10 indicate examples of the phase diagrams when $s_w \geqq s_w^1 > s_c$. They show that the anti-Pasinetti equilibrium, D, is unique and globally stable.

This concludes the proof of Theorem 4.

3-2: The Outline of the Problem

Before proceeding to explicit analysis, it will be convenient for us to draw an outline of the problem.

Our problem is to find the set of all vectors, (i, s_c), such that, given (i, s_c), the objective function, $X(s_w; i, s_c)$, is maximized on the set, $\{s_w: 0 \leqq s_w \leqq s_w^1\}$, only when s_w is equal to the critical value,

$s_w^1 = an/i$.

In view of Theorem 4, therefore, our problem is to look for the set of (i, s_w) for which the workers' per capita welfare cannot be maximized without entering the anti-Pasinetti steady state.

In Appendix 8, it will be shown that, if $i = 0$, then $k_c > 0$ in equilibrium. This means that the anti-Pasinetti steady state is not possible if $i = 0$.

Therefore, in this section, we confine our attention on such a case where (i, s_c) belongs to the set

$$T = \{(i, s_c): i > 0, 1 \geqq s_c > 0, n > is_c\} \qquad (73)$$

Given each $(i, s_c) \in T$, we define s_w^* as follows.

$$X(s_w^*; i, s_c) > X(s_w; i, s_c) \qquad (74)$$

$$\text{for all } s_w \text{ such that } 0 \leqq s_w \leqq s_w^1, s_w \neq s_w^*$$

If s_w^* exists for any element of T, it will be expressed as a function,

$$s_w^* = s_w^*(i, s_c) \qquad (75)$$

defined on T.

Our problem, then, can be represented as follows: whether the subset, Q, of T, defined by

$$Q = \{(i, s_c): s_w^*(i, s_c) = s_w^1 = an/i\} \cap T \qquad (76)$$

is empty or not; and, if Q is not empty, what explicit form the set, Q, takes.

We also define the complementary subset, $R = T - Q$, of Q, relative to T.

$$R = \{(i, s_c): s_w^*(i, s_c) < s_w^1 = an/i\} \cap T \qquad (77)$$

DEFINITIONS 11: The set, Q, is called the *anti-Pasinetti area* In the (i, s_c) plane. Also, the set, R, is called *the Pasinetti area.*

3-3: The Pasinetti and the Anti-Pasinetti Areas

3.3.1: The Objective Function in Explicit Form

Under the assumption of the Cobb-Douglas production function, Balestra-Baranzini (1971) derived the workers' per capita consumption (i. e., C_w/L in our notation) in the Pasinetti steady state, which is recapitulated as follows:

$$\begin{aligned} C_w/L = {} & (1-s_w)(1-a)n^{(2a-1)/(a-1)} \\ & \cdot (n-is_w)^{-1/(1-a)} \\ & \cdot \{(1-a)ns_w+ans_c-is_cs_w\}^{a/(1-a)} \end{aligned} \tag{78}$$

(Balestra-Baranzini, 1971).

In the following, we will show that the terms, $n-is_w$, and $(1-a)ns_w+ans_c-is_cs_w$, in the above expression are positive for all relevant areas of the parameters.

Since the optimality condition, (74), is concerned with the domain of s_w such that $0 \leqq s_w \leqq s_w^1 = an/i$, we compare the levels of C_w/L for those values of s_w such that $n-is_w > an-is_w \geqq 0$. Hence, the first term is always positive.

Also, the above second term equals $(1-a)ns_w+s_c(an-is_w)$, which is easily seen to be positive for all s_w in the domain.

Therefore, (78) is well-defined as the objective function, $X(s_w;\ i,\ s_c)$.

3.3.2: The Explicit Derivation of the Pasinetti and Anti-Pasinetti Areas

Since we are here concerned with the maximization of the objective function, $X(s_w;\ i,\ s_c) = (C_w/L)(s_w;\ i,\ s_c)$, on the domain, $0 \leqq s_w \leqq s_w^1$, which is bounded both up-and downward, the solution of the maximization may well be a corner solution, $s_w = 0$ or s_w^1, where the derivative, dX/ds_w, may not equal zero. On the other hand, if it is not a corner solution, it will have to equal a value of s_w such that the derivative equals zero. In short, we are here simply applying to our problem the same idea as used in the Kuhn-Tucker method.

We first note, therefore, that

$$
\begin{aligned}
&\text{the sign of } dX(s_w;\ i,\ s_c)/ds_w = \text{the sign of } Z(s_w;\ i,\ s_c),\\
&\text{where } Z(s_w;\ i,\ s_c)\\
&\quad = \{i(na+n-i)s_c+n(i-ai-n)\}s_w+an^2(1-s_c) \qquad (79)
\end{aligned}
$$

for all $(s_w,\ i,\ s_c) \in \{s_w\!: 0 \leqq s_w \leqq s_w^1\} \times T$. (See Appendix 9.)

When applying the idea like that of the Kuhn-Tucker method to our problem, we are of course interested in which of negative, zero, and positive, is the sign of the derivative, dX/ds_w. Hence, it will be in order for us to consider the sign of Z.

The function, Z, is linear with respect to s_w. Let us therefore first look at the signs of Z at the terminal values, 0 and s_w^1, of s_w, and then, consider the sign of Z at those values of s_w between these terminal values, by a sort of interpolation which is ensured to be possible by the linearity.

Let us begin to consider the particular case of $s_c = 1$. In this case, it was shown in the last section that the objective function is locally maximized at $s_w = 0$. This does not, of course, necessarily mean that $s_w = 0$ also gives its global maximum for $0 \leqq s_w \leqq s_w^1$.

However, in this Cobb-Douglas case, it can be shown that $s_w = 0$ also gives the global maximum.

Because, if $s_c = 1$, we have $Z(0;\ i,\ s_c) = an^2(1-s_c) = 0$. In addition, $Z(s_w^1;\ i,\ 1) = -(i-n)^2 s_w^1$, which must be negative since $n > is_c = i$ by assumption and $s_w^1 > 0$. It follows that Z equals zero only at $s_w = 0$ and negative for all the other s_w, $0 < s_w \leqq s_w^1$. This implies that $s_w = 0$ gives the global maximum.

Let us then suppose $0 < s_c < 1$. In this case, we have

$$Z(0;\ i,\ s_c) = an^2(1-s_c) > 0. \qquad (80)$$

Although the sign of $Z(s_w^1;\ i,\ s_c)$ is not necessarily uniform, the linearily of Z implies that, if $Z(s_w^1;\ i,\ s_c) \geqq 0$, then $Z(s_w;\ i,\ s_c) > 0$ for all s_w, $0 \leqq s_w < s_w^1$, so that the objective function, X, is maximized at $s_w = s_w^1$. By (76), it follows that

$$\{(i,\ s_c)\colon Z(s_w^1;\ i, s_c) \geqq 0\} \subset Q \tag{81}$$

In the case where $Z(s_w^1;\ i,\ s_c) < 0$, (80) and the linearity of Z together imply that there exists a value, s_w^{**}, such that, $0 < s_w^{**} < s_w^1$,

$$Z(s_w^{**};\ i,\ s_c) = 0 \tag{82}$$

$$Z(s_w;\ i,\ s_c) > 0 \text{ for } 0 \leqq s_w < s_w^{**}, \text{ and} \tag{83}$$

$$Z(s_w;\ i,\ s_c) < 0 \text{ for } s_w^{**} < s_w \leqq s_w^1. \tag{84}$$

In this case, therefore, we have $s_w^*(i,\ s_c) = s_w^{**}(i,\ s_c) < s_w^1$. This implies

$$\{(i,\ s_c)\colon Z(s_w^1;\ i,\ s_c) < 0\} \not\subset Q \tag{85}$$

By $s_w^1 = an/i$, we have $Z(s_w;,\ i,\ s_c) = Z(an/i;\ i,\ s_c)$, which can be regarded as a function, denoted by $h(s_c;\ i)$, of only s_c and i, since we are assuming a and n to be fixed.

Then, by (81) and (85), we can write

$$Q = \{(i,\ s_c)\colon h(s_c,\ i) \geqq 0\} \tag{86}$$

$$R = \{(i,\ s_c)\colon h(s_c,\ i) < 0\} \tag{87}$$

By (79) and $s_w^1 = an/i$, we know that $h(s_c;\ i)$ is a linear function with respect to s_c on $0 < s_c \leqq 1$. In order to use the interpolation method, let us extend the domain for s_c of $h(s_c;\ i)$ to include the terminal value, 0. Let us then first look at the signs of $h(s_c;\ i)$ at the two terminal values, $s_c = 0$ and 1.

$$\begin{aligned} h(0;\ i) &= n(i-ai-n)(an/i)+an^2 \\ &= an^2\{(2-a)i-n\}/i \end{aligned} \tag{88}$$

$$\begin{aligned} h(1;\ i) &= \{i(na+n-i)+n(i-ai-n)\}(an/i) \\ &= (2ni-i^2-n^2)(an/i) \\ &= -(i-n)^2(an/i) \end{aligned} \tag{89}$$

First, let us consider the particular subset, $\{(i,\ s_c):\ s_c = 1\} \cap T$, of T. Let us denote this subset by T_1. Now, $(i,\ s_c) \in T_1$ implies, of course, $s_c = 1$ and $n > is_c$, so that $n > i$. Then, this inequality and (89) together imply $h(1;\ i) < 0$. By (87), this means $T_1 \subset R$.

Then, let us consider the subset, $T - T_1$, of T. For the interpolation, let us note here that $h(1;\ i)$ is always non-positive, whereas the sign of $h(0;\ i)$ depends on whether i is greater than, equal to or less than, $n/(2-a)$.

Suppose $i > n/(2-a)$. Then, we have $h(0;\ i) > 0$ and $h(1;\ i) \leqq 0$. By the linearity of $h(s_c;\ i)$ with respect to s_c, there must exist a value, s_c^*, $0 < s_c^*$, $\leqq 1$, such that

$$h(s_c^*;\ i) = 0 \tag{90}$$

$$h(s_c;\ i) > 0 \text{ for } 0 < s_c < s_c^*, \text{ and} \tag{91}$$

$$h(s_c;\ i) < 0 \text{ for } s_c^* < s_c < 1. \tag{92}$$

s_c^* is a function of i, which we denote by $s_c^*(i)$. By (86) and (87), it follows that, if $s_c < 1$ and $i > n/(2-a)$, then

$$(i,\ s_c) \in Q \text{ for } 0 < s_c \leqq s_c^*(i) \tag{93}$$

$$(i,\ s_c) \in R \text{ for } s_c^*(i) < s_c < 1 \tag{94}$$

Suppose $i \leqq n/(2-a)$. This means $h(0;\ i) \leqq 0$. On the other hand, if $h(1;\ i) = 0$, then, by (89), we have $i = n$, which leads to a contradiction. Because, since $1 > a$, we have $2-a > 1$, so that $n/(2-a) < n = i$, to the violation of $i \leqq n/(2-a)$. Hence, we must have $h(1;\ i) < 0$. Therefore, in this case, we have $h(s_c;\ i) < 0$ for $0 < s_c < 1$. It follows that, if $s_c < 1$ and $i \leqq n/(2-a)$, then $(i,\ s_c) \in R$.

As will be proved in Appendix 10, we have

$$s_c^*(i) = n\{(2-a)i - n\} / \{i(i-an)\} \tag{95}$$

By all of these considerations, we can visualize the sets, Q and R, in the $(i,\ s_c)$ plane, as indicated in Fig. 10. See Appendix 10 for details.

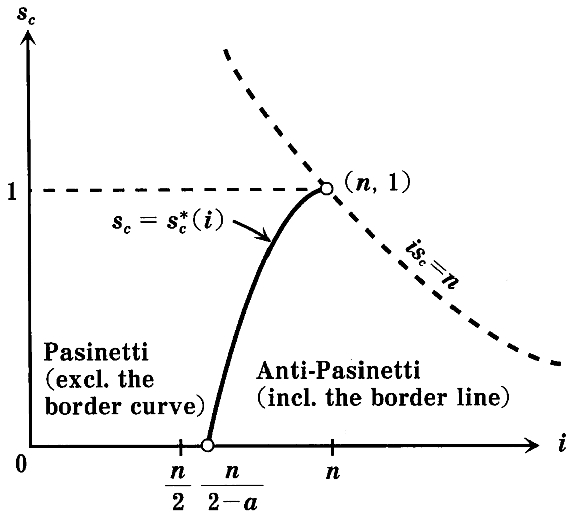

Fig. 11

In summary, we have the following theorem.

THEOREM 5: Under the assumption of the Cobb-Douglas production function, the anti-Pasinetti area, Q, is non-empty, and its position is like that shown in Fig. 11.

3.3.3: The Golden Rule in the Balestra-Baranzini Model

In this subsection, the golden rule of capital accumulation will be considered in the Balestra-Baranzini model with the rate of interest, i, to be a constant.

The argument of this subsection will hold in both the cases where $s_w^1 \geqq s_c$ and $< s_c$.

For such (i, s_c) that belongs to the anti-Pasinetti area, the economy is in an anti-Pasinetti steady state, so that $P_w = P$ in equilibrium. This is a Solow equlibrium with only one class, in which the Harrod-Domar condition $k/f(k) = s_w/n$ holds.

The comparative statics of the Phelps golden rule of capital accumulation applies in the comparative statics concerning such Solow steady states.

So, let us assume that $(i,\ s_c)$ is in the anti-Pasinetti area and consider the relationship between our above welfare results and the golden rule theorem.

We will pose the following problem: cannot we say anything in a little more detail about the anti-Pasinetti area by applying the golden rule theorem to our present model?

The answer to this problem is expected to be in the affirmative since the optimization problem considered in the discussion of the golden rule is the same as that we have considered in the anti-Pasinetti case. Both consider the maximization of the total consumption per unit of labor.

PROPOSITION 14: Suppose $(i,\ s_c)$ belongs to the anti-Pasinetti area. In such a case, if $s_c = 1$, then the golden-rule average propensity to save, s_g, equals s_w^1, at which C_w/L is maximized and $S = P$; and if $s_c < 1$, there can be two cases: (i) if $i \leqq n$ then, $s_g = s_w^1$; and (ii) if $i > n$, then, $s_g > s_w^1$.

Proof. Phelps (1961, 1965) showed the following result: C_w/L for a given average propentity to save, s_w, is a strictly increasing (constant, strictly decreasing) function of the given parameter s_w if and only if $S <$ (=, and >, respectively) P, with the understanding that the functional relationship between C_w/L and s_w is in the sense of the comparative statics.

For each fixed $(i,\ s_c)$, we will compare the magnitudes of S and P at $s_w = s_w^1$.

By Definitions 4, 6 and 8, we have the relationships, $k_2/f(k_2) = s_w^1/n$ and $f'(k_2) = i$. It follows that, at $s_w = s_w^1$, S/Y in equilibrium equals $nk_2/f(k_2)$.

If $s_c = 1$, then, the assumption of $n > is_c$ implies $n > i$. Since $f'(k_2) = i$, this implies

$$S/Y = nk_2/f(k_2) > f'(k_2)k_2/f(k_2) = (P/Y)_{s_w = s_w^1}. \quad (96)$$

Hence, C_w/L is strictly maximized at $s_w = s_w^1$ over the anti-Pasinetti region, $s_w \geqq s_w^1$. Possibly, C_w/L may be strictly increased by reducing s_w from the level of s_w^1 (although the economy inevitably enters the Pasinetti case).

Suppose $s_c < 1$ and $i \leqq n$. Then, the same reasoning and result apply as those in the above case of $s_c = 1$.

Suppose $s_c < 1$ and $i > n$. Then, the opposite of the above inequlity, (96), holds, and C_w/L can surely be strictly increased by raising s_w from the level of s_w^1 to *some* optimal golden-rule level, s_g, of s_w. Q. E. D.

In summary, we can depict such a figure as Fig. 12, in which the anti-Pasinetti area is divided into two subareas, in one of which C_w/L is maximized at $s_w = s_w^1$ and in the other of which it is maximized at some $s_g > s_w^1$.

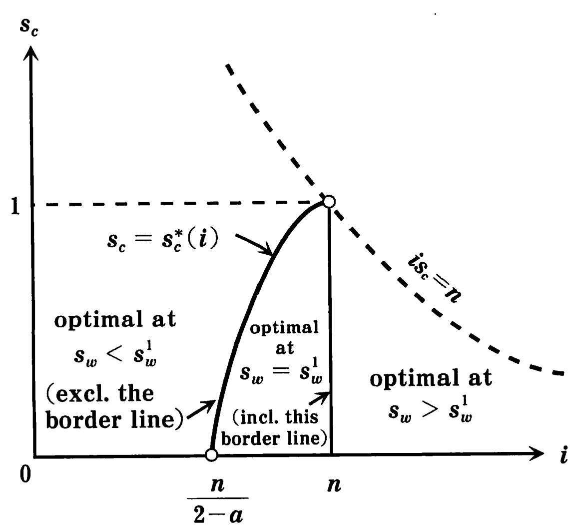

Fig. 12

APPENDICES

Appendix 6

In this appendix, we will prove the existence, uniqueness, and global stability of the Pasinetti steady state in the case where $s_w = 0$.

By $s_w = 0$ and (7), we have $\dot{k} = s_c\{kf'(k) - ik + ik_c\} - nk = \{s_c f'(k) - (n + is_c)\}k + is_c k_c$. Hence, the equation for Curve 1 becomes $k_c = \{(n + is_c) - s_c f'(k)\}k/(is_c)$.

Let k_3 denote the horizontal intercept of Curve 1. Then k_3 is the value of k such that $f'(k) = i + (n/s_c)$. So, we have $f'(k) > n/s_c = f'(k^*)$. This means that the intercept k_3 is less than k^*.

The k-coordinate of the intersection point between Curve 1 and the 45-degree line equals the value of k such that $\{(n + is_c) - s_c f'(k)\}/(is_c) = 1$, so that $f'(k) = n/s_c$. This means that the k-coordinate equals k^*.

Furthermore, Curve 1 is always upward-sloped. This is because the equation for Curve 1 may be arranged into the form, $k_c/k = \{(n + is_c) - s_c f'(k)\}/(is_c)$, the right-hand side of which is a strictly increasing function of k. This increasing property of the function

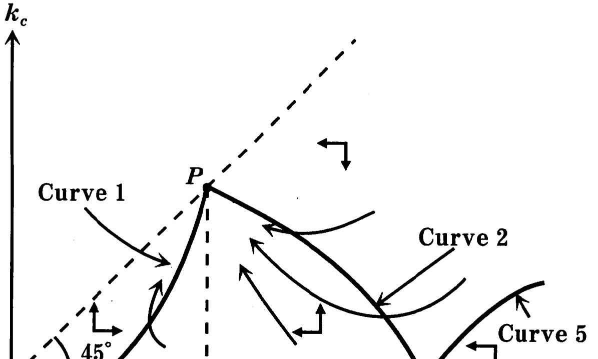

Fig. A8

implies, of course, that the tangent of the straight line from the origin to the point on Curve 1 increases as k rises. It will follow that Curve 1 is upward-sloped for all k such that $k_3 \leqq k \leqq k^*$.

The form of Curve 2 is the same as in the case where $s_w > 0$.

Therefore, the phase diagram in this case looks like that indicated in Fig. A8. The equilibrium is the point (k^*, k^*), whose k_c-coordinate $(= k^*)$ is of course positive. This means that it is a Pasinetti steady state. This point is a unique steady state. Furthermore, Fig. A8 shows that it is globally stable. Q. E. D.

Appendix 7

In this appendix, we will assume that $s_w^1 > s_c$, and consider the phase diagrams in the case when $i \geqq f'(k^{**})$.

If $i = f'(k^{**})$, then the phase diagram will look like that in Fig. A9. If $i > f'(k^{**})$, it will look like that in Fig. A10.

Both Figs. A9 and A10 show that the equilibrium point, D, is on the horizontal axis, so that it is the anti-Pasinetti equilibrium.

They also show that the anti-Pasinetti steady state is uniquely determined and globally stable.

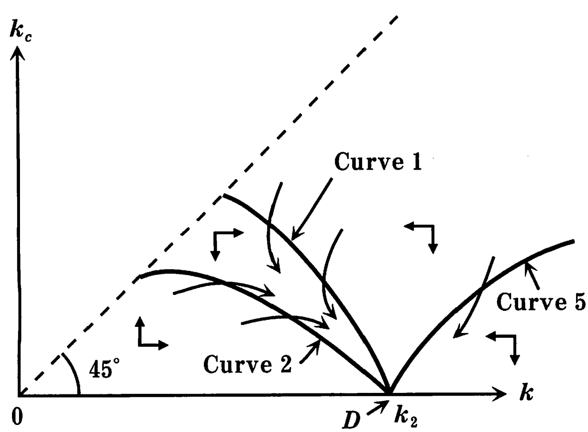

Fig. A9

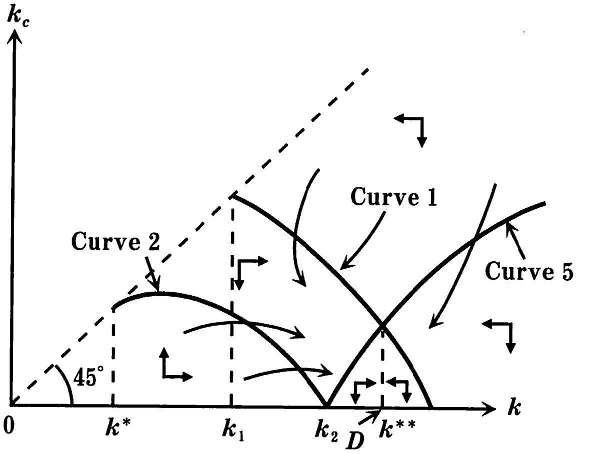

Fig. A10

Appendix 8

Here, we will show that, if $i = 0$, then $k_c > 0$ in equilibrium. Substituting i=0 in (7) and (8), we have

$$\dot{k} = (s_c - s_w)kf'(k) + s_w f(k) - nk \tag{a41}$$

$$\dot{k}_c = s_c kf'(k) - nk_c \tag{a42}$$

Let us denote by $\eta(k)$ the right-hand side of the above first equation, viewed as a function of k. Then, by $s_c f'(k^*) - n = 0$, we have $\eta(k^*) = s_w\{f(k^*) - k^* f'(k^*)\} > 0$. Hence, of course we have $\eta(k^*)/k^* > 0$. This holds true irrespectively of $s_w < s_c$ or $s_w \geqq s_c$.

The function, $\eta(k)/k$ converges to $-n$ as $k \to \infty$, which, again, holds in either case of $s_w < s_c$ or $s_w \geqq s_c$. Indeed, $\eta(k)/k = (s_c - s_w) f'(k) + s_w\{f(k)/k\} - n \to -n$ as $k \to \infty$, in either case, because we have assumed that $\lim_{k\to\infty} f'(k) = 0$. We can prove $\lim_{k\to\infty} f(k)/k = 0$ from this assumption.

By the continuity of the function $\eta(k)/k$, it follows that there exists at least one value, say k_4, such that $k_4 > k^*$ and $\eta(k_4)/k_4 = 0$. For such a value k_4, we have $\dot{k} = 0$. (In the case of $s_w < s_c$, the monotonely decreasing properties of the functions $f'(k)$ and $f(k)/k$ imply that k_4 is uniquely determined and $\dot{k} > 0$ if $k < k_4$ and $\dot{k} < 0$ if $k > k_4$.)

In either case, Curve 1 is vertical: a vertical line in the case of $s_w < s_c$, or vertical lines as in the other case, at $k = k_4$ (there may exist *plural* k_4's in the latter case).

On the other hand, Curve 2 has such a form as indicated in Fig. A11.

Firstly, Curve 2 intersects the 45-degree line at $k = k^*$. Because by putting the second equation for $\dot{k}_c$ equal to zero, we have the equation for Curve 2. Then we know that the function for the tangent of the straight line from the origin to the point on Curve 2 is $k_c/k = (s_c/n)f'(k)$. This tangent equals unity if and only if Curve 2 intesects the 45-degree line. But then, we have $f'(k) = n/s_c$, so that $k = k^*$.

Secondly, the tangent obviously decreases as k increases.

Finally, since the function for the tangent is always positive, Curve 2 is always above, and does not intercept, the horizontal axis.

Therefore, as Fig. A11 indicates, the intersection point between Curve 1 and Curve 2 must be above the horizontal axis. This means that any equilibria in this case of $i = 0$ must be Pasinetti equilibria

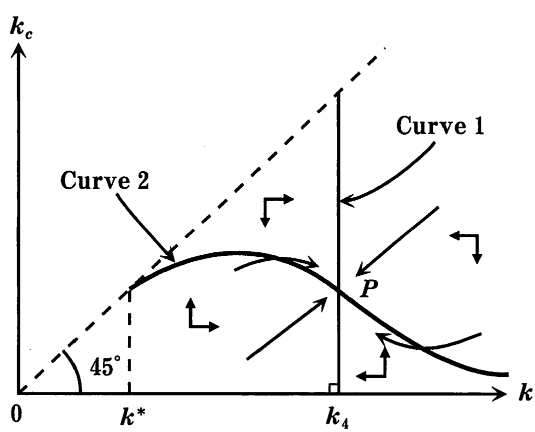

Fig. A11

and not anti- Pasinetti.

Appendix 9

In this appendix, we will prove (79), which says that:

$dX(s_w;\ i,\ s_c)/ds_w$ is always of the same sign as that of the function $Z(s_w;\ i,\ s_c)$, defined as

$$Z(s_w;\ i,\ s_c) = \{i(na+n-i)s_c+n(i-ai-n)\}s_w+an^2(1-s_c), \tag{a43}$$

for all $(s_w,\ i,\ s_c) \in \{s_w\colon 0 \leqq s_w \leqq s_w^1\} \times T$, where $T = \{(i,\ s_c)\colon i > 0,\ 1 \geqq s_c > 0,\ n > is_c\}$.

By (40), $X(s_w;\ i,\ s_c) = (C_w/L)(s_w;\ i,\ s_c)$, that is, a function of $s_w;\ i,\ s_c)$, whose explicit form is

$$(C_w/L)(s_w;\ i,\ s_c) = (1-s_w)(1-a)n^{(2a-1)/(a-1)}(n-is_w)^{-1/(1-a)} [\{(1-a)n-is_c\}s_w+ans_c]^{a/(1-a)}. \tag{a44}$$

Then, we have

$$\begin{aligned}\log(C_w/L) = {} & \log(1-s_w)+\log(1-a) \\ & +\{(2a-1)/(a-1)\}\log n \\ & -\{1/(1-a)\}\log(n-is_w) \\ & +\{a/(1-a)\}\log[\{(1-a)n-is_c\}s_w+ans_c]\end{aligned} \tag{a45}$$

so that

$$\begin{aligned} & d\{\log(C_w/L)(s_w;\ i,\ s_c)\}/ds_w \\ & = -\{1/(1-s_w)\}+\{i/(1-a)\}/(n-is_w) \\ & \quad +\{a/(1-a)\}\{(1-a)n-is_c\}/ \\ & \quad [\{(1-a)n-is_c\}s_w+ans_c]\end{aligned} \tag{a46}$$

Reducing to a common denominator, the right-hand side becomes one fraction, whose denominator (denoted by H) equals

$$H = (1-s_w)(n-is_w)(1-a)[\{(1-a)n-is_c\}s_w+ans_c], \tag{a47}$$

and whose numerator equals

$$
\begin{aligned}
&= -(1-a)[\{(1-a)n-is_c\}s_w+ans_c](n-is_w) \\
&\quad +(1-s_w)[\{(1-a)n-is_c\}s_w+ans_c]i \\
&\quad +(1-s_w)(n-is_w)a\{(1-a)n-is_c\} \\
&= -(1-a)[\{(1-a)n-is_c\}s_w(n-is_w)+ans_c(n-is_w)] \\
&\quad +(1-s_w)[\{(1-a)n-is_c\}s_w+ans_c]i \\
&\quad +(1-s_w)(n-is_w)a\{(1-a)n-is_c\} \\
&= -[\{(1-a)n-is_c\}s_w(n-is_w)+ans_c(n-is_w)] \\
&\quad +a[\{(1-a)n-is_c\}s_w(n-is_w)+ans_c(n-is_w)] \\
&\quad +(1-s_w)[\{(1-a)n-is_c\}s_w+ans_c]i \\
&\quad +(n-is_w)a\{(1-a)n-is_c\} \\
&\quad -s_w(n-is_w)a\{(1-a)n-is_c\}. \qquad \text{(a48)}
\end{aligned}
$$

The second term after the last equality contains the term, $s_w(n-is_w)a\{(1-a)n-is_c\}$, which is cancelled out by the last term. Hence, the numerator

$$
\begin{aligned}
&= -[\{(1-a)n-is_c\}s_w(n-is_w)+ans_c(n-is_w)] \\
&\quad +a^2ns_c(n-is_w) \\
&\quad +(1-s_w)[\{(1-a)n-is_c\}s_w+ans_c]i \\
&\quad +(n-is_w)a\{(1-a)n-is_c\}. \qquad \text{(a49)}
\end{aligned}
$$

We rearrange the whole term with respect to s_w, in the form, $A_2s_w^2+A_1s_w+A_0$ where

$$A_2 = \{(1-a)n-is_c\}i-\{(1-a)n-is_c\}i = 0, \qquad \text{(a50)}$$

$$
\begin{aligned}
A_1 &= -\{(1-a)n-is_c\}n+ans_ci-a^2ns_ci \\
&\quad +\{(1-a)n-is_c\}-ans_ci-ai\{(1-a)n-is_c\}, \qquad \text{(a51)}
\end{aligned}
$$

$$A_0 = -an^2s_c+a^2n^2s_c+ans_ci+na\{(1-a)n-is_c\}. \qquad \text{(a52)}$$

Therefore, since

$$
\begin{aligned}
A_1 &= \{(1-a)n-is_c\}(-n+i-ai)-a^2ns_ci \\
&= \{(1-a)n-is_c\}\{-n+(1-a)i\}-a^2ns_ci \\
&= (1-a)\{-n^2+n(1-a)i-i^2s_c\}+is_cn-a^2ns_ci \\
&= (1-a)\{-n^2+n(1-a)i-i^2s_c\}+(1-a)(1+a)is_cn
\end{aligned}
$$

$$= (1-a)\{-n^2+n(1-a)i-i^2s_c+(1+a)is_cn\}$$
$$= (1-a)\{i(n+an-i)s_c+n(-n+i-ai)\} \quad \text{(a53)}$$

and

$$A_0 = an^2(a-1)s_c+n^2a(1-a) = an^2(1-a)(1-s_c), \quad \text{(a54)}$$

we have

$$d\{\log(C_w/L)(s_w;\ i,\ s_c)\}/ds_w$$
$$= (1-a)[\{i(n+na-i)s_c+n(-n+i-ai)\}s_w$$
$$+an^2(1-s_c)]/H. \quad \text{Q. E. D} \quad \text{(a55)}$$

Appendix 10

In this appendix, (95) will be proved, and the form of the graph of $s_c^*(i)$ will be considered.

In Section 3.3.2, we have defined $h(s_c;\ i) \equiv Z(s_w^1;\ i,\ s_c)$. By $s_w^1 = an/i$, we have

$$Z(s_w^1;\ i,\ s_c) = \{i(na+n-i)s_c$$
$$+n(i-ai-n)\}(an/i)+an^2(1-s_c). \quad \text{(a56)}$$

By (90), we qualified $s_c^*(i)$ by $h(s_c^*,\ i) = 0$

It follows that, for any given i, we have

$$0 = \{i(na+n-i)s_c^*(i)+n(i-ai-n)\}(an/i)+an^2\{1-s_c^*(i)\}$$
$$= an(na+n-i)-an^2\}s_c^*(i)+(an/i)\{n(i-ai-n)+in\}$$
$$= an(na-i)s_c^*(i)+(an/i)n(2i-ai-n) \quad \text{(a57)}$$

Hence $s_c^*(i) = n\{(2-a)i-n\}/\{i(i-an)\}$, which is (93). Q. E. D.

Let us proceed to consider the properties of the graph of $s_c^*(i)$.

By (90), (91), and (92), it is obvious that $s_c^*(i)$ is meaningful only when $i > n/(2-a)$. Hence, the meaningful part of the graph of $s_c^*(i)$ is only that for $i > n/(2-a)$.

Also, $s_c^*(i)$ has actual meaning only when s_w can rise up to the value of $s_w^1 = an/i$, because $s_c^*(i)$ represents the critical level of s_c such that, for the given i, if s_c is equal to, or less than this $s_c^*(i)$, s_w will rise (as a result of the workers' utility-maximization) so much as the anti-Pasinetti steady state is attained, and s_w^1 is nothing but the minimum level to which s_w must at least rise, in order for the economy to reach an anti-Pasinetti steady state.

If s_w^1 equals unity, s_w cannot reach the level of s_w^1, because s_w cannot reach unity, since the workers' average propensity to consume, c_w, cannot fall to zero.

And if s_w cannot rise up to s_w^1, then no anti-Pasinetti steady state will be attained, and hence no actual meaning of $s_c^*(i)$.

It follows that $s_w^1 < 1$ is necessary for $s_c^*(i)$ to have an actual meaning.

$s_w^1 < 1$ implies $an/i < 1$, or $i-an > 0$.

For these reasons, we have $(2-a)i-n > 0$ and $i-an > 0$, implying that both the numerator and denominator of the equation for $s_c^*(i)$ must be positive at all i for which the graph of $s_c^*(i)$ can be drawn.

This allows us to take logarithm of $s_c^*(i)$ as follows:

$$\begin{aligned}\log s_c^*(i) &= (\log n) + \log\{(2-a)i-n\} \\ &\quad -(\log i) - \{\log(i-an)\}. \end{aligned} \tag{a58}$$

Hence, we have

$$\begin{aligned} d\{\log s_c^*(i)\}/di &= \{ds_c^*(i)/di\}/s_c^*(i) \\ &= [(2-a)/\{(2-a)i-n\}] - (1/i) - \{1/(i-an)\} \end{aligned} \tag{a59}$$

Reducing the right-hand side to a common denominator, the common denominator equals $\{(2-a)i-n\}i(i-an) > 0$.

On the other hand, the numerator is calculated as follows:

$$\begin{aligned}(2-a)i(i-an) - \{(2-a)i-n\}(i-an) - i\{(2-a)i-n\} \\ = n(i-an) - i\{(2-a)i-n\} \\ = 2ni - 2i^2 + a(i^2-n^2) \\ = 2(n-i)i + a(i-n)(i+n) \\ = (i-n)\{(a-2)i+an\}. \end{aligned} \tag{a60}$$

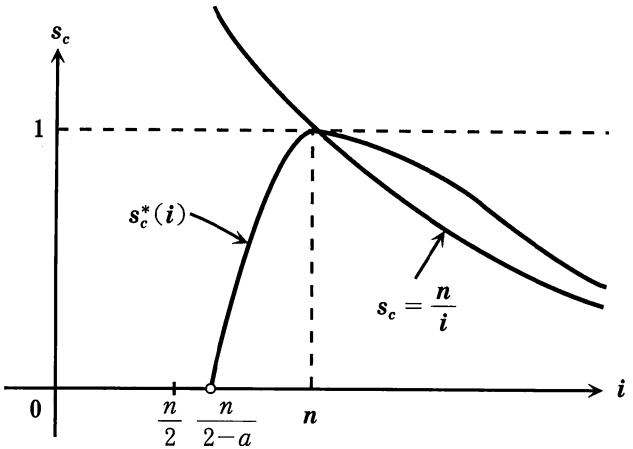

Fig. A12

Since $n/(2-a) < n$ we have $an/(2-a) < n$. Therefore, $an/(2-a) < i < n$ is a possible interval for i, and for all i belonging to this interval, $s_c^*(i)$ is increasing. Because, the above numerator is indeed positive so that the first derivative of $s_c^*(i)$ is positive for this interval.

For $i > n$, the numerator, and hence $ds_c^*(i)/di$, are negative, so that $s_c^*(i)$ is decreasing.

Also, let us remark here that $s_c^*(n) = 1$, so that the graph of $s_c^*(i)$ goes through the point, $(n, 1)$.

Therefore, the graph of $s_c^*(i)$ has such a form as indicated in Fig. A12.

As for the relationship between the positions of the graph of $s_c^*(i)$ and the rectangular hyperbola, $s_c = n/i$ (Fig. 11), we can say that the former is above the latter whenever $i > n$. Because the ratio of $s_c^*(i)$ to n/i equals $\{(2-a)i-n\}/(i-an)$, which is greater than unity since $\{(2-a)i-n\}-(i-an)=(1-a)i-(1-a)n=(1-a)(i-n) > 0$ if $i > n$.

Appendix 11: On the Concepts of the Aggregative Capital and the Neoclassical Production Function

In this appendix, we will consider the assumption of the existence of the aggregative capital, K, and the assumption of the neoclassical, well-behaved production function both of which are made in this paper.

In Section 1 of this appendix, it will be argued that, once the aggregation of capital is admitted, then it will be almost straightforward for us to reach the assumption of the well-behaved neoclassical production function.

Though the present writer does not necessarily regard the problem concerning the aggregation of capital to be formidable, however, the aggregation itself has been disputed by many economists.

In Section 2, we will consider the opinions of a few economists concerning the capital controversy, and try to show that their views concerning this problem are at least ambivalent, and are never decisively negative to the concept of the neoclassical production with only one kind of capital.

1. The Aggregative Capital Almost Implies the Neoclassical Production Function

In this section, we will make the following assumptions.

ASSUMPTION A1: The aggregation of capital (K) is possible.

ASSUMPTION A2: The aggregation of commodities (Y) is possible.

ASSUMPTION A3: The aggregation of labor (L) is possible.

ASSUMPTION A4: The production is done by use of only capital and labor.

ASSUMPTION A5: The production function obeys the constant returns to scale.

and, finally,

ASSUMPTION A6: The marginal product of labor is decreasing.

Under these assumptions, we can prove the following theorem.

PROPOSITION a5: Let the production function, $Y = F(K, L)$, be twice-differentiable with respect to both K and L. Then, $F_{KK}(K, L) < 0$ whenever $F_L > 0$, where F_L and F_{KK} denote the partial derivative of F with respect to L and the second order partial derivative of F with respect to K.

This proposition has been substantially well known. E. g., Sato (1968) uses the function, $g(e) = F(K, L)/K = F(1, e)$, where $e = L/K$. He of course assumes $g'(e) < 0$, that is, the marginal product of labor is decreasing.

Proof. Let $g(e) = F(K, L)/K$ and $e = L/K$. Then, by Assumption A5, we have $g(e) = F(1, e)$. By $F_L > 0$, we have $g'(e) = F_L(1, e)/K > 0$.

By Assumption A6, we also have $F_{LL}(K, L) < 0$. Hence, $g''(e) = F_{LL}(1, e)/K^2 < 0$.

Since $F_K = (Kg(e))_K = g(e) - Kg'(e)(-(L/K^2)) = g(e) - g'(e)e$, we have $F_{KK} = (g(e) - g'(e)e)_K = -g''(e)e(-L/K^2) = g''(e)e^2/K < 0$. Q. E. D.

This proposition ensures that the marginal productivity of capital exists and is decreasing under the above assumptions and the additional (trivial) assumption of $F_L > 0$.

Most importantly, this theorem ensures that the marginal product of capital must be decreasing once we accept the assumptions of the aggregations of capital, labor and product, of constant returns to scale, and of the decreasing marginal product of *labor*.

This is important because the decreasing property of the marginal product of capital is not anything but what has been the most seriously disputed point in the capital controversy (Harcourt, 1969 and

1972).

If all the assumptions of Proposition a5 are accepted, the most seriously attacked property of the marginal product of capital would come to be defended.

Therefore, it will be in order for us to examine the validity of the assumptions of Proposition a5, one by one.

In the opinion of the present writer, Assumption A1, saying that the aggregation of capital is possible, is a *premise of macroeconomics.* In this view, this assumption is a starting point of macroeconomics just as are Assumptions A2 and A3, saying that the aggregations of product and labor are possible.

If the validity of the aggregation of capital is doubted, the validity of the aggregations of product and labor will be also doubted, since the aggregations of product and labor involve the for midable problem of the price indeces concerning the prices of the various kinds of commodities and labor which prevents any clear-cut and consistent aggregations of product and labor, just as the problem involved in the aggregation of capital prevents any clear-cut and consistent aggregation of capital.

Such a view of the present writer is also stated by Hicks. He says, "The measurement of capital and the measurement of product are at bottom two aspects of the same problem..." (p. 190, line 8, Hicks, 1981).

Assumption A4, saying that the production is done only by capital and labor, seems to be a universally accepted assumption in the modern theory of growth. Indeed, this assumption was made in the original Harrod-Domar models, and also in the works on economic growth by such anti-neoclassical writers as Robinson (1956), Kaldor (1956), and Pasinetti (1962) who were the major opponents against the decreasing property of marginal product of capital.

Assumption A5, of constant returns to scale, is very much doubted by Kaldor (as pointed out in Hicks, 1989), though Kaldor uses this assumption in his major contribution to the neo-Keynesian theory of economic growth (Kaldor, 1956). Robinson and Pasinetti however, seem to accept it. This assumption, therefore, may be regarded as an almost universally accepted assumption in the growth theory.

Finally, the validity of Assumption A6, saying that the marginal product of labor is decreasing, is difficult even for the neo-Keynesian school to refute. Indeed, Paul Davidson uses this assumption for one of his papers (Davidson, 1983), though he there uses this assumption in order to maintain that the marginal product curve of labor is not the demand curve for labor. Davidson seems to take the position of accepting the decreasing property of the marginal product of labor but of maintaining that the labor market is almost always in disequilibrium.

However, this disequilibrium argument of Davidson is a short-run analysis which is not necessarily relevant to the theory of growth in the long-run. In the theory of growth, irrespective of whether it is neoclassical or neo-Keynesian or whatever, quite properly abstracts from such a short-run disequilibrium.

Even such a neo-Keynesian, or Post Keynesian, writer as Davidson accepts the decreasing property of the marginal product of labor, although such neo-Keynesian theorists have been the main critics in the capital controversy against the decreasing marginal product of capital.

It will follow that Assumption A6 is also an almost universally accepted assumption.

All the assumptions for Proposition a5 are assumptions which are accepted not only by the neoclassical but also the neo-Keynesian schools, and in this sense almost universally accepted assumptions in the modern theory of economic growth.

The decreasing property of the marginal product of capital is therefore difficult to refute even to the neo-Keynesian school who has so furiously attacked it in the capital controversy.

2. One-Capital-Good vs. Many-Capital-Good

In this section, we will briefly consider what a few famous economists say on the well-behaved neoclassical production function.

It was an unfortunate event that the so-called nonreswitching theorem enunciated by Levhari proved false in such a very severe and dramatic manner as it was attacked by the many bullets of the counter examples made by Morishima (1966), Bruno-Burmeister-Sheshinski (1966), and Pasinetti (1966). (Also see Levhari and Samuelson, 1966).

For Levhari's nonreswitching theorem had tried to refute the anti-neoclassical criticism against the monotonely decreasing property of the marginal product of capital, by asserting that the reswitching — that is, the phenomenon in which lowering the interest rate does not raise the capital-output ratio but reduces it — is impossible "in an indecomposable' technology (which means a situation in which every single output requires, directly or indirectly as input for its production, something positive of every single other output)" (Merton, ed., 1972, Chapter 149, pp. 246-247, which is a reprint of Levhari and Samuelson, 1966).

The fact that Samuelson and Levhari admitted that the nonreswitching theorem was in error has been taken by many economists to mean a complete defeat of the neoclassical, American Cambridge side against the neo-Keynesian, British Cambridge opponent.

This interpretation is very often the case for economists who are much acquainted with,and consonant to, the latter school of thought.

However, Samuelson's position concerning the neoclassical production function is not totally and decisively negative but is certainly ambivalent.

Let us follow what Samuelson says in "A Summing Up" (Merton, ed., 1972, Chapter 148, which is a reprint of Samuelson, 1966)

> Whether it is empirically rare for [the phenomenon of reswitching] to happen is not an easy question to answer. My suspicion is that a modern mixed economy has so many alternative techniques that it can, so to speak, use time usefully, but will run out of new

> equally profitable uses and is likely to operate on a curve of diminishing returns' (at least after non-constant-returns-to-scale opportunities have been exhausted). In any case, by the time one reaches a zero interest rate (or more generally the Golden Rule state where the interest and growth rates are equal), this kind of diminishing returns must have set in (p. 244, line 3-11, Merton, ed., 1972, a reprint of Samuelson, 1966).

In the conclusion of the paper, Samuelson also says, "Pathology illuminates healthy physiology." Needless to say, the "pathology" indicates the phenomenon of reswitching, and the "healthy physiology" means the monotonely decreasing property of the marginal product of capital. What he is trying to say by this phrase is that the marginal product of capital can be believed to be *ordinarily* decreasing, so that there ordinarily holds the diminishing returns to capital. Samuelson's more recent attitude to the neoclassical production function is that it is "over-simplifying." Samuelson uses this adjective to the neo-classical production function in the title of a paper (Nagatani and Crowley, eds., 1977, Chapter 215, a reprint of Samuelson, 1976) and in another paper (Nagatani and Crowley, eds., 1977, Chapter 216, p. 42, line 3 from below). In these papers, he presented a deeper criticism to the neoclassical production function which is concerned with a different point from that connected with the reswitching. Of course, many-capital-good model will need to be studied in order to answer such a criticism to the one-capital-good model as being too simple.

As early as in 1965, Hicks had already explored the many-capital-good model, and reached his own position to the neoclassical production function (Hicks, 1965). In short, Hicks's position is that "the one-capital-good model is "a fair simplification of the ... general [many-capital-good] model" (pp. 165-166, footnote 2, Hicks, 1965). After discussing the possibility of the factor-price curves for different techniques intersecting more than once, Hicks comments, "[in the many-capital-good model, the possibility of the more-than-once intersections] has to be taken rather (I do not think one need say more than *rather*) more seriously" (parentheses and italic are Hicks's, p. 166, line 6–9, Hicks, 1965). Here, Hicks seems to be warning the reader that the possibility of reswitching should *not* be too much exaggerated.

Finally, let us see what Solow says about the capital controversy. Solow (1988) says, "[the] whole episode [of the Cambridge capital controversy] now seems to me to have been a waste of time, a playing-out of ideological games in the language of analytical economics. ... [T] he argument was about marginalism, about smooth marginalism" (p. 309, Solow, 1988). Solow regards the capital controversy as merely sterile discussions.

All in all, in this section, we have seen that all of Samuelson, Hicks, and Solow are of the opinion that the mere possibility of reswitching is far from being a final destructive blow against the neo-classical, well-behaved production function.

In concluding this appendix, let us cite the following passage at the end of Solow's comment on "The Unimportance of Reswitching" by Robinson (1975).

> ... Suppose that, long ago and in another country, I had accepted the standard theory of consumer behavior — utility maximization subject to a budget constraint — but I had somehow thought that this theory implied that all demand curves were downward sloping. Then someone showed me that the Giffen good was a clear possibility within the theory. I would have to kiss a neat generalization good-by, and its immediate consequences too, but the theory of consumer demand would evidently not tumble on that account. (p. 329, line 8 from below, Solow, No. 70, Vol. II of Wood and Woods, eds., 1989, a reprint of Solow, 1975).

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