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Noisy Screening Models for the Educational Cramp in Korea

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Abstract
This paper develops pedagogical job-matching screening models in order to explain the educational cramp in Korean National College Entrance Examination. The models developed here incorporate two kinds of factors into the standard screening model, which potentially make the examination in Korea noisy as a screening device. The first factor is measurement error associated with the examination itself. It is demonstrated that the matching efficiency from screening is aggravated not only by the measurement error itself but also by the individuals' self-selections in their applications. The second factor is the moral hazard problem caused by private tutoring. It is demonstrated that at reasonable screening equilibria overspending in private tutoring by low productivity types and underspending by high productivity types may exist simultaneously. These individuals' adverse selections in private tutoring may also aggravate the matching efficiency. Under these circumstances, the economic impact of Korean 7.30 Education Reform in 1980 is discussed. This reform initiated the prohibition of private tutoring for high school students in Korea.

I. Introduction

When individuals have comparative advantages in different jobs and the individual productivity types are private information, there is a matching problem in their choice of jobs. The population distribution of existing individual types may be well known in the market, but individual productivity may not be directly observable. Individuals who know themselves, or at least know more about themselves than do the others, have an incentive to pretend to be efficient in a high wage job. Because of this asymmetry of information, the privately optimal choice of jobs may not be socially optimal, therefore there is a social need for a selection mechanism in order to screen the efficient type from the inefficient type.

In Korea, the college entrance examination system plays the role of the social selection mechanism, and it can be said that the individuals' scores in the college entrance examination determine a private success in society. However, the excessive demand for the college education in Korea has forced students to struggle desperately to obtain sufficient scores on the college entrance examination. This

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struggle has resulted in the over-heated pursuit of private tutoring just for the sake of passing the examination. This specialized private tutoring is believed to prevent the educational system from serving as a fair and rational social mechanism for selection. For example, in 1979, total private expenditure for tutoring was estimated to be 165 million dollars which is equivalent to 14% of the annual budget of the Korean Education Ministry [1].

In order to reduce the social waste in private tutoring, the Korean government established the 7.30 Reform of Education in 1980, which contained the elimination of the over-heated pursuit for private tutoring. Hence, since August, 1981, any kind of private tutoring for high school students[2] was legally prohibited. Less surprisingly, after the policy was initiated, underground markets for private tutoring became well developed and it is estimated that social spending for private tutoring has never been reduced and people have questioned the validity of the policy.

This paper is intended to develop pedagogical job-matching screening models in order to explain the educational struggle in Korea. In the mode, two kinds of aspects of the college entrance examination are considered: (a) The college entrance examination as a test dividing the population into those who succeed and those who fail (the screening aspect). (b) The college entrance examination as a self-selection device where individuals' costs of taking the examination sort the population into applicants (more able) and non-applicants (less able).

The paper employs a simple job-matching screening model with two kinds of jobs where people may differ in their productivities. The circumstances considered in this paper are that an individual productivity type is initially unknown but may be revealed to a certain extent by some socially costly actions. In our model, people choose their jobs according to the individual types they declare, but people are tested by the pass- fail college entrance examination in order to check their declarations with their true types: People declaring themselves to be efficient types for job 1 must pass the college entrance examination in order to prove it. If they fail, they have to choose job 2. People declaring themselves as efficient types in job 2 are not tested. The paper assumes a continuum of individual types with a hierarchical property in which thee is an unambiguous ranking of individual types in terms of their productivity. In this context, if individual productivity types cannot be observed at all, a low productivity type will always have an incentive to declare himself as a high productivity type.

The theoretical base of the matching model borrows from the previous screening models including Guash et al. (1981), Weiss (1985) and Stiglitz (1975). In particular, this paper closely follows Stiglitz's analysis (1975) to show how the equilibrium price and the matching efficiency are determined. Compared to Stiglitz's model, the models in this paper incorporate the following potential factors which make the college entrance examination in Korea incomplete as a social selection device.

1 Source: 'Korean Educational Development (1980)' published by the National Institute of Korean Educational Development.
2 private tutoring for fine arts including art and music was exempted from the 7.30 Education Reform.

Measurement error

One such factor is measurement error associated with the examination itself. In Korea, the college entrance examination is given once a year and is administered by the Education Ministry. The main reason that such an important social selection mechanism is administered in a single day is to save the enormous cost of administering the examination. If a higher selection accuracy of the examination requires the Korean government to pay a greater cost to administer the examination, the perfect accuracy may be uneconomic, and measurement error is introduced into the scores of the examination.

With measurement error in the examination, the matching efficiency is reduced by two forces. The first force is the adverse selection of the examination itself. With more measurement error introduced into the examination, the probability of the high productivity types' passing the examination decreases and the probability of the low productivity types' passing the examination increases. Then, the adverse selection of the examination is more likely. The second force is the individual self-selections. As more measurement error is introduced, the low productivity types are more willing to infiltrate the examination and these infiltrations aggravate the matching efficiency. Therefore, the matching efficiency is aggravated not only by the noisy examination itself but also by the individual self-selection results.

If the examination has measurement error, individuals' cost of taking the examination which includes the direct application fee and the spending in private tutoring plays an effective role in sorting out the population. For example, the larger spending in private tutoring is more disadvantageous to the low productivity types than to the high productivity types because the high productivity types have higher probability of passing the examination than the low productivity types.

Under these circumstances, it is demonstrated that the 7.30 Education Reform may aggravate the matching efficiency because the smaller spending in private tutoring may stimulate the low productivity types to infiltrate the examination.

Private tutoring

If individuals are allowed to choose private tutoring which improves the score in the examination but does not affect individuals' attributes of economic productive skill, adverse selections in taking private tutoring may occur. If everybody spends an equal amount on private tutoring and the score in the examination is without measurement error, the examination can sort out the high productivity types with perfect accuracy. However, it is demonstrated that at reasonable screening eqilibrium overspending in private tutoring by low productivity types but underspending by high productivity types may exist simultaneously. These individuals' adverse selections will aggravate the matching efficiency.

Under these circumstances, it is suggested: if the 7.30 Education Reform had been successful in eliminating private tutoring for the inefficient type, it could have enhanced the matching efficiency and could have reduced social spending for private tutoring at the same time. However, since the 7.30 Education Reform could not prevent the underground market for private tutoring from developing, the matching efficiency gain from the reform is reduced and the social spending in private tutoring may even increase even though the fraction of low productivity types taking private tutoring decreases.

In this paper, two kinds of models are developed. The first one is a model for private tutoring causing the moral hazard problem in the college entrance examination and the second one is a model for measurement error.

II. The College Entrance Examination with a Moral Hazard Problem Caused by Private Tutoring

Variables used in this paper are defined as follows.

x: Index of individual's ability, $x \in A = \{x \mid 0 \le x \le 1\}$

$f(x)$: Productivity of individuals with ability x

r: Individuals' reservation wage

τ: Screening level of the examination ($0 \le \tau \le 1$)

s: Individuals' spending in private tutoring

Basic assumptions are made as follows.

Assumption 1: Individuals' ability types (x) and individuals' spending in private tutoring (s) are known only to themselves.

Assumption 2: x is distributed uniformly in A.

Assumption 3: $f(x) = \alpha + \beta x$ for $x \ge \tau$ where $\alpha > 0$, $\beta > 0$ and $(\alpha + \beta x) > r$, and $f(x) = 0$ for $x < \tau$.

Assumption 4: Individuals have to take a pass-fail examination in order to self-select to the job.

Assumption 5: If there is no examination required for the job, average productivity of individuals self-selecting to the job is equal to the reservation wage r.

The following assumptions are used only for this part.

Assumption 6: The individuals' examination scores are determined by their ability x and the spending in the private tutoring s. In order to improve the score by 'a' point above their ability, the individuals must spend $s = \theta a$ in the private tutoring. $\theta\,(\ge 0)$ is determined in the market for private tutoring.

Since the individuals have no motivation to perform above the screening level τ, s is specified as follows:

(i) If $x \ge \tau$, $s = 0$

(ii) If $x < \tau$, $s = \theta(\tau - x) > 0$

The high ability types ($x \ge \tau$) will not take private tutoring and their scores are determined by their true ability. However, the low ability types with x less than τ must take private tutoring in order to pass the examination.

The equilibrium pool of applicants for the examination is characterized by the minimum ability type y of the pool of applicants satisfying:

$$\int_y^1 (\alpha + \beta x) / (1 - y)\, dx = r + \theta(\tau - y) \tag{1}$$

Since the examination is a pass-fail test, applicants can signal the market on the basis of pass-fail test results. Then, the compensation for applicants who passed the examination is determined by their average productivity. The left hand side of (1) is the average productivity of the applicant pool with the minimum ability y. The right hand side of (1) is the opportunity cost of ability y individuals' applying for the examination, which is the sum of the reservation wage r and the spending in private tutoring $\theta(\tau - x)$. Therefore, in (1) the equilibrium is determined when the average statistic is equal to the marginal statistic. In order to understand the equilibrium mechanism[3] in (1), first suppose that the left hand side is greater than the right hand side. Then, a lower ability type with a higher opportunity cost will infiltrate the examination, which will reduce the average productivity until the equilibrium relationship (1) is achieved. On the other hand, suppose that the right hand side is greater than the left hand side. Then, low ability types with high opportunity cost will be discouraged from applying the examination, which will increase the average productivity until the equilibrium relationship (1) is achieved.

Figure 1 displays (1). In figure 1, $G(x) = \int_x^1 (\alpha + \beta z) / (1 - x)\, dz$ and $H(x) = r + \theta(\tau - x)$. $G(x)$ is the average productivity of the applicant pool with the minimum ability x and $H(x)$ is the opportunity cost of ability x individuals' applying for the examination, which is the sum of the reservation wage r and the spending in private tutoring $\theta(\tau - x)$. y is determined by the intersection point between $G(x)$ and $H(x)$.

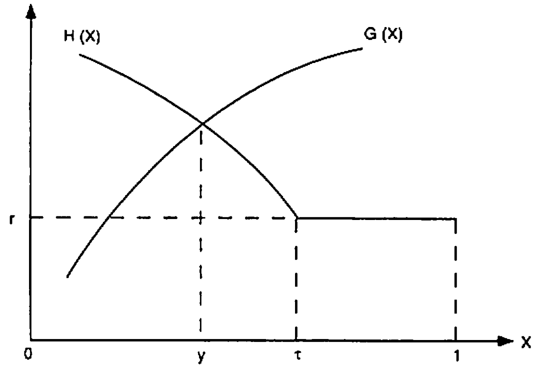

Figure 1

From (1), y (> 0) is calculated as (2)

$$y = \{2(\theta\tau + r - \alpha) - \beta\} / (\beta + 2\theta) \tag{2}$$

The social efficiency gain from screening is:

$$E = \int_\tau^1 (\alpha + \beta x - r)\, dx - \int_y^\tau r\, dx \tag{3}$$

The comparative statics associated with (2) and (3) give the following results[4]:

3 In (1), the equilibrium y can not be greater than nor equal τ under the assumption that $(\alpha + \beta x) > r$ for $x \geq \tau$. For $x \geq \tau$, the opportunity cost of applying the test is r. Then, it always holds that

$$\int_y^1 (\alpha + \beta x) / (1 - y)\, dx > r$$

Therefore, the equilibrium can not be obtained for $y \geq \tau$.

4 Note that $y > 0$ in (2) gives $2(\theta\tau + r - \alpha) > \beta > 0$ which leads to $(\theta\tau + r) > \alpha$. Also from the assumption that $\alpha + \beta x > r$ for $x \geq \tau$, it is clear that $\alpha + \beta > r$. The signs for $dy/d\beta < 0$ and $dy/d\theta > 0$ were determined according to these results.

$$dy/d\alpha = (-2) / (\beta + 2\theta) < 0$$

$$dy/d\beta = -2 \{\theta + (\theta\tau + r - \alpha)\} / (\beta + 2\theta)^2 < 0$$

$$dy/dr = 2 / (\beta + 2\theta) > 0$$

$$dy/d\theta = 4\tau (\alpha + \beta - r) / (\beta + 2\theta)^2 > 0$$

$$dE/d\alpha = (1 - \tau) + (dy/d\alpha)\, r$$

$$dE/d\beta = (1 - \tau^2) /2 + (dy/d\beta)\, r$$

$$dE/dr = (dy/dr)\, r - (1 - y)$$

$$dE/d\theta = (dy/d\theta)\, r > 0$$

Since $dy/d\alpha < 0$, $dy/d\beta < 0$ and $dy/dr < 0$, the sings of $dE/d\alpha$, $dE/d\beta$ and dE/dr are ambiguous.

The above results should be contrasted with the case where the examination screens ability types with perfect accuracy. In the case, θ goes to the infinity and (2) becomes

$$\lim_{\theta \to \infty} y = \tau$$

At the limit point, it is obtained that

$$dy/\mathrm{d}\alpha = dy/d\beta = dy/dr = 0$$

$$dE/d\alpha = (1 - \tau) > 0$$

$$dE/d\beta = (1 - \tau^2) / 2 > 0$$

$$dE/dr = - (1 - \tau) < 0$$

The above results give the following insights:

(i) If the examination screens individual types accurately, the more productive the high ability types are (greater α and β), the larger the efficiency gain from screening. However, if the low ability types are allowed to take private tutoring and the private tutoring does not affect the productive attributes of the low ability types, the efficiency gain is not uniformly determined because the low ability types are more eager to take private tutoring with a greater productivity of the high ability types.

(ii) If there exists no good alternative for the low ability types to opt out of the screening mechanism (i.e. the lower value of r), the low ability types are more inclined to take private tutoring, and the efficiency gain is aggravated.

(iii) θ determines the effectiveness of the examination as a screening device. As θ increases, the cost of the low ability types' pretending to be the high ability types increases. Then, the examination becomes more effective as a screening device, and the efficiency gain from screening increases.

The above results lead to the following proposition.

Proposition: *Suppose that private tutoring improves the examination score but not* *the individuals' productive ability. If the 7.30 Education Reform had been successful in eliminating private tutoring for the low ability types (i.e., $y = \tau$), it could have enhanced the matching efficiency and could have reduced the social spending in private tutoring at the same time. However, since the 7.30 Education Reform can not prevent the underground market for private tutoring from developing ($y<\tau$), the matching efficiency gain from the Reform is reduced and the change of social spending in private tutoring is not uniformly determined because some low ability types spend more in private tutoring even if the fraction of low ability types taking private tutoring decreases.*

III. The College Entrance Examination with Measurement Error

In the previous part, individual application decisions were equivalent to their self-selection decisions for the job. However, this is not the case if measurement error is introduced. With measurement error, the individuals' application decisions depend on the probability of their passing the examination.

Additional variables used in this part are defined as follows.

u: Measurement error which is independent of x

$I(u)$: Density of u which is randomly distributed in [−1, 1]

σ: Variance of u

$Pr(x)$: Probability that the individuals with ability x pass the examination

The following assumptions are used for this part.

Assumption 8: $I(u)$ is symmetric with respect to 0.

Assumption 9: The score of the individuals with x is determined by $x+u$.

Assumption 10: Individuals spend an equal amount in private tutoring ($d > 0$), and the direct cost of applying the examination is 0.

Asumption 11: $f(x) = (\alpha + \beta x) > (r + s)$ for $x \geq \tau$ and $f(x) = 0$ for $x < \tau$.

$Pr(x)$ is calculated as follows:

$Pr(x) = \text{Probability}(x + u \geq \tau) = \text{Probability}(u \geq \tau - x) = \int_t^1 I(e)de$ where $t = \tau - x$

The equilibrium pool of applicants[5] is also characterized by a minimum ability type y ($0 \leq y \leq 1$). With given y and σ, the expected average productivity of the pool conditional on passing the examination is:

$$AP(y) = \int_y^1 f(x)\, h(x)\, dx \qquad (4)$$

$$\text{where } h(x) = \frac{Pr(x)}{\int_y^1 Pr(x)\, dx}$$

$h(x)$ is the conditional weighted density which is the probability that individuals with x pass the examination amog the pool of applicants.

5 Individual score on the examinations is measured as $x+u$ which is distributed in [−1, 2]. Let's take simple examples. The first assumes that x=1 and u=−1. Then, $x+u$=0 where the highest ability applicant picked up the worst luck. If $\tau > 0$, every applicant (even the highest ability applicant) has a chance to fail the test. On the other hand, if $\tau \leq 0$, at least the highest ability applicant passes the test with the probability 1.

Lemma: *As σ increases, the high ability type with $x>\tau$ has a lower probability of passing the examination, while the low ability type with $x<\tau$ has a higher probability of passing the examination. Further, with a given pool of applicants, as σ increases, the expected average productivity of the pool decreases.*

(Proof) See appendix.

y must satisfy the following (5):

$$Pr(y)AP(y) + [1 - Pr(y)]r = r + d \tag{5}$$

where d is the cost of private tutoring. The left hand side of (5) is the expected compensation for the applicants and the right hand side of (5) is the opportunity cost of applying the examination. By arranging terms of (5), it is obtained

$$Pr(y)[AP(y) - r] = d \tag{6}$$

Figure 2 displays (6). In figure 2, $F(x) = Pr(x)[AP(x) - r]$ which is the expected wage increase of individuals with the ability x when applying for the examination.

The expected efficiency gain from screening is specified by E:

$$E = \int_\tau^1 Pr(x)(\alpha + \beta x - r)dx - \int_y^\tau Pr(x)r\,dx$$

The comparative statics give the followings:

$$dy/d\alpha = \frac{-Pr(y)[dAP(y)/d\alpha]}{dPr(y)/dy \cdot [AP(y) - r] + Pr(y)[dAP(y)/dy]} < 0$$

$$dy/d\beta = \frac{-Pr(y)[dAP(y)/d\beta]}{dPr(y)/dy \cdot [AP(y) - r] + Pr(y)[dAP(y)/dy]} < 0$$

$$dy/dr = \frac{Pr(y)}{dPr(y)/dy \cdot [AP(y) - r] + Pr(y)[dAP(y)/dy]} > 0$$

$$dy/dd = \frac{1}{dPr(y)/dy \cdot [AP(y) - r] + Pr(y)[dAP(y)/dy]} > 0$$

$$dE/d\alpha = \int_\tau^1 Pr(x)dx + dy/d\alpha \cdot Pr(y) \cdot r$$

$$dE/d\beta = \int_\tau^1 Pr(x)x dx + dy/d\beta \cdot Pr(y) \cdot r$$

$$dE/dr = Pr(y)r \cdot dy/dr - \int_y^1 Pr(x) \cdot dx$$

$$dE/dd = dy/dd \cdot Pr(y)\,r > 0$$

Since $dy/d\alpha < 0$, $dy/d\beta < 0$ and $dy/dr > 0$, the signs of $dE/d\alpha$, $dE/d\beta$ and dE/dr are ambigu-ous.

These results should be contrasted with the case with the perfect screening

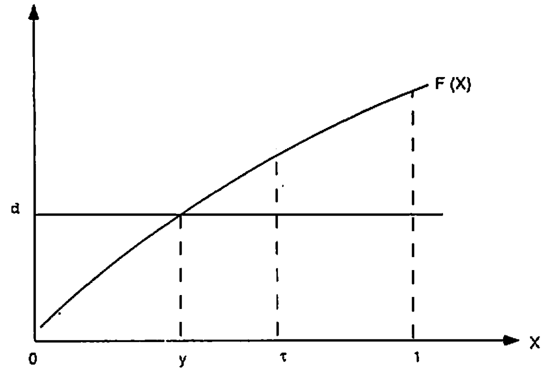

Figure 2

accuracy (i.e. $\sigma = 0$). If the examination screens the ability types with perfect accuracy, $Pr(x) = 1$ for $x \geq \tau$ and $Pr(x) = 0$ for $x < \tau$. Under assumption 11, $AP(\tau) > r + s$ and $y = \tau$. Then, it is obtained that

$$dy/d\alpha = dy/d\beta = dy/dr = 0,$$

$$dE/d\alpha = \int_{\tau}^{1} Pr(x)\, dx > 0$$

$$dE/d\beta = \int_{\tau}^{1} Pr(x)\, x\, dx > 0$$

$$dE/dr = \int_{\tau}^{1} Pr(x)\, dx < 0$$

The above comparisons give the following insights:

(i) If the examination screens individual types accurately, the more productive the high ability types are (greater α or β), the larger the efficiency gain from screening is. However, if measurement error is introduced into the examination, the results are not uniformly determined because the low ability types are more eager to infiltrate the examination with the greater productivity of the high ability types.

(ii) If there exists no good alternative for low ability types to opt out of the screening mechanism (i.e. the lower value of r), the low ability types are more inclined to infiltrate the examination, and the efficiency gain is aggravated.

(iii) As the cost of private tutoring (d) increases, the low ability types are more discouraged to apply for the examination than the high ability types because the low ability types have a lower probability of passing the examination. Then, the efficiency gain from screening increases.

The above results lead to the following proposition.

Proposition 2: *If the college entrance examination plays the role of self-selection device where individuals' cost of taking the private tutoring sort the population into applicants (more able) and non-applicants (less able), the 7.30 Education Reform may aggravate the efficiency gain from screening because the smaller spending in private tutoring may stimulate the low ability types to infiltrate the examination.*

IV. Conclusion

In Korea, the college entrance examination system plays a role of social selection mechanism, and it can be said that the individuals' scores in the college entrance examination determine private success in society. However, the excessive demand for higher education in Korea has created strong competition in taking the college entrance examination and has resulted in the over-heated pursuit of private tutoring, which is believed to prevent the educational system from serving as a fair and rational social mechanism of selection. In order to reduce the social waste of private tutorings, the Korean government established the 7.30 Reform of Education in 1980, which contained the elimination of the over-heated pursuit of private tutoring. Hence, since August 1981, any kind of private tutoring for high school students was legally prohibited. Less surprisingly, after the policy was initiated, underground markets for private tutoring became well developed and social spending in private tutoring was estimated to have never been reduced and a lot of people have questioned the accomplishment of the policy.

This paper is intended to develop pedagogical job-matching screenig models in order to explain the educational struggle in Korea and the economic impact of the 7.30 Education Reform in Korea. The models incorporate two kinds of factors into the standard screening model, which potentially makes the college entrance examination in Korea incomplete as a social selection mechanism. One such factor is measurement error associated with the examination itself. If higher selection accuracy of the examination requires the Korean government to pay a greater cost to administer the examination, the perfect accuracy may not be attainable. Then, the prohibitive cost of perfect accuracy introduces measurment error into the scores of the examination. The matching efficiency is aggravated not only by the measurement error itself but also by the individual self-selection results. If this is the case, it is demonstrated that the 7.30 Education Reform may be successful in reducing the social spending in private tutoring but may aggravate the matching efficiency because the smaller spending in private tutoring may stimulate the low productivity types to infiltrate the examination. Another factor is private tutoring. It is demonstrated that at reasonable screening equilibrium overspending in private tutoring by low productivity types but underspending by high productivity types may exist simultaneously. These individuals' adverse selections will also aggravate the job-matching efficiency. If this is the case, it is demonstrated that if the 7.30 Education Reform had been successful in eliminating the private tutoring for the low productivity types, it could have enhanced the matching efficiency and could have reduced the social spending in private tutoring at the same time. However, since the 7.30 Education Reform could not prevent the underground market for private tutoring from developing, the efficiency gain from the reform is reduced, and the social spending in private tutoring may even increase even if the fraction of low productivity types taking private tutoring decreases.

Appendix

Proof of lemma

This is proved by two steps.

(i) $dPr\ (x)/d\sigma < 0$ for $x > \tau$ and $dPr\ (x)/d\sigma > 0$ for $x < \tau$

first, it is proven that $dPr\ (x)/d\sigma < 0$ for $x > t$. Let t be $t = \tau - x$.

$$Pr\ (x) = \int_t^1 I\ (e)\ de$$

Because I(u) is symmetric with respect to 0,

$$\text{for } x > \tau,\ \ Pr\ (x) = \int_t^0 I\ (e)\ de + 1/2$$

This leads to:

$$\text{for } x > \tau,\ \ dPr\ (x)/d\sigma = \int_t^0 dI\ (e)\ /\ d\sigma\ \ de$$

It must be shown that the above has negative sign, and it is proven by the contradiction method. Suppose that there exists $x(> \tau)$ satisfying

$$dPr\ (x)/d\sigma = \int_t^0 dIb\ (e)\ /\ d\sigma\ \ de \geq 0 \tag{7}$$

Since it is true that

$$\int_1^0 dI\ (e)/d\sigma\ \ de = \int_1^t dI\ (e)/d\sigma\, de + \int_t^0 dI\ (e)/d\sigma\ \ de = 0$$

x in (7) also satisfies

$$\int_1^t dI\ (e)/d\sigma\ \ dx < 0 \tag{8}$$

By the facts that

$$\sigma^2 = 2\int_t^0 e^2 I\ (e)\ de$$

and that e^2 increases with $e(< 0)$, (7) and (8) give the result that σ^2 decreases with σ. This is contradictory. Therefore, $dPr\ (x)/d\sigma < 0$ for all $x(> \tau)$.

Similarly, it can be proved that $dPr(x)/d\sigma > 0$ for all $x(<\tau)$.

(ii) with given y, $dAP\ (y)/d\sigma < 0$

$$dh(x)\ /\ d\sigma = dPr\ (x)/d\sigma - \int_y^1 dPr\ (x)/d\sigma\, dx$$

$$\int_y^1 dPr\ (x)/dx$$

From (9), for $x > \tau$, $dh(x)/d\sigma < 0$ and for $x < \tau$, $dh(x)/d\sigma > 0$.

Recall that

$$AP\ (y) = \int_y^1 f\ (x)\ h(x)\ dx = \int_\tau^1 (\alpha + \beta x)\ h\ (x)\ dx$$

By the fact that $dh(x)/d\sigma < 0$ for $x > \tau$,

$$dAP\,(y)/d\sigma = \int_{\tau}^{1} (\alpha + \beta x) \cdot dh(x)\,/d\sigma\,dx < 0$$

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