Price Flexibility and Instability in a Macrodynamic Model with a Debt Effect

ASADA, Toichiro

(出版者 / Publisher)
Institute of Comparative Economic Studies, Hosei University / 法政大学比較経済研究所

(雑誌名 / Journal or Publication Title)
Journal of International Economic Studies

(巻 / Volume)
18

(開始ページ / Start Page)
41

(終了ページ / End Page)
60

(発行年 / Year)
2004-03

(URL)
https://doi.org/10.15002/00002492

Journal of International Economic Studies (2004), No.18, 41-60

Price Flexibility and Instability in a Macrodynamic Model with a Debt Effect

Toichiro Asada
Faculty of Economics, Chuo University, Tokyo, Japan

Abstract

In this paper, we investigate the impact of price flexibility on macroeconomic instability using as our analytical framework a macrodynamic model with a Fisher debt effect. We introduce the expectations-augmented Phillips curve and the adaptive expectations hypothesis into a macrodynamic model with a Fisher debt effect, as developed by Asada (2001). We demonstrate analytically that both of the increase of the speed of price adjustments and the increase of the speed of expectations adaptation contribute to destabilizing rather than stabilizing the economy, and we also show that at the intermediate ranges of the parameter values, cyclical fluctuations occur by means of the Hopf bifurcation theorem. We also present some numerical examples which support our analytical results.

1. Introduction

In the standard version of macroeconomics, it is taken for granted that the full employment equilibrium will be attained automatically if wages and prices are flexible. According to this view, a sufficient *reduction* of nominal wages and price levels will contribute to an *increase* of output and employment when the economy is in depression. If this view is correct, the cause of persistent unemployment must be the rigidities of wages or prices. This notion is not unique to the so called 'classical' or 'neoclassical' school, but is also shared by the textbook version of the 'Keynesian' IS-LM model and the 'new Keynesian' school which concentrates on the microeconomic interpretation of wage or price rigidities. However, this conventional view apparently contradicts the widespread experiences of the Japanese and Asian economies in the 1990s. In particular, in the Japanese economy in the late 1990s, the downward pressure of nominal wages and prices seemed to contribute to aggravating rather than mitigating the depression, and seemed to destabilize rather than stabilize the economy. If we trace back economic history to the period of the Great Depression in the 1930s, we can observe essentially the same phenomenon in many countries including the United States, Britain, Germany and Japan. This phenomenon, which is sometimes called a 'deflationary spiral', is generally ignored in the orthodox economic literature, which stress the stabilizing effect of price flexibility, although it has recently attracted some attention among economic journalists. In both 1930s and the 1990s, the impact of financial factors such as debt on the real economy seems to have played an important role in macroeconomic performance. Nevertheless, the impact of debt on macroeconomic performance is practically ignored in the standard versions of macroeconomics[1].

However, there is a somewhat heretical tradition in economic thought, which stresses the importance of the impact of debt on macroeconomic performance. The most famous examples are Fisher (1933), Keynes (1936) and Minsky (1975 and 1986). For example, Fisher (1933) argued that a decrease of prices in a period of depression will cause a further decline of the demand for goods through an increase of the real debt burden. We shall call this negative impact of the increase of the real debt burden on effective demand (in particular, on investment) the 'Fisher debt effect'. Recently, some authors have developed various mathematical models which try to capture the economic interaction between real and financial factors, under the inspiration of the earlier works by Fisher, Keynes, and Minsky[2]. In particular, Keen (2000) and Chiarella, Flaschel and Semmler (2000 and 2001) studied the dynamics of debt deflation in macroeconomic models with variable prices and a Fisher debt effect.

In this paper, we also investigate the impact of price flexibility on macroeconomic stability or instability, using as our analytical framework of a macrodynamic model with a Fisher debt effect. It must be noted that our model is somewhat similar to those of Chiarella, Flaschel and Semmler (2000 and 2001), but the structure is much simpler. In fact, our model consists of an at most five-dimensional (and at least three-dimensional) system of differential equations, while those of Chiarella, Flaschel and Semmler (2000 and 2001) consist of an at least seven-dimensional (and at most twenty-dimensional) system of differential equations. Although our model is less general than theirs, it has merit in the sense that it can focus more sharply and clearly on the essence of the problem. We introduce the expectations-augmented Phillips curve and the adaptive expectations hypothesis into a macrodynamic model with a Fisher debt effect as developed by Asada (2001). The Introduction of price flexibility into the model increases the dimensions, so that the basic model in this paper, as presented in Section 2, consists of a three-dimensional system of nonlinear differential equations instead of a two dimensional system as in the original model by Asada (2001). In Section 3, we show analytically that both an increase of the speed of price adjustments and an increase of the speed of expectations adaptation contribute to destabilizing rather than stabilizing the economy, and we also show that at the intermediate range of the parameter values, cyclical fluctuations occur by means of the Hopf bifurcation theorem. In other words, in our model an increase of the price flexibility is responsible for macroeconomic instability. This is contrary to the conclusion of the textbook version of macroeconomics which ignores the debt effect. In Section 4, we present some numerical examples which support our analytical results. In Section 5, we show that the main conclusion of our basic model is robust even if we consider a more complicated model consisting of a four-dimensional system of nonlinear differential equations. The final section is devoted to an economic interpretation of the results which we obtained analytically and numerically.

2. Formulation of the basic model

The basic model in this paper consists of the following system of equations[3].

$$\dot{d} = \phi(g(\beta y, \rho-\pi^e, d)) - s_f\{\beta y - i(\rho, d)d\} - \{g(\beta y, \rho-\pi^e, d)+\pi\}d \quad (1)$$

$$\dot{y} = \alpha[\phi(g(\beta y, \rho-\pi^e, d)) + (1-s_f)\{\rho\nu + i(\rho, d)d\} - \{s_f + (1-s_f)s_r\}\beta y] \quad (2)$$

$$\pi = \varepsilon(y-y^*) + \pi^e \quad (3)$$

$$\dot{\pi}^e = \gamma(\pi - \pi^e) \quad (4)$$

where the meanings of the symbols are as follows. $d = D/(pK)$ = debt-capital ratio, $y = Y/K$ = output-capital ratio, which is also called the 'rate of capital utilization', y^* = equilibrium level of the output-capital ratio, which corresponds to the 'natural rate of employment', D = nominal stock of firms' private debt, p = price level, K = real capital stock, Y = real output (real national income), $g = \dot{K}/K$ = rate of capital accumulation, ρ = nominal rate of interest of interest-bearing safe assets, i = nominal rate of interest which is applied to firms' private debt, $\pi = \dot{p}/p$ = rate of price inflation, π^e = expected rate of price inflation. The function $\phi(g)$ is the adjustment cost function of investment, and, α, β, γ, ε, ν, s_f, and s_r are parameters which will be explained later.

Next, let us explain how these equations are derived. We can express the dynamic law of motion of private debt as follows.

$$\dot{D} = \phi(g)pK - s_f(rpK - iD) \quad (5)$$

where r is the rate of profit (i.e., $r = P/K$, where P is the real profit), and s_f is the rate of internal retention of firms, which is assumed to be constant $(0 < s_f \leqq 1)$[4]. The function $\phi(g)$ is the adjustment cost function of investment with the properties $\phi'(g) \geqq 1$ and $\phi''(g) \geqq 0$, as introduced by Uzawa (1969)[5]. By the definition of d, we have

$$\dot{d}/d = \dot{D}/D - \dot{p}/p - \dot{K}/K = \dot{D}/D - \pi - g. \quad (6)$$

Substituting Eq. (5) into Eq. (6), we have

$$\dot{d} = \phi(g) - s_f(r - id) - (g+\pi)d. \quad (7)$$

As for the dynamic of the goods market, we assume the following Keynesian quantity adjustment process.

$$\dot{y} = \alpha(c+h-y);\; c = C/K,\; h = E/K \quad (8)$$

where C is real consumption expenditures, $E = \phi(g)K$ is real investment expenditures including adjustment cost, and α is the speed of adjustments in the goods market, which is assumed to be a positive parameter[6]. For consumption expenditures, we adopt the following Kaleckian formulation of the two-class economy[7].

$$C = C_w + C_r \quad (9)$$

$$C_w = W = Y - P \tag{10}$$

$$C_r = (1-s_r)\{(1-s_f)P + \rho(V/p) + i(D/p)\};\ 0 < s_r \leqq 1 \tag{11}$$

where C_w is workers' real consumption, C_r is capitalists' real consumption, W is real wage income, V is the nominal value of interest-bearing safe assets, and s_r is the capitalists' average propensity to save, which is assumed to be constant. Eq. (10) implies that workers do not save, so that workers' real consumption expenditures are identical to real wage income. Eq. (11) implies that the capitalists save a part of their real income, which consists of three parts, i.e., the receipt of real dividends $((1-s_f)P)$, real interest receipts on interest-bearing safe assets $(\rho V/p)$, and real interest receipts on corporate debt (iD/p)[8].

Substituting equations (9), (10), and (11) into Eq. (8), we obtain

$$\dot{y} = \alpha[\phi(g) + (1-s_r)\{\rho(V/pK) + id\} - \{s_f + (1-s_f)s_r\}r]. \tag{12}$$

Usually, i, the rate of interest of 'risky' assets, will be higher than ρ, and the difference between i and ρ will reflect the degree of risk. To capture this fact, let us assume that

$$i = \rho + \xi(d) = i(\rho, d);\ \xi(d) \geqq 0,\ i_d = \xi'(d) > 0 \text{ for } d > 0,$$
$$i_d < 0 \text{ for } d < 0. \tag{13}$$

To simplify the analysis, we further assume that

$$P/Y = \beta = \text{constant},\ 0 < \beta < 1, \tag{14}$$

$$V/pK = \nu = \text{constant} > 0, \tag{15}$$

$$\rho = \text{constant} > 0. \tag{16}$$

Eq. (14) means that the share of profit in national income (P/Y) is constant. As is well known, P/Y is an increasing function of the mark up of the price on the average wage cost. In fact, by definition we have

$$p = z(wN/Y) = zw/a;\ z > 1 \tag{17}$$

where z is the mark up, w is the nominal wage rate, N is labor employment, and $a = Y/N$ is average labor productivity, so that we have the following relationship.

$$\beta = P/Y = (Y-W)/Y = 1-(W/Y)$$
$$= 1 - \{(w/p)N/Y\} = 1-(1/z) \tag{18}$$

Therefore, β is determined if we suppose that z is determined, for example, by the 'degree of monopoly' in the sense of Kalecki (1971). We assume that z is constant so that β is also constant. In this case, we can express the rate of profit as

$$r = P/K = \beta Y/K = \beta y. \tag{19}$$

In other words, the rate of profit is proportional to the output-capital ratio in our model.

Eq. (15) is merely a simplifying assumption to keep the structure of the model as simple as possible and avoid unnecessary complications. Eq. (16) implies that the nominal rate of interest (ρ) is constant through time. This means that we are implicitly assuming that the monetary authority (central bank) passively accommodates the money supply to the demand for money, to keep ρ constant. This hypothesis is in line with a Post Keynesian 'horizontalist' view in the sense of Moore (1988)[9].

To close the system, we must specify the investment function of the corporate sector. We assume the following investment function with a Fisher debt effect.

$$g = g(r, \rho-\pi^e, d);\ g_r = \partial g/\partial r > 0,\ g_{\rho-\pi} = \partial g/\partial(\rho-\pi^e) < 0,$$
$$g_d = \partial g/\partial d < 0 \tag{20}$$

This type of investment function has some microeconomic foundations. For example, we can derive it from firms' maximization behavior of the expected net cash flow by using both Uzawa (1969)'s hypothesis of increasing adjustment cost (the so called Penrose effect) and Kalecki (1937)'s hypothesis of increasing risk of investment[10].

Substituting equations (13), (14), (15), (16), (19) and (20) into equations (7) and (12), we obtain equations (1) and (2).

Now, let us turn to price dynamics. We assume the following type of expectations-augmented wage Phillips curve.

$$\dot{w}/w = \varepsilon(y-y^*)+\pi^e \tag{21}$$

where ε is the speed of the wage adjustment, which is assumed to be a nonnegative parameter. In this formulation, we adopt the rate of capital utilization (y) as a proxy for the rate of employment (e) or tightness of the labor market. This procedure, which is derived from Franke and Asada (1994), can simplify the analysis by saving one state variable, the rate of employment. We can justify this simplification due to the high correlation between the two variables y and e over the business cycle (cf. Franke and Asada (1994)). In fact, we shall show in Section 5 that the explicit introduction of the variable e into the model only adds some complexity to the model without affecting the main conclusion.

It follows from Eq. (17) that

$$\pi = \dot{p}/p = \dot{w}/w \tag{22}$$

if we assume that the markup (z) and the average labor productivity (a) are constant. Substituting Eq. (22) into Eq. (21), we have Eq. (3). Eq. (4) is a formalization of the adaptive expectations hypothesis of the price inflation, and γ is the speed of adaptation, which is assumed to be a nonnegative parameter.

Substituting Eq. (3) into equations (1) and (4), we obtain the following system of three-dimensional nonlinear differential equations.

$$\text{(i)}\quad \dot{d} = \phi(g(\beta y, \rho-\pi^e, d))-s_f\{\beta y-i(\rho, d)d\}$$
$$-\{g(\beta y, \rho-\pi^e, d)+\varepsilon(y-y^*)+\pi^e\}d \equiv f_1(d, y, \pi^e; \varepsilon)$$
$$\text{(ii)}\quad \dot{y} = \alpha[\phi(g(\beta y, \rho-\pi^e, d))+(1-s_r)\{\rho\nu+i(\rho, d)d\}$$
$$-\{s_f+(1-s_f)s_r\}\beta y] \equiv f_2(d, y, \pi^e; \alpha)$$
$$\text{(iii)}\quad \dot{\pi}^e = \gamma\varepsilon(y-y^*) \equiv f_3(y; \gamma, \varepsilon) \tag{S_1}$$

In this system, we can adopt the parameters ε and γ as measures of the degree of price flexibility. We can assume that the larger ε and γ are, the more flexible are the prices. If we assume that $\varepsilon = \gamma = \pi^e = 0$, this system is reduced to a two-dimensional system with respect to d and y. This special case with fixed prices was studied by Asada (2001). Asada (2001) showed that $\dot{d}$ becomes an increasing function of y and $\dot{y}$ becomes a decreasing function of d under some reasonable assumptions. This implies that the formal structure of this two-dimensional system is similar to the famous Volterra-Lotka system of predator-prey[11]. In fact, in this model d plays the role of predator and y the role of prey, and the interaction between these two state variables causes cyclical fluctuations (the so called Minsky cycle) under some range of the parameter value α. We can also show that the increase of α tends to destabilize the system (cf. Asada (2001)). In the next section, we will study analytically how the increase of price flexibility (increase of ε or γ) affects the dynamic stability or instability of the three-dimensional system (S_1).

3. Analysis of the basic model

First, let us consider the equilibrium solution of the system (S_1) which satisfies the condition $\dot{d} = \dot{y} = \dot{\pi}^e = 0$. The equilibrium values of d, y, and π^e are determined by the following system of equations.

$$\begin{aligned} &(\text{i}) \quad f_1(d, y^*, \pi^e) = 0 \\ &(\text{ii}) \quad f_2(d, y^*, \pi^e) = 0 \\ &(\text{iii}) \quad y = y^* > 0 \end{aligned} \tag{23}$$

We will study the local stability/instability of an equilibrium point by *assuming* that the equilibrium solution such that $d > 0$ exists. It is worth noting that the equilibrium values of d, y and π^e are *independent* of the parameter values α, γ and ε. The Jacobian matrix of this system *at the equilibrium point* can be expressed as follows[12].

$$J_1 = \begin{bmatrix} f_{11} & f_{12}(\varepsilon) & f_{13} \\ f_{21}(\alpha) & f_{22}(\alpha) & f_{23}(\alpha) \\ 0 & f_{32}(\gamma, \varepsilon) & 0 \end{bmatrix} \tag{24}$$

where

$$\begin{aligned}
f_{11} &= \partial f_1/\partial d = (\underset{(-)}{\phi'}(g) - d)\underset{(-)}{g_d} - g - \pi^e + s_f(\underset{(+)}{i_d} d + i), \\
f_{12}(\varepsilon) &= \partial f_1/\partial y = \beta\{(\underset{(+)}{\phi'}(g) - d)\underset{(-)}{g_r} - s_f\} - \varepsilon d, \\
f_{13} &= \partial f_1/\partial \pi^e = -(\underset{(+)}{\phi'}(g) - d)\underset{(-)}{g_{\rho-\pi}} - d, \\
f_{21}(\alpha) &= \partial f_2/\partial d = \alpha[\underset{(+)}{\phi'}(g)\underset{(-)}{g_d} + (1 - s_r)(\underset{(+)}{i_d} d + i)], \\
f_{22}(\alpha) &= \partial f_2/\partial y = \alpha\beta[\underset{(+)}{\phi'}(g)\underset{(+)}{g_r} - \{s_f + (1 - s_f)s_r\}], \\
f_{23}(\alpha) &= \partial f_2/\partial \pi^e = -\alpha\underset{(+)}{\phi'}(g)\underset{(-)}{g_{\rho-\pi}} > 0, \\
f_{32}(\gamma, \varepsilon) &= \partial f_3/\partial y = \gamma\varepsilon \geqq 0.
\end{aligned} \tag{25}$$

Now, let us assume as follows.

Assumption 1.
$f_{11} < 0,\ f_{12}(0) > 0,\ f_{13} > 0,\ f_{21}(\alpha) < 0$, and $f_{22}(\alpha) > 0$.

Assumption 2.
$f_{11}f_{22}(\alpha) - f_{12}(0)f_{21}(\alpha) > 0$, and $f_{11}f_{23}(\alpha) - f_{13}f_{21}(\alpha) > 0$.

The inequalities in **Assumption 1** will in fact be satisfied if $\phi'(g)$, g_r, $|g_{\rho-\pi}|$, and $|g_d|$ are sufficiently large *at the equilibrium point.* We can interpret **Assumption 2** in economic terms as follows. It is easy to show that the following relationships are satisfied.

$$\lim_{s_r \to 1}\{f_{11}f_{22}(\alpha) - f_{12}(0)f_{21}(\alpha)\} = \alpha\beta[\{-d\underset{(-)}{g_d} - g + (d\underset{(+)}{g_r} + s_f)(\underset{(+)}{i_d} + i)\}\{\underset{(+)}{\phi'}(g)\underset{(+)}{g_r} - 1\} + s_f\underset{(+)}{\phi'}(g)\underset{(+)}{g_r}] \quad (26)$$

$$\lim_{s_r \to 1}\{f_{11}f_{23}(\alpha) - f_{13}f_{21}(\alpha)\} = \alpha\underset{(+)}{\phi'}(g)[\{g + \pi^e - (\underset{(+)}{i_d} + i)\}\underset{(-)}{g_{\rho-\pi}} - \underset{(-)}{g_d}d] \quad (27)$$

The right hand side of Eq. (26) will become positive if $\phi'(g)$, g_r, $|g_d|$, and i_d are sufficiently large. The right hand side of Eq. (27) will become positive if $|g_{\rho-\pi}|$, $|g_d|$, and i_d are sufficiently large. That is to say, the two inequalities in **Assumption 2** will in fact be satisfied if $\phi'(g)$, g_r, $|g_{\rho-\pi}|$, $|g_d|$, i_d, and s_r are sufficiently large *at the equilibrium point.*

The characteristic equation of this system becomes

$$\Delta_1(\lambda) \equiv |\lambda I - J_1| = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0 \quad (28)$$

where

(i) $a_1 = -traceJ_1 = -\underset{(-)}{f_{11}} - \underset{(+)}{f_{22}}(\alpha)$,

(ii) $a_2 = \begin{vmatrix} f_{22}(\alpha) & f_{23}(\alpha) \\ \gamma\varepsilon & 0 \end{vmatrix} + \begin{vmatrix} f_{11} & f_{13} \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} f_{11} & f_{12}(\varepsilon) \\ f_{21}(\alpha) & f_{22}(\alpha) \end{vmatrix}$

$= -\gamma\varepsilon\underset{(+)}{f_{23}}(\alpha) + \underset{(-)}{f_{11}}\underset{(+)}{f_{22}}(\alpha) - f_{12}(\varepsilon)\underset{(-)}{f_{21}}(\alpha)$,

(iii) $a_3 = -\det J_1 = \gamma\varepsilon\{\underset{(-)}{f_{11}}\underset{(+)}{f_{23}}(\alpha) - \underset{(+)}{f_{13}}\underset{(-)}{f_{21}}(\alpha)\}$,

(iv) $a_1a_2 - a_3 = \gamma\varepsilon\{\underset{(+)}{f_{13}}\underset{(-)}{f_{21}}(\alpha) + \underset{(+)}{f_{22}}(\alpha)\underset{(+)}{f_{23}}(\alpha)\}$
$+ \{-\underset{(-)}{f_{11}} - \underset{(+)}{f_{22}}(\alpha)\}\{\underset{(-)}{f_{11}}\underset{(+)}{f_{22}}(\alpha) - f_{12}(\varepsilon)\underset{(-)}{f_{21}}(\alpha)\}$. (29)

Now, we can prove the following proposition under Assumptions 1 and 2.

Proposition 1.
(i) The equilibrium point of the system (S_1) is locally asymptotically stable if all of the parameters α, γ and ε are positive and sufficiently small.
(ii) The equilibrium point of the system (S_1) becomes locally unstable if either

of the parameters α, γ, or ε is sufficiently large.

(Proof.)

(i) We can easily see, in view of Assumptions 1 and 2, that all of the Routh-Hurwitz conditions for stable roots (a set of inequalities (A2) in the mathematical appendix) are satisfied if α, γ, and ε are sufficiently close to zero.

(ii) We have $a_1 < 0$ for a sufficiently large α. On the other hand, we have $a_2 < 0$ if γ or ε is sufficiently large. In these cases, at least one of the inequalities (A2) in the mathematical appendix is violated. □

This proposition implies that the increase of price flexibility tends to *destabilize* rather than stabilize the economy in our model with a Fisher debt effect, contrary to the teaching of 'classical' and 'neoclassical' macroeconomics. We will attempt to provide the intuitive economic explanation of this proposition in Section 6.

Next, let us investigate the effect of the changes of the parameter value γ on the dynamic properties of the system, by assuming that α and ε are relatively small. It is easy to show that

$$\lim_{s_r \to 1}\{f_{13}f_{21}(\alpha)+f_{22}(\alpha)f_{23}(\alpha)\} = -\alpha\underset{(+)}{\phi'(g)}[\{(\underset{(+)}{\phi'(g)}-d)\underset{(-)}{g_{\rho-\pi}}+d\}\underset{(-)}{g_d} + \alpha\beta\{\underset{(+)}{\phi'(g)}-1\}\underset{(-)}{g_{\rho-\pi}}]. \tag{30}$$

The right hand side of Eq. (30) becomes negative if $\phi'(g)$, $|g_{\rho-\pi}|$, and $|g_d|$ are sufficiently large and α is sufficiently small. In sum,

$f_{13}f_{21}(\alpha)+f_{22}(\alpha)f_{23}(\alpha)$ will become negative if $\phi'(g)$, $|g_{\rho-\pi}|$, $|g_d|$, and s_r are sufficiently large and α is sufficiently small. Now, let us posit the following additional assumption.

Assumption 3.

$\alpha > 0$, $\varepsilon > 0$, $a_1 = -f_{11}-f_{22}(\alpha) > 0$, $f_{11}f_{22}(\alpha)-f_{12}(\varepsilon)f_{21}(\alpha) > 0$, and $f_{13}f_{21}(\alpha)+f_{22}(\alpha)f_{23}(\alpha) < 0$.

Assumption 3 means that the values of the parameters α and ε are relatively small. Under Assumptions 1, 2, and 3, we obtain the following proposition.

Proposition 2.

There exists a parameter value $\gamma_0 > 0$ which satisfies the following properties.

(i) The equilibrium point of the system (S_1) is locally asymptotically stable for all $\gamma \in (0, \gamma_0)$, and is locally unstable for all $\gamma \in (\gamma_0, \infty)$,

(ii) At $\gamma = \gamma_0$, the Hopf bifurcation occurs. In other words, there exist some non-constant periodic solutions of the system (S_1) at some parameter values $\gamma > 0$ which are sufficiently close to γ_0.

(Proof.)

(i) Let us define the value γ_0 as follows.

$$\gamma_0 = \frac{\{-f_{11}-f_{22}(\alpha)\}\{f_{11}f_{22}(\alpha)-f_{12}(\varepsilon)f_{21}(\alpha)\}}{-\varepsilon\{f_{13}f_{21}(\alpha)+f_{22}(\alpha)f_{23}(\alpha)\}} \tag{31}$$

which is positive from **Assumption 3.** It is easy to show that $a_1a_2-a_3>0$ if $0 \leqq \gamma < \gamma_0$. On the other hand, we have $a_1>0$ and $a_3>0$ under Assumptions 2 and 3. Then, we also have $a_2 > a_3/a_1 > 0$. Therefore, all of the Routh-Hurwitz conditions for stable roots (a set of inequalities (A2) in the mathematical appendix) are satisfied if $0 \leqq \gamma < \gamma_0$. Next, let us suppose that $\gamma > \gamma_0$. In this case, we have $a_1a_2-a_3<0$, which implies that one of the Routh-Hurwitz conditions is violated.

(ii) At $\gamma = \gamma_0$ we have a set of conditions $a_1>0$, $a_3>0$, and $a_1a_2-a_3=0$, which also implies that $a_2 = a_3/a_1 > 0$. In this case, the characteristic equation (28) has a set of pure imaginary roots and a negative real root in view of **Theorem 2** in the mathematical appendix. Furthermore, we have $\partial(a_1a_2-a_3)/\partial\gamma < 0$, which implies that the real part of the imaginary roots becomes an increasing function of γ at $\gamma=\gamma_0$. This means that all of the conditions for the occurrence of the Hopf bifurcation (conditions (1)-(3) in **Theorem 5** in the mathematical appendix) are satisfied at $\gamma=\gamma_0$, since the equilibrium solution of the system (S_1) is independent of the value of γ. □

Proposition 2 (ii) implies that the cyclical fluctuation occurs at some intermediate values of the parameter γ. In other words, the endogenous alternations of booms which entail debt finance, and depressions which entail debt deflation (debt cycles or so called Minsky cycles) occur at some set of parameter values in our model.

4. A numerical illustration

In this section, we present some numerical examples which illustrate the main conclusions of the previous section. Let us assume the following parameter values and the functional forms[13].

$$s_f = s_r = 1,\ \beta = 0.3,\ i = \rho + 0.1d^2,\ \rho = 0.08,\ y^* = 0.25,\ \phi(g) = g,$$
$$g = y^{0.3} - (\rho - \pi^e) - 2d - 0.12. \tag{32}$$

In this case, the system (S_1) becomes as follows.

$$\text{(i)}\quad \dot{d} = (y^{0.3}+\pi^e-2d-0.2)(1-d)-0.3y+0.08d+0.1d^3 -\varepsilon(y-0.25)d-\pi^e d$$
$$\text{(ii)}\quad \dot{y} = \alpha(y^{0.3}+\pi^e-2d-0.2-0.3y)$$
$$\text{(iii)}\quad \dot{\pi}^e = \gamma\varepsilon(y-0.25) \tag{33}$$

Fig. 1 through Fig. 5 show the time trajectories of y and d corresponding to the various parameter values of α, γ, and ε, and the common initial condition $d(0)=0.1$, $y(0)=0.2$, and $\pi^e(0)=0.05$.[14]

It seems that the long run equilibrium point is dynamically stable in all examples. In other words, the conditions for local stability seem to be satisfied in these examples. Nevertheless, endogenous cyclical fluctuations of y and d occur in all cases. That is to say, the endogenous alternations of periods of boom, which are

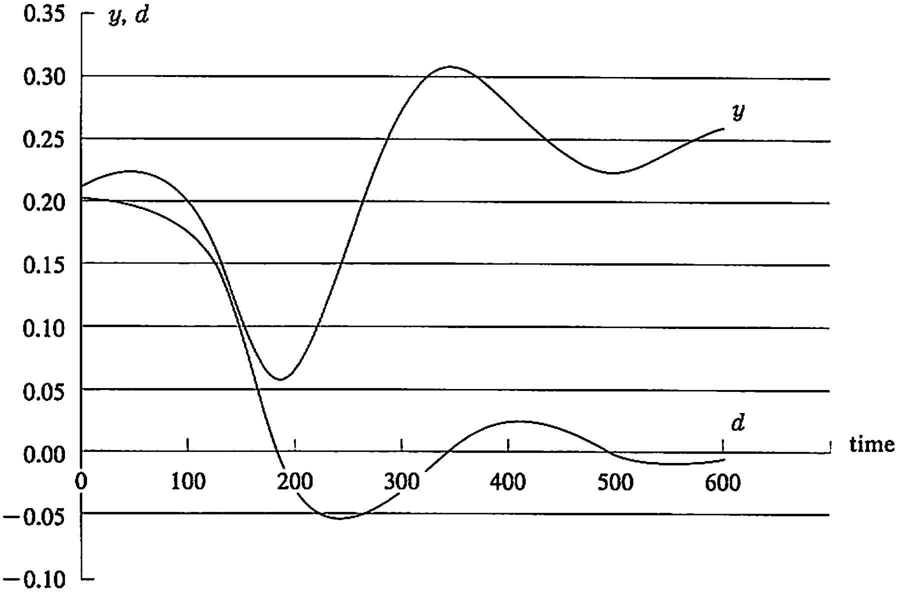

Fig. 1 $\alpha = 0.1$, $\gamma = 0.1$, $\varepsilon = 0.2$

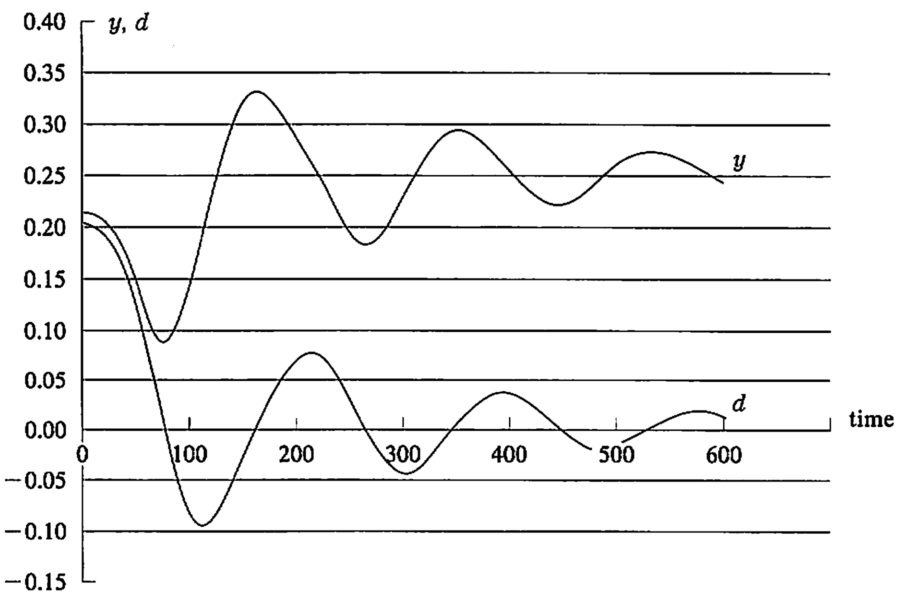

Fig. 2 $\alpha = 0.1$, $\gamma = 0.1$, $\varepsilon = 0.5$

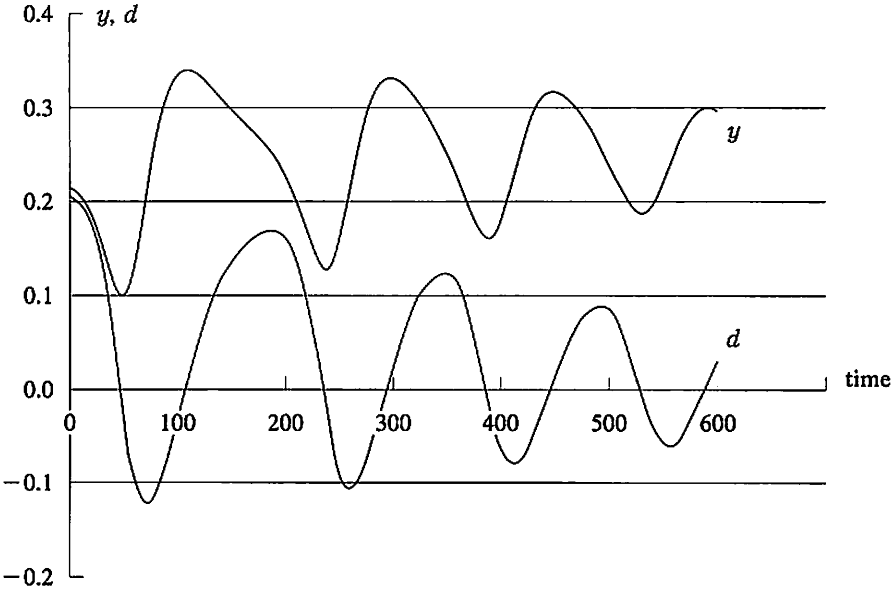

Fig. 3 $\alpha = 0.1,\ \gamma = 0.1,\ \varepsilon = 0.88$

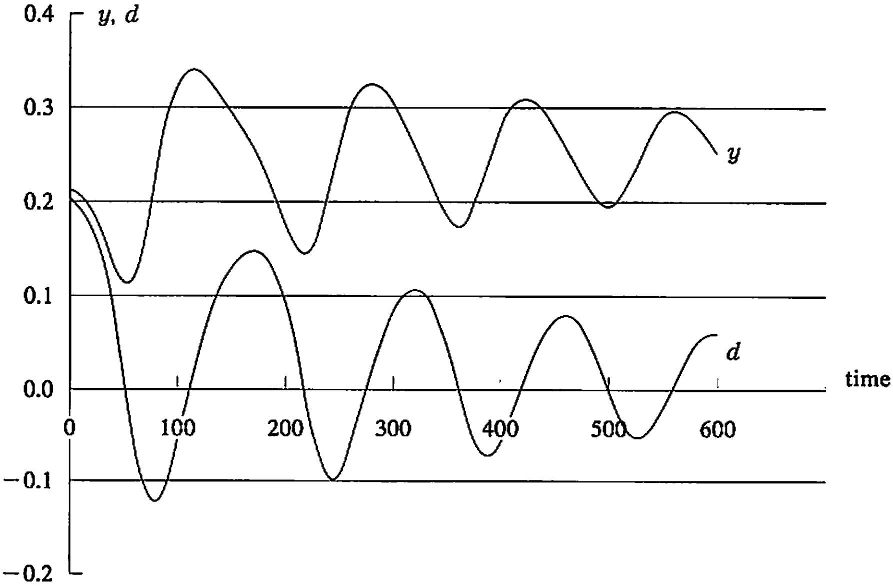

Fig. 4 $\alpha = 0.1,\ \gamma = 0.18,\ \varepsilon = 0.5$

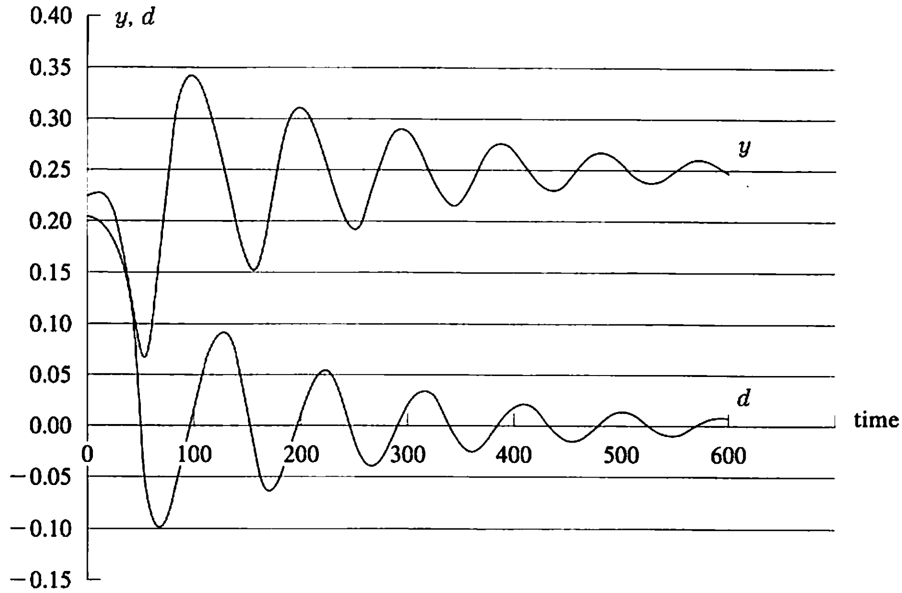

Fig. 5 $\alpha = 0.2$, $\gamma = 0.18$, $\varepsilon = 0.5$

supported by debt financing, and periods of debt deflation (so called debt cycles or Minsky cycles) occur in these examples. We observe that the turning points of d always lag behind those of y in these examples. This means that at the upper turning point of the business cycle, the debt burden still continues to increase, and at the lower turning point the debt burden continues to decrease for some time[15]. A comparison of Figures 1, 2, and 3 suggests that the increase of the value of parameter ε tends to shorten the period of the cycle and expand its amplitude. A comparison of Figures 2 and 4 suggests that the increase of the value of γ also has a similar effect. These observations lead us to the conclusion that roughly speaking, increased price flexibility tends to destabilize the system. This is precisely the main proposition of the previous section. A comparison of Figures 4 and 5 suggests that the increase of the value of parameter α tends to shorten the period of the cycle. These figures also show that an increase of the value of α lessons the amplitude of the business cycle, at least at some range of the combination of parameter values. In other words, the increase of the value of α has a stabilizing effect at least at some range of α, although we already know that a further increase of α will ultimately destabilize the system (cf. **Proposition 1** (ii) in the previous section).

5. Extension of the model

In this section, we formulate an extended model which explicitly introduces the fourth state variable, the rate of employment. Let us replace Eq. (21) in section 2 with the following more 'natural' form of the wage Phillips curve.

$$\dot{w}/w = \varepsilon(e-\bar{e})+\pi^e;\ 0 < \bar{e} = \text{constant} < 1 \tag{34}$$

where $e = N/N_s$ is the rate of employment, N is labor employment, and N_s is labor supply. By definition, we have

$$N = \frac{\{Y/K)K}{Y/N} = (yK)/a, \tag{35}$$

where $a = Y/N$ is average labor productivity. Substituting Eq. (35) into the definition of e and differentiating with respect to time, we obtain the following law of motion of the variable e.

$$\begin{aligned} \dot{e}/e &= (\dot{N}/N)-(\dot{N}_s/N_s) = (\dot{y}/y)+(\dot{K}/K)-(\dot{a}/a)-(\dot{N}_s/N_s) \\ &= (\dot{y}/y)+g(\beta y, \rho-\pi^e, d)-(\dot{a}/a)-(\dot{N}_s/N_s) \end{aligned} \tag{36}$$

On the other hand, we have the following expression from Eq. (17) if we assume a constant mark up.

$$\pi = \dot{p}/p = (\dot{w}/w)-(\dot{a}/a) \tag{37}$$

From equations (34) and (37) we obtain the following 'price Phillips curve'.

$$\pi = \varepsilon(e-\bar{e})-(\dot{a}/a)+\pi^e \tag{38}$$

In this section, we adopt the simplifying assumption of constant growth rates of labor supply and labor productivity, following the tradition of the 'old' growth theory. This implies that[16]

$$\dot{N}_s/N_s = n_1 = \text{constant} \geqq 0, \tag{39}$$

$$\dot{a}/a = n_2 = \text{constant} \geqq 0. \tag{40}$$

In this case, the system (S_1) in Section 2 must be modified to the following four-dimensional system of nonlinear differential equations.

$$\begin{aligned}
&\text{(i)} && \dot{d} = \phi(g(\beta y, \rho-\pi^e, d))-s_f\{\beta y-i(\rho, d)d\} \\
& && \quad -\{g(\beta y, \rho-\pi^e, d)+\varepsilon(e-\bar{e})-n_2+\pi^e\}d \equiv F_1(d, y, \pi^e, e; \varepsilon) \\
&\text{(ii)} && \dot{y} = \alpha[\phi(g(\beta y, \rho-\pi^e, d))+(1-s_r)\{\rho\nu+i(\rho, d)d\} \\
& && \quad -\{s_f+(1-s_f)s_r\}\beta y] \equiv F_2(d, y, \pi^e; \alpha) \\
&\text{(iii)} && \dot{\pi}^e = \gamma\{\varepsilon(e-\bar{e})-n_2\} \equiv F_3(e; \gamma, \varepsilon) \\
&\text{(iv)} && \dot{e} = \{F_2(d, y, \pi^e; \alpha)/y+g(\beta y, \rho-\pi^e, d)-(n_1+n_2)\}e \\
& && \quad \equiv F_4(d, y, \pi^e, e; \alpha)
\end{aligned} \tag{S_2}$$

The equilibrium point of this system $(d^*, y^*, \pi^{e*}, e^*)$ is determined by the following system of equations.

$$\begin{aligned}
&\text{(i)} && \phi(n_1+n_2)-s_f\{\beta y-i(\rho, d)d\}-(n_1+n_2+\pi^e)d = 0 \\
&\text{(ii)} && \phi(n_1+n_2)+(1-s_r)\{\rho\nu+i(\rho, d)d\}-\{s_f+(1-s_f)s_r\}\beta y = 0 \\
&\text{(iii)} && g(\beta y, \rho-\pi^e, d) = n_1+n_2 \\
&\text{(iv)} && e = \bar{e}+(n_2/\varepsilon)
\end{aligned} \tag{41}$$

The Jacobian matrix of this system *at the equilibrium point* may be written as

$$J_2 = \begin{bmatrix} F_{11} & F_{12} & F_{13} & -\varepsilon d \\ F_{21}(\alpha) & F_{22}(\alpha) & F_{23}(\alpha) & 0 \\ 0 & 0 & 0 & \gamma\varepsilon \\ F_{41} & F_{42} & F_{43} & 0 \end{bmatrix} \quad (42)$$

where $F_{11} = (\underset{(+)}{\phi'(g)} - d)\underset{(-)}{g_d} - (n_1 + n_2) - \pi^e + s_f(\underset{(+)}{i_d} d + i)$,

$F_{12} = \beta\{(\underset{(+)}{\phi'(g)} - d\}\underset{(+)}{g_r} - s_f\}$,

$F_{13} = -(\underset{(+)}{\phi'(g)} - d)\underset{(-)}{g_{\rho-\pi}} - d$, $F_{21}(\alpha) = \alpha[\underset{(+)}{\phi'(g)}\underset{(-)}{g_d} + (1-s_r)(\underset{(+)}{i_d} d + i)]$,

$F_{22}(\alpha) = \alpha\beta[\underset{(+)}{\phi'(g)}\underset{(+)}{g_r} - \{s_f + (1-s_f)s_r\}]$, $F_{23}(\alpha) = -\alpha\underset{(+)}{\phi'(g)}\underset{(-)}{g_{\rho-\pi}} > 0$,

$F_{41} = \{(F_{21}/y) + \underset{(-)}{g_d}\}e$, $F_{42} = \{(F_{22}/y) + \beta \underset{(+)}{g_r}\}e$, and $F_{43} = \{(F_{23}/y) - \underset{(-)}{g_{\rho-\pi}}\}e$.

Eq. (41) (iii) and (iv) imply that at the long run equilibrium point of this extended system, the rate of capital accumulation is equal to the exogenously determined 'natural rate of economic growth' $g^* = n_1 + n_2$, and the rate of employment is also equal to the exogenously determined 'natural rate of employment' $e^* = \bar{e} + (n_2/\varepsilon)$. This means that the equilibrium growth rate and equilibrium rate of employment are determined *independently* of financial factors, and the effective demand as far as the values of the parameters such as n_1, n_2, $\bar{e}$, or ε are not influenced by the financial factors or effective demand. In this sense, the extended version of our model has a somewhat 'classical' flavor, as far as we concentrate on the long run equilibrium point. However, the concentration on the long run equilibrium is *not* justified in our model, because the out- of-steady-state dynamics in this model are heavily influenced by the complex interaction of financial factors and effective demand. In fact, if we carry out some somewhat tedious calculations, we can prove that essentially the same results as **Proposition 1** and **Proposition 2** in the previous section also apply to this extended version of the model[17]. In other words, the qualitative behavior of the out-of-steady-state dynamics in the extended four-dimensional version of the system (S_2) is almost identical to that in the simpler three-dimensional version of the system (S_1). This means that the simpler version is sufficient for our purpose of formalizing the dynamics of the cycles of debt and income in a flexible price economy.

6. Discussion of the results

In Chapter 19 of *The General Theory,* Keynes wrote as follows.

"The depressing influence on entrepreneurs of their greater burden of debt may partly offset any cheerful reactions from the reduction of wages. Indeed if the fall of wages and prices goes far, the embarrassment of those entrepreneurs who are heavily indebted may soon reach the point of insolvency, —with severely adverse effect on investment." (Keynes (1936) p. 264) "It would be much better that wages should be rigidly fixed and deemed incapable of material changes, than that depressions should be accompanied by a gradual downward tendency of money wages." (Keynes (1936) p. 265) "There is, therefore, no ground for the belief that a flexible wage policy is capable of maintaining a state of continuous full employment; ... The economic system cannot be made self-adjusting along these lines." (Keynes (1936) p. 267)

It is clear from these quotations that Keynes did not consider the cause of the persistent unemployment and the economic fluctuation to be rigidities of wages or prices. This assertion by Keynes himself is contrary to the teachings of classical, neoclassical, and of the so called new Keynesian macroeconomics. In this paper we have formulated a macrodynamic model which provides some theoretical foundations to this somewhat heretical assertion by Keynes. The main destabilizing mechanism of price flexibility in our model is based on the 'Fisher debt effect', which may be represented schematically as follows[18].

$$(y\downarrow)\Rightarrow\pi\downarrow\Rightarrow d=D/(pK)\uparrow\Rightarrow g\downarrow\Rightarrow(y\downarrow) \qquad \text{(FDE)}$$

Furthermore, we have the following other destabilizing effect through changes of the expected real rate of interest, via changes of the price expectations, which is called the 'Mundell effect'[19].

$$(y\downarrow)\Rightarrow\pi\downarrow\Rightarrow\pi^e\downarrow\Rightarrow(\rho-\pi^e)\uparrow\Rightarrow g\downarrow\Rightarrow(y\downarrow) \qquad \text{(ME)}$$

These two effects have positive feedback mechanisms which are undoubtedly destabilizing. In other words, in these processes, a decrease of y induces a further decrease of y[20].

Needless to say, there also exist counteracting negative feedback mechanisms which have stabilizing effects. First, the increase of real corporate debt due to the fall of the price level during the period of depression implies an increase of the real wealth of creditors. Thus, it does not only have a negative impact on the corporate investment expenditure, but will also induce an increase of the consumption expenditure of creditors. This 'wealth effect' is explicitly incorporated in our model. However, it is doubtful whether the latter effect is strong enough to that it outweigh the former. In fact, in our model the stabilizing 'wealth effect' is relatively weak compared to the destabilizing 'debt effect'. The second stabilizing mechanism, which has been neglected in this paper up to now, is the so called 'Keynes effect' which works through the change of the nominal rate of interest. If we assume the standard Keynesian real demand function for money $L^D=\varphi(\rho)Y$ ($\varphi'(\rho)<0$), the equilibrium condition for the money market can be expressed as

$$M/p=\varphi(\rho)Y \qquad (43)$$

where M is the nominal money supply. Solving this equation with respect to ρ, we have the following standard type of the 'LM equation'.

$$\rho=\rho(m);\rho'(m)=1/\varphi'(\rho)<0,\ m=M/(pY) \qquad (44)$$

In our model, it was assumed that the monetary authority adopts the policy of keeping the nominal rate of interest (ρ) constant, which implicitly means that the monetary authority adopts a monetary policy of keeping m constant. It is obvious that the 'Keynes effect' does not work in this case. However, there is a room for the

Keynes effect if the monetary authority adopts an alternative policy, for example, to keep the growth rate of nominal money supply $(\dot{M}/M)$ constant rather than the nominal rate of interest. In this case, we obtain the following law of motion for the variable m by differentiating the definitional equation $m = M/(pY) = M/(pyK)$ with respect to time.

$$\begin{aligned}\dot{m}/m &= (\dot{M}/M)-(\dot{p}/p)-(\dot{y}/y)-(\dot{K}/K)\\ &= \mu-\pi-(\dot{y}/y)-g(\beta y, \rho-\pi^e, d)\end{aligned} \tag{45}$$

where $\mu = \dot{M}/M =$ constant. If we replace the assumption $\rho =$ constant with equations (44) and (45), the system (S_1) is transformed to a four-dimensional system, and the system S_2 becomes a five-dimensional system. First, let us consider the long run equilibrium solution. Substituting $\dot{m} = \dot{y} = 0$ and $g = n_1+n_2$ into Eq. (45), we obtain the following expression.

$$\mu-\pi-(n_1+n_2) = 0 \tag{46}$$

Eq. (46) characterizes the long run equilibrium position of both of the system with a fixed nominal rate of interest and with a fixed growth rate of the nominal money supply. However, the causality of determination is different between two systems. In the system with a fixed nominal rate of interest, this equation becomes the formula which determines the equilibrium growth rate of nominal money supply corresponding to the equilibrium rate of price inflation, which is pre-determined, while in the system with a fixed growth rate of nominal money supply, the equation becomes the formula which determines the equilibrium rate of price inflation corresponding to the exogenously given growth rate of money supply. What can we say about the out-of-steady-state dynamics of the system with a constant growth rate of money supply and variable nominal rate of interest? How does the change of the assumption from constant nominal rate of interest to constant growth rate of nominal money supply affect the dynamic stability of the system?

Without committing ourselves to a formal analysis, we can observe that this modification of the model will increase the stability of the system, at least potentially. In fact, the following stabilizing negative feedback effect, which is called the 'Keynes effect', will work if the sensitivity of money demand with respect to the change of the rate of interest $|\varphi'(\rho)|$ is so small that the decrease of π can induce a decrease of the expected real rate of interest $\rho-\pi^e$ through the sharp decrease of the nominal rate of interest ρ, in spite of the fact that π^e also falls.

$$(y\downarrow) \Rightarrow \pi\downarrow \Rightarrow m\uparrow \Rightarrow \rho\downarrow \Rightarrow (\rho-\pi^e)\downarrow \Rightarrow g\uparrow \Rightarrow (y\uparrow) \tag{KE}$$

But, it is quite unlikely that the Keynes effect will outweigh the Mundell effect when the economy is in depression. In general, the *nominal* rate of interest cannot fall below zero. In fact, the sensitivity of ρ with respect to the change of m will approach to zero because $|\varphi'(\rho)|$ becomes almost infinite if ρ is already close to zero. In this situation of the so called 'liquidity trap', the Keynes effect does not work. For example, in the Japanese economy in the late 1990s, the expected real rate of interest was still relatively high because of the fear of price deflation, although the *nominal* rate of interest was nearly zero. Furthermore, the corporate debt burden was high enough. As a result, corporate investment was held down to quite a low level. This recent Japanese experience is consistent with the theoretical conjecture derived from our model.

Our theoretical investigation of the dynamic interaction between real and financial factors suggests that the policy recommendation to increase the flexibility of wages and prices during a period of depression, without an active counter cyclical demand management policy, is likely to lead to disastrous results.

Acknowledgment

This is a revised version of a paper which was written while the author was a visiting scholar at the University of Bielefeld in Germany. An earlier version of this paper was published as Discussion Paper No. 483, Department of Economics, University of Bielefeld (February 2002), and was presented at the 6th International Conference on Macroeconomic Analysis and International Finance which was held at the University of Crete in Greece (25 May, 2002). The author is grateful to the University of Bielefeld for the generous provision of its facilities. This research was financially supported by the Japan Ministry of Education, Culture, Sports, Science and Technology (Grant-in-Aid for Scientific Research (B) 15330037) and Chuo University Joint Research Grant 0382.

Mathematical Appendix

In this appendix, we present some mathematical theorems which are useful for the derivation of the propositions in the text.

Theorem 1. (Routh-Hurwitz conditions for a three-dimensional system)

All of the real parts of the roots of the characteristic equation

$$\Delta_1(\lambda) = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0 \tag{A1}$$

are negative if and only if a set of inequalities

$$a_j > 0 \ (j = 1, 2, 3),\ a_1a_2 - a_3 > 0 \tag{A2}$$

is satisfied.

Theorem 2.

The characteristic equation (A1) has a pair of pure imaginary roots $\pm hi(i = \sqrt{-1}, h \neq 0)$ *if and only if* a set of conditions

$$a_2 > 0,\ a_1a_2 - a_3 = 0 \tag{A3}$$

is satisfied. In this case, we have the explicit solution $\lambda = -a_1, \pm i\sqrt{a_2}$.

Theorem 3. (Routh-Hurwitz conditions for a four-dimensional system)

All of the real parts of the roots of the characteristic equation

$$\Delta_2(\lambda) = \lambda^4 + b_1\lambda^3 + b_2\lambda^2 + b_3\lambda + b_4 = 0 \tag{A4}$$

are negative if and only if a set of inequalities

$$b_j > 0 (j = 1, 2, 3, 4,),\ b_1b_2b_3 - b_1^2b_4 - b_3^2 > 0 \tag{A5}$$

is satisfied.

Theorem 4.

The characteristic equation (A4) has a pair of pure imaginary roots and two roots with non-zero real parts *if and only if* either of the following set of conditions (A) or (B) is satisfied.

(A) $b_1 = 0$, $b_3 = 0$, and $b_4 < 0$.

(B) $b_1 b_3 > 0$, $b_4 \neq 0$, and $b_1 b_2 b_3 - b_1^2 b_4 - b_3^2 = 0$.

Theorem 5. (Hopf bifurcation theorem for an n-dimensional system)

Let $\dot{x} = f(x\,;\varepsilon)$, $x \in R^n$, $\varepsilon \in R$ be a system of an n-dimensional differential equations with a parameter ε. Suppose that the following properties (1)-(3) are satisfied.

(1) This system has a smooth curve of equilibria $f(x^*(\varepsilon);\varepsilon) = 0$.

(2) The characteristic equation $|\lambda I - Df(x^*(\varepsilon_0);\varepsilon_0)| = 0$ has a pair of pure imaginary roots $\lambda(\varepsilon_0)$, $\bar{\lambda}(\varepsilon_0)$ and no other roots with zero real parts, where $Df(x^*(\varepsilon_0);\varepsilon_0)$ is the Jacobian matrix of the above system at $(x^*(\varepsilon_0),\varepsilon_0)$ with the parameter value ε_0.

(3) $d\{\mathrm{Re}\lambda(\varepsilon)/d\varepsilon\}|_{\varepsilon=\varepsilon_0} \neq 0$, where $\mathrm{Re}\lambda(\varepsilon)$ is the real part of $\lambda(\varepsilon)$.

Then, there exists a continuous function $\varepsilon(\gamma)$ with $\varepsilon(0) = \varepsilon_0$, and for all sufficiently small values of $\gamma \neq 0$ there exists a continuous family of non-constant periodic solutions $x(t,\gamma)$ for the above dynamical system, which collapses to the equilibrium point $x^*(\varepsilon_0)$ as $\gamma \to 0$. The period of the cycle is close to $2\pi/\mathrm{Im}\lambda(\varepsilon_0)$, where $\mathrm{Im}\lambda(\varepsilon_0)$ is the imaginary part of $\lambda(\varepsilon_0)$.

Theorems 1, 3, and 5 in this appendix are well known among researchers of dynamic economics (cf. Gandolfo (1996) and Lorenz (1994)). On the other hand, Theorems 2 and 4 (and in particular, Theorem 4) may be relatively little known among economists, although their results are quite useful for our purposes. For the proof of Theorem 2, see Asada (1995) and Asada and Semmler (1995). Asada and Yoshida (2003) contains the proof of Theorem 4.

Notes

1 For a history of financial crises, see Kindleberger (1989) and Wolfson (1994).

2 See, for example, Asada (1999 and 2001), Chiarella, Flaschel and Semmler (2000 and 2001), Chiarella, Flaschel, Groh and Semmler (2000) Chap. 2, Delli Gatti and Gallegati (1994), Flaschel, Franke and Semmler (1997) Chap. 12, Franke and Semmler (1989), Keen (2000), and Palley (1996). Nasica (2000) contains an excellent survey of recent developments in this area of research.

3 A dot over a symbol denotes the derivative with respect to time.

4 We assume that there are no issues of new shares, and for simplicity we neglect the repayment of debt.

5 See also Asada (1999 and 2001) and Asada and Semmler (1995).

6 In this formulation, we neglect government expenditures and international trade for simplicity. In other words, we abstract the problems of public debt and foreign debt to concentrate on that of domestic private debt.

7 See Kalecki (1971).

8 If the capitalists are debtors and the corporate sector as a whole is a creditor, we must have $D < 0$, but we can expect that in a 'normal' situation, we have $D > 0$, meaning

that, the corporate sector as a whole becomes a debtor and the capitalists become creditors.

9 Lavoie (1995) asserts that the real rate of interest should be treated as an exogenous variable for Post Keyesian analysis. However, we do not follow his approach, because it is in fact more difficult for the monetary authority to control the real rather than nominal rate of interest in an economy with variable prices. Incidentally, in the final section, we will briefly discuss the 'verticalist' case in which the growth rate of nominal money supply rather than the nominal rate of interest is assumed to be constant.

10 For the derivation, see Asada (1999 and 2001). See also Asada and Semmler (1995).

11 For the Volterra-Lotka system, see Gandolfo (1996) Chap. 24 and Lorenz (1993) Chap. 2.

12 All values of f_{ij} are evaluated at the equilibrium point.

13 We do not claim that our simple numerical examples are quantitatively realistic. The purpose of this section is only to illustrate and make visible the main qualitative conclusions which were derived analytically in the previous section.

14 We adopted Euler's algorithm and the time interval $\Delta t = 0.1$ for the computer simulation. However, we found that we can obtain essentially the same results even if we adopt the time interval $\Delta t = 0.01$.

15 In these examples, in some periods of the business cycle D becomes negative. In other words, the corporate sector becomes a creditor rather than debtor in these periods. However, this feature is not essential for the qualitative pattern of movement of the main variables.

16 The recently developed 'new growth theory' focuses on the endogenous determination of n_1 and n_2, but this is not the theme of this paper. Incidentally, Eq. (40) implies that we are assuming a 'Harrodian neutral' exogenous technical progress.

17 Due to lack of space, the proof is omitted. We can make use of Theorems 3, 4, and 5 in the mathematical appendix to prove these propositions.

18 For the Fisher debt effect, see Chiarella, Flaschel and Semmler (2000 and 2001).

19 For the Mundell effect, see Flaschel, Franke and Semmler (1997) Chap. 7.

20 In some recent economic literature, the importance of positive feedback mechanisms or self-reinforcing mechanisms in the dynamic context is well recognized. See, for example, Agliardi (1998). It is worth noting, incidentally, that these positive feedback mechanisms work in periods of inflationary boom as well as in periods of depression.

References

Agliardi, E. (1998), *Positive Feedback Economics*, Macmillan, London.

Asada, T. (1995), Kaldorian Dynamics in an Open Economy, *Journal of Economics/ Zeitschrift für Nationalökonomie*, Vol. 62, pp. 239–269.

Asada, T. (1999), Investment and Finance: A Theoretical Approach, *Annals of Operations Research*, Vol. 89, pp. 75–89.

Asada, T. (2001), Nonlinear Dynamics of Debt and Capital: A Post-Keynesian Analysis, in Y. Aruka and Japan Association for Evolutionary Economics (ed.) *Evolutionary Controversies in Economics: A New Transdisciplinary Approach*, Springer-Verlag, Tokyo, pp. 73–87.

Asada, T. and W. Semmler (1995), Growth and Finance: An Intertemporal Model, *Journal of Macroeconomics*, Vol. 17, pp. 623–649.

Asada, T. and H. Yoshida (2003), Coefficient Criterion for Four-dimensional Hopf Bifurcations: A Complete Mathematical Characterization and Applications to Economic Dynamics, *Chaos, Solitons and Fractals*, Vol. 18, pp. 525–536.

Chiarella, C., P. Flaschel, G. Groh and W. Semmler (2000), *Disequilibrium, Growth and*

Labor Market Dynamics. Springer-Verlag, Berlin.

Chirella, C., P. Flaschel and W. Semmler (2000), The Macrodynamics of Debt Deflation, in R. Bellofiore and P. Perri (ed.) *Financial Fragility and Investment in the Capitalist Economy,* Edward-Elgar, Cheltenheim, U. K., pp. 133–183.

Chiarella, C., P. Flaschel and W. Semmler (2001), Price Flexibility and Debt Dynamics in a High Order AS-AD Model, *Central European Journal of Operations Research,* Vol. 9, pp. 119–145.

Delli Gatti, D. and M. Gallegati (1994), External Finance, Investment Expenditure and the Business Cycles, in W. Semmler (ed.) *Business Cycles: Theory and Empirical Methods,* Kluwer Academic Publishers, Boston, pp. 269–288.

Fisher, I. (1933), The Debt-deflation Theory of Great Depressions, *Econometrica,* Vol. 1, pp. 337–357.

Flaschel, P., R. Franke and W. Semmler (1997), *Dynamic Macroeconomics: Instability, Fluctuations, and Growth in Monetary Economies,* MIT Press, Cambridge, Massachusetts, U. S. A.

Franke, R. and T. Asada (1994), A Keynes-Goodwin Model of the Business Cycle, *Journal Economic Behavior and Organization,* Vol. 24, pp. 273–295.

Franke, R. and W. Semmler (1989), Debt-Financing of Firms, Instability, and Cycles in a Dynamical Macroeconomic Growth Model, in W. Semmler (ed.) *Financial Dynamics and Business Cycles: New Perspectives,* M. E. Sharpe, New York, pp. 38–64.

Gandolfo, G. (1996), *Economic Dynamics* (Third Edition), Springer-Verlag, Berlin.

Kalecki, M. (1937), The Principle of Increasing Risk, *Economica,* Vol. 4, pp. 440–447.

Kalecki, M. (1971), *Selected Essays on the Dynamics of the Capitalist Economy,* Cambridge University Press, Cambridge, U. K.

Keen, S. (2000), The Nonlinear Economics of Debt Deflation, in W. A. Barnett et. al. (ed.) *Commerce, Complexity, and Evolution,* Cambridge University Press, Cambridge, U. K., pp. 83–110.

Keynes, J. M. (1936), *The General Theory of Employment, Interest and Money,* Macmillan, London.

Kindleberger, C. (1989), *Manias, Panics, and Crashes: A History of Financial Crises* (Revised Edition), Basic Books, New York.

Lavoir, M. (1995), Interest Rates in Post-Keynesian Models of Growth and Distribution, *Metroeconomica,* Vol. 46, pp. 146–177.

Lorenz, H. W. (1993), *Nonlinear Dynamical Economics and Chaotic Motion* (Second Edition), Springer-Verlag, Berlin.

Minsky, H. P. (1975), *John Maynard Keynes,* Columbia University Press, New York.

Minsky, H. P. (1986), *Stabilizing an Unstable Economy,* Yale University Press, New Haven.

Moore, B. J. (1988), *Horizontarists and Verticalists: The Macroeconomics of Credit Money,* Cambridge University Press, Cambridge, U. K.

Nasica, E. (2000), *Finance, Investment and Economic Fluctuations: An Analysis in the Tradition of Hyman P. Minsky,* Edward Elgar, Cheltenheim, U. K.

Palley, T. (1996), *Post Keynesian Economics: Debt, Distribution and the Macro Economy,* Macmillan, London.

Uzawa, H. (1969), Time Preference and the Penrose Effect in a Two-Class Model of Economic Growth, *Journal of Political Economy,* Vol. 77, pp. 628–652.

Wolfson, M. H. (1994), *Financial Crises: Understanding the Postwar U. S. Experience,* M. E. Sharpe, New York.