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INTERNATIONAL TRADE WITH CAPITAL MOBILITY AND A NONTRADED GOOD

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The results of international trade theory which incorporates international capital mobility and a nontraded good are investigated. First, the effects of changes in the price of an imported good, a tariff and foreign investment are analyzed with respect to imports. Then a sufficient condition for local dynamic stability (l.d.s.) is studied. Finally, assuming l.d.s., consideration is given to the effects of changes in both the tariff on imports and a tax on foreign investment, on the world and domestic prices of the imported good, foreign investment, domestic imports, and the level of social welfare.

I. Introduction

The purpose of this paper is to investigate the topics of international trade theory with capital mobility and a nontraded good. Such attempts to generalize international trade theory with capital mobility are not new; significant contributions have been made by MacDougall (1960), Kemp (1966, 1969), Gehrels (1971), Chipman (1971, 1972), Uekawa (1972), Batra and Casas (1976), Das and Lee (1979), and Brecher and Feenstra (1983). Its normative aspects have been considered by Jones (1967), Gehrels (1971), Kemp (1966, 1969) and Chipman (1972). One of the basic problems of this topic is the existence of the world production possibility frontier (p.p.f.) (i.e., the efficient set of world production) with incomplete specialization. After earlier discussion by Kemp (1966) and Jones (1967), Chipman (1971) and Uekawa (1972) established sufficient conditions for its existence in the two country, two commodity and two factor case. This was generalized by Tawada (1982) who gave necessary and sufficient conditions for its existence in the many country, commodity and factor case. Based on the existence of the world p.p.f. with incomplete specialization in the two country, two commodify and two factor case, Brecher and Feenstra (1983) analyzed such positive aspects of international trade as stability, and (assuming stability), the effects of changes in the tariff and a tax on foreign investment on the terms of trade, level of foreign investment and domestic price of goods. Our model follows this conventional framework and generalizes its conclusions to a nontraded good. Neverthless, there is a significant difference between our concept of stability and that used by the above authors. As was lucidly pointed out by Das and Lee (1979), what was adopted by Kemp (1969) was Hicksian imperfect stability, in which the rentals of capital in both countries are the same continuously. Brecher and Feenstra (1983) made the same assumption of Hicksian imperfect stability. However, whether perfect or imperfect, Hicksian stability is a static stability. In its place we assume local dynamic stability (1.d.s.) as was assumed by Das and Lee (1979). While Das and Lee deal with the sector-specific model of Batra and Casas (1976), our model is not sector-specific. The rest of this paper proceeds as follows. In section II, the notation and assumptions are introduced. In section III, the effects on imports of changes in the imported good's price (Marshall-Lerner condition), its tariff and the level of foreign investment are analyzed. Then a sufficient condition for 1.d.s. is considered. Next, assuming 1.d.s., the effects of changes in the tariff and a tax on foreign investment, on the world and domestic price of the imported good and the level of foreign investment are considered. Finally the effects on domestic imports and the level of social welfare of changes in the tariff and a tax are considered.

II. Notations and Assumptions

The world is assumed to be composed of two countries; the home country and the foreign country. There are three goods; two traded goods (goods 1 and 2) and a nontraded good (good 3). Two factors of production, labor and capital, are employed for goods production, where capital is internationally mobile but labor is not. The production function of each good is neoclassical; constant returns to scale, concave and twice continuously differentiable. Let

y_i = the quantity of production of good i, $i = 1,2,3$,
x_i = the quantity of consumption of good i, $i = 1,2,3$,
p_i = the price of good i, $i = 1,2,3$,
z_i = the quantity of imports of good i, if it is positive. If it is negative, $-z_i$ is the quantity of export,
L = the endowment of labor,
C = the quantity of capital owned by home residents,
w = the wage rate,
r = the rental of capital,

for the home country, For the corresponding variables and figures of the foreign country, a (*) superscript is added. Let π be the gross domestic products. Then by definition,

$$\pi = p_1 y_1 + p_2 y_2 + p_3 y_3. \qquad (1)$$

If the home country is a creditor (debtor), then the income from the home residents' (foreigners') investment abroad (at home) is the return on the level of investment $K > 0$ ($K < 0$) with the rental r^*, i.e., $r^* K$. The gross national income I is the sum of gross domestic income and the return on foreign investment, i.e.,

$$I = \pi + r^* K. \qquad (2)$$

Since the gross domestic product is obtained by employing a quantity of labor, L, and a quantity of capital C-K, and the factor prices are w and r respectively, the following identity

$$\pi = w L + r (C - K)^{1)} \tag{3}$$

holds. Furthermore, since the gross national income is spent on the consumption of goods,

$$I = p_1 x_1 + p_2 x_2 + p_3 x_3. \tag{4}$$

Here the demand for good i is assumed to be generated by the maximization of an aggregate utility function so that

$$x_i = h_i [p_1, p_2, p_3, I], \; i = 1,2,3 \tag{5}$$

where h_i is the demand function of good i. Here non-specialization in consumption, i.e., $x_i > 0$, $i = 1,2,3$ whenever $I > 0$ is assumed. Let b_{ij} be the quantity of input j used to produce one unit of good i where $i = 1,2,3$ and $j = 1,2$ ($j = 1$ refers to labor and $j = 2$ to capital). Let $G_i (w, r)$ be the unit cost function of good i. Then

$$p_i = G_i (w, r), i = 1,2,3 \tag{6}$$

if $y_i > 0$, and $p_i < G_i (w, r)$ means $y_i = 0$. Furthermore by the duality theorem (see, e.g., Samuelson (1953) and Shephard (1970), the unit cost function is homogeneous of degree one and concave. It is assumed to be twice continuously differentiable. Then the following holds;

$$b_{i1} = \partial G_i (w, r)/\partial w, \;\; b_{i2} = \partial G_i (w, r)/\partial r, \;\; i = 1,2,3. \tag{7}$$

Since good 3 is nontraded, $y_3 = x_3$ holds always. The equality between demand and supply for factors implies

$$b_{11} y_1 + b_{21} y_2 + b_{31} y_3 = L, \tag{8}$$

$$b_{12} y_1 + b_{22} y_2 + b_{32} y_3 = C - K. \tag{9}$$

III. Results

Chipman (1971) first established that W_1, the world *p.p.f.* with diversification in both countries (i.e., y_i, $y_i^* > 0$, $i = 1,2)^{2)}$ exists for some factor endowments for the two country, two commodity, two factor case if and only if (*iff*) the system of equations $p_i = G_i (w, r) = G_i^* (w^*, r)$, $i = 1,2$ has a solution $p_i > 0$, $i = 1,2$, w, w^*, $r \geq 0$. With the introduction of a nontraded good, this is readily generalized as

Lema 1.

W_2, the world *p.p.f.* with capital mobility and complete diversification in both countries (i.e., y_i, $y_i^* > 0$, $i = 1,2,3$) exists for some factor endowments for the two country, three commodity (two traded goods and a non-traded good), two factor case *iff* the system of equations $p_i = G_i (w, r) = G_i^* (w, r)$, $i = 1,2$, $p_3 = G_3 (w, r)$, $p_3^* = G_3 (w^*, r)$ has a solution $p_i > 0$, $i = 1,2$, p_3, $p_3^* > 0$, and w, w^*, $r \geq 0$.

As is easily seen, the introduction of a nontraded good does not change the necessary and sufficient condition for the existence of W_2, i.e., W_1 exists *iff* W_2 exists. Denote Case 1 if the home country does not completely diversity, i.e., if $y_i = 0$ for at least $i = 1$ or 2, and Case 2 if there is complete diversification, i.e., $y_i > 0$ for $i = 1,2,3$. Denote Case 1* and Case 2* for the corresponding cases in the foreign country. Consider Case 2. For this case, $p_i = G_i (w, r)$, $i = 1,2,3$ holds. Now let $p_i = G_i(w, r)$, $i = 1,2$ define w and r implicitly to be functions of p_1 and p_2 so that $w = W(p_1, p_2)$, $r = R(p_1, p_2)$, which then define p_3 as a function of p_1, p_2;

$$p_3 = \psi(p_1, p_2). \tag{10}$$

For Case 1, if $y_1 = 0$, $y_2, y_3 > 0$, then $w = W(p_2, p_3)$ and $r = R(p_2, p_3)$, and if $y_1 = y_2 = 0$, $y_3 > 0$, then $w = W(p_3, K)$, $r = R(p_3, K)$ with $\partial W/\partial p_3$, $\partial R/\partial p_3 > 0$ and $\partial R/\partial K > 0$[3]. Therefore for both cases denote

$$w = w(p_1, p_2, p_3, K), \quad r = R(p_1, p_2, p_3, K) \tag{11}$$

Henceforth let $p_1 = p_1{}^* = 1$ without loss of generality. From Eq.s (8) and (9) it follows

$$y_1 = \xi\, y_3 + \lambda_1, \tag{12}$$
$$y_2 = \eta\, y_3 + \lambda_2 \tag{13}$$

where

$$\begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{bmatrix} \begin{bmatrix} \xi \\ \eta \end{bmatrix} = -\begin{bmatrix} b_{31} \\ b_{32} \end{bmatrix}, \tag{14}$$

$\lambda_1 = \{ b_{22} L - b_{21}(C-K)\}/|B|$, $\lambda_2 = \{ -b_{12} L + b_{11}(C-K)\}/|B|$ and $|B| = \begin{vmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{vmatrix}$

Then Eq.s (1) through (5), (11) through (13), and $x_3 = y_3$ define x_i, y_i, $i = 1,2,3$ as functions of p_2, p_3 and K so that

$$\left\{ \begin{array}{ll} x_i = X_i(p_2, p_3, K), \quad i = 1,2,3 & (15) \\ y_i = Y_i(p_2, p_3, K), \quad i = 1,2,3. & (16) \end{array} \right.$$

For Case 2, observing Eq. (10), the above is rewritten as

$$\left\{ \begin{array}{ll} x_i = \hat{X}_i(p_2, K) = X_i(p_2, \psi(1, p_2), K), & (17) \\ y_i = \hat{Y}_i(p_2, K) = Y_i(p_2, \psi(1, p_2), K). & (18) \end{array} \right.$$

Similarly z_i, $i = 1,2$ is defined to be the function of p_i's and K so that

$z_i = Z_i(p_2, p_3, K) = X_i(p_2, p_3, K) - Y_i(p_2, p_3, K)$ for Case 1,
$z_i = \hat{Z}_i(p_2, K) = \hat{X}_i(p_2, K) - \hat{Y}_i(p_2, K)$ for Case 2.

Now we are ready to examine the effects on imports of a change in the price of the imported good, i.e., the Marshall-Lerner condition. Henceforth if any good is exported (imported) by the home country, let it be good 1 (2).

Theorem 1.1. For Case 1

$$\frac{\partial Z_2(p_2, p_3, K)}{\partial p_2} = S_{22} + \frac{m_2}{p_2}\left\{ -z_2 + K\frac{\partial R^*(p_2, p_3{}^*, K)}{\partial p_2}\right\} - \frac{\partial Y_2(p_2, p_3, K)}{\partial p_2} \tag{19}$$

where $S_{22} = \partial h_2/\partial p_2 + x_2 \cdot \partial h_2/\partial I$, $m_2 = p_2 \cdot \partial h_2/\partial I$.

Proof

By partially differentiating $z_2(p_2, p_3, K) = h_2[1, p_2, p_3, I(p_2, p_3, K)] - Y_2(p_2, p_3, K)$ where $I = I(p_2, p_3, K) = \pi(p_2, p_3, K) + R^*(p_2, p_3{}^*, K)K$[4] with respect to p_2, we obtain $\partial z_2(p_2, p_3, K)/\partial p_2 = \partial h_2/\partial p_2 + (\partial h_2/\partial I)\{y_2 + K \cdot \partial R^*(p_2, p_3^*, K)/\partial p_2\} - \partial Y_2(p_2, p_3, K)/\partial p_2 = \partial h_2/\partial p_2 + (\partial h_2/\partial I)$

$\{-z_2 + x_2 + K \cdot \partial R^*(p_2, p_3^*, K)/\partial p_2\} - \partial Y_2(p_2, p_3, K)/\partial p_2 = S_{22} + \frac{m_2}{p_2}$
$\{-z_2 + K \cdot \partial R^*(p_2, p_3^*, K)/\partial p_2\} - \partial Y_2(p_2, p_3, K)/\partial p_2$ where $S_{ij} = \partial h_i/\partial p_j + x_j \cdot \partial h_i/\partial I$ is the Slutsky substitution term and $m_i = p_i \cdot \partial h_i/\partial I$ is the marginal propensity to consume good i.

Q.E.D.

Hence observing that

$$\partial R^*(p_2, p_3^*, K)/\partial p_2 = \begin{cases} -b_{31}^* / \begin{vmatrix} b_{21}^* & b_{31}^* \\ b_{22}^* & b_{32}^* \end{vmatrix} & \text{if } y_1^* = 0,\ y_2^*, y_3^* > 0 \\ 0 & \text{if } y_2^* = 0 \\ b_{11}^* / |B^*| & \text{for Case } 2^*, \end{cases}$$

we obtain the following;

Corollary 1.1.

For Case 1, assume that good 2 is imported by the home country and there are no inferior goods (i.e., $m_i \geqq 0, i = 1,2,3$). If one of the following two conditions holds, then $\partial z_2/\partial p_2 < 0$, i.e., the increase in the price of the imported good decreases its demand.

Condition 1. The home country is a creditor, i.e., $K > 0$ and in the foreign country $\partial R^*/\partial p_2 < 0$, i.e., good 2 is more labor intensive than good 1 for Case 2*, or good 2 is more labor intensive than good 3 when only goods 2 and 3 are produced.

Condition 2. The home country is a debtor, i.e., $K < 0$, and in the foreign country $\partial R^*/\partial p_2 > 0$, i.e., good 2 is more capital intensive than good 1 for Case 2*, or good 2 is more capital intensive than good 3 when only goods 2 and 3 are produced.

Corollary 1.1 supplies two sufficient conditions for the Marshall-Lerner condition i.e., $\partial z_2/\partial p_2 < 0$ for Case 1. These are (1) the home country imports the labor intensive good of the foreign country when the home country is a creditor and (2) the home country imports the capital intensive good of the foreign country when the home country is a debtor. If we can assume that the home country is well endowed with capital (labor), is a creditor (debtor), and the rank of the relative factor intensities of the goods of both countries remains the same, then these two sufficient conditions seem plausible in view of the Heckscher-Ohlin Theorem. Next, we generalize the above results for Case 2. Let $z_2 = \widetilde{Z}_2(I) = h_2[1, p_2, p_3, I] - (\eta \cdot h_3[1, p_2, p_3, I] + \lambda_2)$ and $M_2 = p_2 \cdot \frac{d\widetilde{Z}_2(I)}{dI}$ be the *marginal propensity to import good 2*. Then we obtain

Theorem 1.2. For Case 2,

$$\frac{\partial \widehat{Z}_2(p_2, K)}{\partial p_2} = [1, -\eta] \begin{bmatrix} S_{22} & S_{23} \\ S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} 1 \\ -\eta \end{bmatrix} + \frac{M_2}{p_2}(-z_2 + K \cdot \partial R^*/\partial p_2) - E \tag{20}$$

where $E = h_3 \cdot \partial\eta/\partial p_2 + \partial\lambda_2/\partial p_2 > 0$ and $M_2 = p_2 \dfrac{d\tilde{Z}_2(I)}{dI} = p_2\ (\partial h_2/\partial I - \eta \cdot \partial h_3/\partial I)$.

Proof. See Appendix I.

Q.E.D.

Observing that the first term of the right hand side of Eq. (20) is negative since the matrix (S_{ij}) is negative semi-definite and $E > 0$[5] and if (1) $b_{11}/b_{12} > \text{Max}\ (b_{21}/b_{22},\ b_{31}/b_{32})$ or $b_{11}/b_{12} < \text{Min}\ (b_{21}/b_{22},\ b_{31}/b_{32})$ and if (2) $m_i \geqq 0, i = 2,3,$ and $M_2 > 0$[6], then we obtain the following;

Corollary 1.2.

For Case 2, assume that good 2 is imported by the home country and goods 2 and 3 are not inferior. Then if one of the two conditions of Corollary 1.1. holds *and* if in the home country *either* good 1 is the most labor intensive *or* the most capital intensive, then $\partial\hat{Z}_2/\partial p_2 < 0$, i.e., the increase in the price of the imported good 2 decreases its demand.

By comparing Corollaries 1.1 and 1.2, we see that the sufficient conditions for $\partial\hat{Z}_2/\partial p_2 < 0$ for Case 2 are composed of two parts; i) the sufficient conditions for Case 1 and ii) good 1 must be either the most labor intensive or capital intensive good in the home country. Since the latter part is automatically satisfied in Case 1, we could regard Corollary 1.2 as holding for both Cases 1 and 2. Das and Lee (1979) obtained Eq. (19) for the case of two traded goods. Next we introduce a tariff and examine the Marshall-Lerner condition, and the effects on imports of a change in the tariff for Cases 1 and 2. Let τ be the tariff on good 2. Then the gross national income I is represented by

$$I = p_1 y_1 + T p_2 y_2 + p_3 y_3 + \tau p_2 z_2 + r^* K \tag{21}$$

where $p_1 = 1$, $T = \tau + 1$ and p_3 is the domestic price of the nontraded good. Let $I = I(p_2, p_3, K, \tau)$. Henceforth the foreign country is assumed to retain free trade. Then for Cases 2 and 2* respectively

$$p_3 = \psi\ (1, Tp_2) \ \text{ and } \ p_3^* = \psi^*(1, p_2)$$

hold. Now we show the expressions for $\partial Z_2/\partial p_2$ and $\partial Z_2/\partial\tau$ for Case 1. Let $z_2 = \underset{\sim}{Z}_2\ (p_2, p_3, K, \tau)$ be defined implicitly by Eq.s (15), (16) and (21) where $x_i = X_i\ (Tp_2, p_3, K)$ and $y_i = Y_i\ (Tp_2, p_3, K)$. Then the following holds;

Theorem 2.1. For Case 1,

$$\frac{\partial\tilde{Z}_2(p_2, p_3, K, \tau)}{\partial p_2} = T S_{22} + \frac{m_2}{p_2}\left\{-z_2 + \frac{K}{T}\frac{\partial R^*(p_2, p_3^*, K)}{\partial p_2}\right\} - T\frac{\partial Y_2(Tp_2, p_3, K)}{\partial p_2} \tag{22}$$

$$\frac{\partial Z_2(p_2, p_3, K, \tau)}{\partial\tau} = p_2 S_{22} - z_2 m_2 - p_2\frac{\partial Y_2(Tp_2, p_3, K)}{\partial p_2} \tag{23}$$

where $m_2 = T p_2\ \partial h_2/\partial I$.

Proof.

Partially differentiate $z_2 = \tilde{Z}_2(p_2, p_3, K, \tau) = h_2[1, Tp_2, p_3, I(p_2, p_3, K, \tau)] - Y_2(Tp_2, p_3, K)$ with respect to p_2 and τ respectively, and obtain the desired results.

Q.E.D.

From Theorem 2.1, the following is immediate;

Corollary 2.1.

(1) Corollary 1.1 still holds with the introduction of a tariff.

(2) The increase in the tariff on good 2 decreases (increases) its imports (exports) if good 2 is not inferior.

Next we analyze Case 2. Let $z_2 = \tilde{Z}_2(p_2, K, \tau)$. Then

Theorem 2.2. For Case 2,

$$\frac{\partial \tilde{Z}_2(p_2, K, \tau)}{\partial p_2} = \left\{ T[1, -\eta] \begin{bmatrix} S_{22} & S_{23} \\ S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} 1 \\ -\eta \end{bmatrix} - TE + \frac{M_2}{Tp_2}\left(-z_2 + K\frac{\partial \tilde{R}^*}{\partial p_2}\right)\right\} / \left(1 - \frac{T-1}{T} M_2\right), \quad (24)$$

$$\frac{\partial \tilde{Z}_2(p_2, K, \tau)}{\partial \tau} = p_2 \left\{ [1, -\eta] \begin{bmatrix} S_{22} & S_{23} \\ S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} 1 \\ -\eta \end{bmatrix} - E \right\} / \left(1 - \frac{T-1}{T} M_2\right). \quad (25)$$

Proof. See Appendix II.

Q.E.D.

As is easily seen by comparing Eq.S(20) and (24), as far as $1 - \frac{T-1}{T} M_2 > 0$ the sufficient condition for $\partial \tilde{Z}_2 / \partial p_2 < 0$ remains unchanged with the introduction of a tariff, i.e., $M_2 > 0$ and $K \cdot \partial \tilde{R}^* / \partial p_2 < 0$. Furthermore $\partial \tilde{Z}_2 / \partial \tau < 0$ if $\frac{T}{T-1} M_2 > 0$. Hence we obtain

Corollary 2.2

Assume free trade prevails initially (or more generally $1 - \frac{T-1}{T} M_2 > 0$) Then

(1) Corollary 1.2 still holds with the introduction of a tariff:

(2) The increase in the tariff on good 2 decreases (increases) its imports (exports) if, in the home country, goods 2 and 3 are non-inferior and good 1 is either the most capital intensive or the most labor intensive.

For the same reasons as before, Corollary 2.2 could be regarded as holding for both Cases 1 and 2. Furthermore, notice the following; assume that there are no inferior goods in the home country, and that the rank of the relative factor intensities of the goods remains the same in both countries. Then if the home country is a creditor (debtor), i.e., $K > 0$ (< 0) – which implies that it is well endowed with capital (labor) –, and if the home country imports good 2, which is relatively more labor (capital) intensive than good 3, and if good 1 – which is exportable for the home country – is the most capital (labor) intensive, then the increase in the price of the imported good decreases its imports. In other words, if the rank of the relative factor intensities of the goods remains the same in both countries, and trade occurrs according to the prediction of the Heckscher-Ohlin Theorem, then the increase in the price of the imported good decreases its imports. Furthermore, the sufficient condition for an increase in the tariff on the imported good (good 2) to reduce its imports is seen to be weaker than that for an increase in its price to decrease imports. In fact, the sufficient condition is that good 1 be either the most labor intensive or capital intensive. Now let $\rho = -\frac{p_2}{Z_2}\frac{\partial \tilde{Z}_2}{\partial p_2}$, $\zeta = -\frac{T}{Z_2}\frac{\partial \tilde{Z}_2}{\partial p_2}$, i.e., ρ and ζ are respectively, the *import elasticity* and *tariff elasticity of the home offer curve.* Then

$$\zeta = \left\{ \rho \frac{M_2}{T} \left(1 - \frac{K}{z_2}\frac{\partial R^*}{\partial p_2}\right) \right\} / \left(1 - \frac{T-1}{T} M_2\right). \tag{26}$$

This is regarded as a generalization of the relation between the two elasticities ρ and ζ to the case where capital is mobile. The expression when $K = 0$, i.e., there is no capital mobility is shown by Komiya's (1967) Eq. (70), Kemp's (1969) Eq. (6.30) and Inoue's (1985) Eq. (20). Before we analyze the stability condition, the expressions for $\partial Z_2^*/\partial p_2$, $\partial Z_2/\partial K$ and $\partial Z_2^*/\partial K$ are required.

Theorem 3. For Cases 1 and 2, with the introduction of a tariff

$$\frac{\partial \tilde{Z}_2(p_2, p_3, K, \tau)}{\partial K} = \left\{ \frac{m_2}{Tp_2}\left(R - R^* + K\frac{\partial R^*(p_2, p_3^*, K)}{\partial K}\right) - \frac{\partial Y_2(Tp_2, p_3, K)}{\partial K} \right\} / \left(1 - \frac{T-1}{T} m_2\right) \text{ for Case 1,} \tag{27}$$

$$\frac{\partial \tilde{Z}_2(p_2, K, \tau)}{\partial K} = \left\{ \frac{M_2}{Tp_2}\left(R - R^* + K\frac{\partial R^*(p_2, p_3^*, K)}{\partial K}\right) + b_{11}/|B| \right\} / \left((1 - \frac{T-1}{T} M_2\right) \text{ for Case 2.} \tag{28}$$

Proof.

These can be proved using the method in Appendices I and II respectively.

Q.E.D.

Here notice that when the home country is a creditor, $\partial R^*/\partial K = 0$ must hold since the foreign country has to produce at least two goods. Furthermore, if free trade prevails initially, then $R = R^*$. Moreover $\partial Y_2/\partial K < 0$ (> 0) *iff* good 2 is more capital (labor) intensive than good 3 for Case 1 where only goods 2 and 3 are produced. For Case 2 $|B| > 0$ (< 0) *iff* good 2 is more capital (labor) intensive than good 1. Hence we obtain the following;

Corollary 3.

Assume that free trade prevails initially and that the home country is a creditor or more generally, the foreign country produces at least two goods. Then

(1) for Case 1, provided that good 2 is not inferior, the increase in investment abroad, or the decrease in investment from abroad increases (decreases) imports of good 2 *iff* good 2 is more capital (labor) intensive than good 3 in the home

country.

(2) for Case 2, provided that goods 2 and 3 are not inferior, and good 1 is either the most labor intensive or capital intensive, the increase in investment abroad, or the decrease in investment from abroad increases (decreases) imports of good 2, or decreases (increases) exports of good 2 *iff* good 2 is more capital (labor) intensive than good 1 in the home country.

Here again Case 2 could be regarded as a generalization of Case 1. Part (2) of Corollary 3 implies that under fairly mild conditions, the increase in investment abroad or the decrease in investment from abroad encourages (discourages) the imports of the labor (capital) intensive good. Next, we show the expression for $\partial Z_2^*/\partial p_2$.

Theorem 2*.

$$\frac{\partial Z_2^*(p_2, p_3^*, K)}{\partial p_2} = S_{22}^* + \frac{m_2}{p_2}\left\{-z_2^* - K\frac{\partial R^*(p_2, p_3^*, K)}{\partial p_2}\right\} - \frac{\partial Y_2^*(p_2, p_3^*, K)}{\partial p_2}, \text{ for Case 1*}, \quad (29)$$

$$\frac{\partial \hat{Z}_2^*(p_2, K)}{\partial p_2} = [1, -\eta^*]\begin{bmatrix} S_{22}^* & S_{23}^* \\ S_{32}^* & S_{33}^* \end{bmatrix}\begin{bmatrix} 1 \\ -\eta^* \end{bmatrix} + \frac{M_2^*}{p_2}\left\{-z_2^* - K\frac{\partial R^*(p_2, p_3^*, K)}{\partial p_2}\right\} - E^*, \text{ for Case 2*} \quad (30)$$

Proof.

First observe that for the foreign country gross national income and gross domestic products are respectively expressed by

$$I^* = p_1 x_1^* + p_2 x_2^* + p_3^* x_3^* = p_1 y_1^* + p_2 y_2^* + p_3^* y_3^* - r^* K, \quad (31)$$
$$\pi^* = p_1 y_1^* + p_2 y_2^* + p_3^* y_3^* \quad (32)$$

where $p_1 = 1$. The rest of the proof is the same as that of Theorem 1.1 and Appendix I.

Q.E.D.

The signs of $\partial Z_2^*/\partial p_2$ and $\partial \hat{Z}_2^*/\partial p_2$ are indeterminate. However, what is needed to analyze the stability condition are the signs of $\partial(Z_2+Z_2^*)/\partial p_2$ and $\partial(Z_2+Z_2^*)/\partial K$. Next we show the expression for $\partial Z_2^*/\partial K$.

Theorem 3*.

$$\frac{\partial Z_2^*(p_2, p_3^*, K)}{\partial K} = -\frac{m_2^*}{p_2} K \frac{\partial R^*(p_2, p_3^*, K)}{\partial K} - \frac{\partial Y_2^*(p_2, p_3^*, K)}{\partial K}, \text{ for Case 1*}, \quad (33)$$

$$\frac{\partial \hat{Z}_2^*(p_2, K)}{\partial K} = -b_{11}^*/|B^*|, \text{ for Case 2*}. \quad (34)$$

Proof.

For Case 1*, partially differentiate $Z_2^*(p_2, p_3^*, K) = h_2^*[1, p_2, p_3^*, I^*(p_2, p_3^*, K)] - Y_2^*(p_2, p_3^*, K)$ with respect to K, while observing $\partial I^*/\partial K = \partial \pi^*/\partial K -$

$R^* - K\ \partial R^*/\partial K = -K\ \partial R^*/\partial K$ (since $\partial\pi^*/\partial K = R^*$). For Case 2*, notice that $\partial\hat{X}_2^*(p_2, K)/\partial K = 0$ since $\partial R^*/\partial K = 0$, and $\partial\hat{Y}_2^*(p_2, K)/\partial K = \partial\{\eta^*\hat{X}_3^*(p_2, K) + \lambda_2^*\}/\partial K = \partial\lambda_2^*/\partial K$ (since $\partial\hat{X}_3^*(p_2, K)/\partial K = 0$) $= b_{11}^*/|B^*|$, observing $\lambda_2^* = \{-b_{11}^* L^* + b_{11}^*(C^* + K)\}/|B^*|$.

Q.E.D.

Here again notice that if the home country is a creditor, or more generally if the foreign country produces at least two goods, then $\partial R^*/\partial K = 0$ so that the sign of $\partial Z_2^*/\partial K$ depends only on the relative factor intensities of the goods. then, from Eq.S (33) and (34), we see that for Case 1* the increase in investment abroad or the decrease in investment from abroad increases (decreases) the exports (imports) of good 2 of the foreign country *iff* good 2 is more capital intensive than good 3 in the foreign country. For Case 2*, the increase in investment abroad or the decrease in investment from abroad increases (decreases) the exports (imports) of good 2 of the foreign country *iff* good 2 is more capital intensive than good 1 in the foreign country.

There are four possible combinations of Cases 1, 2 and Cases 1* and 2*. Nevertheless, if (Case 1 and Case 1*) or (Case 2 and Case 2*) occurrs – for Case 1* at least two goods are assumed to be produced in the foreign country – and if the rank of relative factor intensities of the goods remains the same in both countries, and if free trade prevails initially, then under fairly mild conditions such that $m_2 \geqq 0$ and $M_2 \geqq 0$, we observe that $\partial Z_2/\partial K > 0$ *iff* $\partial Z_2^*/\partial K < 0$ and $\partial\hat{Z}_2/\partial K > 0$ *iff* $\partial\hat{Z}_2^*/\partial K < 0$, i.e., the increase in investment abroad or the decrease in investment from abroad increases (decreases) the exports (imports) of good 2 in the foreign country. Now we are ready to analyze the stability condition. Let

$$J_1(p_2, K) = \hat{Z}_2(p_2, K) + \hat{Z}_2^*(p_2, K), \tag{35}$$

$$J_2(p_2) = R^*(p_2) - R(p_2). \tag{36}$$

That is, J_1 is the world excess demand for good 2, and J_2 is the difference in the foreign and home rentals of capital for Cases 2 and 2*. Henceforth we are concerned with only this combination, i.e., the complete diversification of both countries, for it is the most general one and all the other cases can be dealt with by an analogous method. Here notice the following; that W_2, the world p.p.f. with capital mobility and complete diversification existing under certain (sufficient) conditions given by Uekawa (1972), implies the existence of an equilibrium defined by $J_1(p_2, K) = 0$ and $J_2(p_2) = 0$. Let $\dot{p}_2 = dp_2/ds$, $\dot{K} = dK/ds$ be the rates of change of p_2 and K with respect to times respectively. With regard to stability it is assumed that

$$\left.\begin{aligned} \dot{p}_2 &= J_1(p_2, K) \\ \dot{K} &= J_2(p_2) \end{aligned}\right\}. \tag{37}$$

That is, the price of good 2 increases if there exists a world excess demand for the goods, and the exports (imports) of capital from (to) the home country increase (decrease) if the foreign rental of capital is higher than the domestic one. Brecher and Feenstra (1983) proposed Hicksian imperfect stability assuming either $J_2(p_2) = 0$ (Walrasian stability) or $J_1(p_2, K) = 0$ (Marshallian stability) hold continuously. Clearly our system of adjustment, Eq. (37), is more general than theirs. Now we assume that the system of adjustment, Eq. (37) satisfies l.d.s. Then by the Routh-Hurwitz Theorem (see, e.g., Gantmacher (1960)), Eq. (37) satisfies l.d.s. *iff* at equilibrium

(i) $J_{11} + J_{22} < 0$,

(ii) $\begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix} = J_{11} J_{22} - J_{12} J_{21} > 0$

where $J_{i1} = \partial J_i / \partial p_2$, $J_{i2} = \partial J_i / \partial K$, $i = 1, 2$[7].

Since $J_{22} = 0$, Eq. (37) does not satisfy Hicksian perfect stability but satisfies l.d.s. *iff* $J_{11} < 0$ and $J_{12} J_{21} < 0$. Here

$$J_{11} = \frac{\partial \hat{Z}_2 (p_2, K)}{\partial p_2} + \frac{\partial \hat{Z}_2^* (p_2, K)}{\partial p_2} = S + S^* - E - E^* + (-z_2 + K \frac{\partial R^*}{\partial p_2}) (M_2 - M_2^*)/p_2 \qquad (38)$$

where $S = [1, -\eta] \begin{bmatrix} S_{22} & S_{23} \\ S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} 1 \\ -\eta \end{bmatrix}$ and $S^* = [1, -\eta^*] \begin{bmatrix} S_{22}^* & S_{23}^* \\ S_{32}^* & S_{33}^* \end{bmatrix} \begin{bmatrix} 1 \\ -\eta^* \end{bmatrix}$,

$$J_{12} = \frac{\partial \hat{Z}_2 (p_2, K)}{\partial K} + \frac{\partial \hat{Z}_2^* (p_2, K)}{\partial K} = \frac{M_2}{p_2} (R - R^*) + b_{11} / |B| - b_{11}^* / |B^*|.$$

Hence at equilibrium

$$J_{12} = b_{11} / |B| - b_{11}^* / |B^*|, \qquad (39)$$

$$J_{21} = \frac{\partial R^*}{\partial p_2} - \frac{\partial R}{\partial p_2} = b_{11}^* / |B^*| - b_{11} / |B|^{8)}. \qquad (40)$$

Therefore $J_{12} J_{21} < 0$ at equilibrium. Within the concept of Hicksian imperfect stability Brecher and Feenstra (1983) showed that stability is obtained *iff* $J_{12} J_{21} < 0$ while assuming $J_{11} < 0$ (which they call the (generalized) Marshall-Lerner condition) without providing sufficient conditions for its negativeness. With these assumptions their necessary and sufficient condition for stability is the same as ours. However, we can obtain the following sufficient conditions for $J_{11} < 0$;

Theorem 4.

Under free trade if good 2 is imported by the home country, and if $M_2 \geqq M_2^*$ and $K\, \partial R^* / \partial p_2 < 0$, then the system of adjustment, Eq. (37), satisfies l.d.s.

Since $M_2 = m_2 - \frac{p_2}{p_3} \eta\, m_3$ when the production technologies of both countries are the same, then $M_2 \geqq M_2^*$ if (1) $m_2 \geqq m_2^*$ and (2) ($\eta < 0$ and $m_3 \geqq m_3^*$) or ($\eta > 0$ and $m_3^* \geqq m_3$). Furthermore if the home country is a creditor, then $z_2 > 0$ and $K > 0$ so that $\partial R^* / \partial p_2 < 0$ for l.d.s. Since $\partial R^* / \partial p_2 < 0$ is consistent with $|B| < 0$, it follows that

Corollary 4.

Under free trade, when the production technologies of both countries are the same, and the home country is a creditor, the system of adjustment, Eq. (37), satisfies l.d.s. if (1) $m_2 \geqq m_2^*$ and (2) either $[b_{11}/b_{12} < \text{Min}\,(b_{21}/b_{22}, b_{31}/b_{32})$ and $m_3 \geqq m_3^*]$ or $[b_{21}/b_{22} > b_{11}/b_{12} > b_{31}/b_{32}$ and $m_3^* \geqq m_3]$.

The above corollary shows that under free trade and identical production technologies, when the home country is a creditor, l.d.s. holds if the marginal propensity to consume the imported good of the home country is not less than that of the foreign country, and if either (i) good 1 is the most capital intensive and the marginal propensity to consume the nontraded good of the home country is not less than that of the foreign country or (ii) the imported good is the most labor intensive, the nontraded good is the most capital intensive, and the marginal propensity to consume the nontraded good of the home country is not more than that of the foreign country.

Next we introduce a tariff on the imported good and a tax on foreign investment, and consider the system of adjustment. Let t be the tax rate on income from the home residents' investment abroad. Then at equilibrium $r = (1-t)r^*$ holds. Hence the system of adjustment is

$$\left.\begin{aligned} \dot{p}_2 &= \tilde{Z}_2(p_2, K, \tau) + \hat{Z}_2^*(p_2, K) \\ \dot{K} &= (1-t)R^*(p_2) - R(Tp_2) \end{aligned}\right\} \qquad (41)$$

Henceforth we assume that the home country is a creditor. When it is a debtor, if t^* is the tax rate on income from foreigners' investment at home, the second equation of the above system would be $\dot{K} = R^* - (1-t^*)R$, which was discussed by Brecher and Feenstra (1983). Here notice that $\tilde{Z}_2(p_2, K, \tau) + \hat{Z}_2^*(p_2, K)$ and $(1-t)R^*(p_2) - R(Tp_2)$ are continuous with respect to τ and t[9]. Hence

Corollary 5.

Assume that the home country is a creditor.Then if $M_2 \geqq M_2^*$, i.e., the marginal propensity to import good 2 of the home country is not less than that of the foreign country, and if good 1 (the exportable good) is more capital intensive than good 2 in the foreign country, then the system of adjustment, Eq. (41), satisfies l.d.s. for small values of τ and t[10].

Next, by assuming the l.d.s. of Eq.(41), we analyze the effects of the change both in the tariff and the tax on the world and domestic price of good 2, and capital exports, etc. First let $J_1(p_2, K, \tau) = \tilde{Z}_2(p_2, K, \tau) + \hat{Z}_2^*(p_2, K)$, $J_2(p_2, \tau, t) = (1-t)R^*(p_2) - R(Tp_2)$. Then $J_1 = J_2 = 0$ defines p_2 and K to be implicit functions of τ and t, so that $dp_2/d\tau = -J_{2\tau}/J_{21} = p_2 R'/J_{21}$, $dp_2/dt = -J_{2t}/J_{21} = R^*/J_{21}$, $dK/d\tau = -\{J_{11}(-J_{2\tau}/J_{21}) + J_{1\tau}\}/J_{12}$, and $dK/dt = J_{11}J_{2t}/J_{12}J_{21}$ where $J_{i\tau} = \partial J_i/\partial\tau$, $i = 1,2$, $J_{2t} = \partial J_2/\partial t$. Furthermore let q be the domestic price of good 2, i.e., $q = Tp_2$. Then $dq/d\tau = p_2(1-t)R^{*\prime}/J_{21}$[11]. Now we assume the l.d.s. of Eq. (41) and obtain the following;

Theorem 5.

Let the system of adjustment, Eq. (41) satisfy l.d.s. Then

(i) if $dp_2/d\tau > 0$ and $M_2 \geqq 0$, then $dK/d\tau \gtreqless 0$ *iff* $J_{12} \lesseqgtr 0$ *iff* $J_{21}(= (1-t)R^{*\prime} - TR') \lesseqgtr 0$.

(ii) $dK/dt < 0$.

Proof.

(i) By l.d.s. $J_{11} < 0$ and $J_{21}J_{12} < 0$ hold. Furthermore by Eq. (25), if $M_2 \geqq 0$, $J_{1\tau} = \partial\tilde{Z}_2/\partial\tau < 0$, $dp_2 d\tau > 0$ means $J_{2\tau}/J_{21} < 0$. Then sign $dK/d\tau$ = sign J_{12}.

(ii) sign dK/dt = sign J_{2t}, $J_{2t} = -R^*$.

Q.E.D.

The assumption of l.d.s. in Eq. (41) helps to determine the signs of the effects on capital exports of changes in the tariff and tax, but not on the world or domestic price of the imported good. The latter depends on the signs of $J_{21} = (1-t)R^{*\prime} - R'$ or $R^{*\prime}$ and R' which cannot be assumed arbitrarily.

Next we consider the effects on social welfare of changes in the tariff and tax. Let the social welfare $V(\tau, t)$ be

$$V(\tau, t) = U\,[h\,\{1, Tp_2, \psi\,(1, Tp_2), I\,\}] \tag{42}$$

where $h(\,.\,) = (h_1(\,.\,), h_2(\,.\,), h_3(\,.\,)) \epsilon R_+^3$, $I = \pi + r^*K + (T-1)\,p_2 z_2$, $\pi = \pi\,(Tp_2, K)$, $r^* = R^*(p_2)$, $p_2 = P_2(\tau, K)$, $K = K(\tau, t)$. Then

Theorem 6.

$$\frac{\partial V}{\partial \tau} = \{\,(\tau-1)\,z_2 + K\frac{dR^*}{dp_2}\,\}\,\frac{\partial p_2}{\partial \tau} + (R^* - R)\,\frac{\partial K}{\partial \tau} + \tau p_2 \frac{\partial \widetilde{Z}_2}{\partial \tau}, \tag{43}$$

$$\frac{\partial V}{\partial t} = (-z_2 + K\frac{dR^*}{dp_2})\,\frac{\partial p_2}{\partial t} + (R^* - R)\,\frac{\partial K}{\partial t} + \tau p_2\,\frac{\partial \widetilde{Z}_2}{\partial t} \tag{44}$$

Proof. See Appendix III.

Q.E.D.

With initially free trade, since $\tau = 0$, $t = 0$, the second and third terms of both Eq.s (43) and (44) vanish, and

$$\frac{\partial V}{\partial \tau} = (-z_2 + K\frac{dR^*}{dp_2})\,\frac{\partial p_2}{\partial \tau}, \tag{45}$$

$$\frac{\partial V}{\partial t} = (-z_2 + K\frac{dR^*}{dp_2})\,\frac{\partial p_2}{\partial t}, \tag{46}$$

which was obtained by Brecher and Feenstra (1983), Eq. (14). From Eq. (45) it follows that if $(1-t)\,R^{*\prime} < R' < 0$ (so that $dp_2/d\tau > 0$) then $\partial V/\partial \tau < 0$, i.e., an increase in the tariff on the imported good, which is more labor intensive than the exportable good 1 in both countries, decreases the social welfare of the home country. Similarly from Eq. (46) if $0 > (1-t)\,R^{*\prime} > R'$ (so that $dp_2/dt > 0$), then an increase in the tax on income from the home residents' investment abroad decreases the social welfare.[12)]

Appendix I.

By partially differentiating the following with respect to p_2, $\widehat{Z}_2(p_2, K) = h_2[1, p_2, \psi\,(1, p_2), I\,(p_2, K)] - \eta\, h_3[1, p_2, \psi\,(1, p_2), I\,(p_2, K)] - \lambda_2$ where $I\,(p_2, K) = \pi\,(p_2, K) + R^*\,(p_2, p_3^*, K)\,K$, we obtain

$$\begin{aligned}\frac{\partial Z_2}{\partial p_2} &= \frac{\partial h_2}{\partial p_2} + \frac{\partial h_2}{\partial p_3}\frac{\partial \psi}{\partial p_2} + \frac{\partial h_2}{\partial I}\,(y_2 + \frac{\partial \psi}{\partial p_2}\,y_3 + K\frac{\partial R^*}{\partial p_2}) \\ &\quad - \eta\Big\{\frac{\partial h_3}{\partial p_2} + \frac{\partial h_3}{\partial p_3}\frac{\partial \psi}{\partial p_2} + \frac{\partial h_2}{\partial I}\,(y_2 + \frac{\partial \psi}{\partial p_2}\,y_3 + K\frac{\partial R^*}{\partial p_2})\Big\} \\ &\quad - (h_3\frac{\partial \eta}{\partial p_2} + \frac{\partial \lambda_2}{\partial p_2}),\end{aligned}$$

observing $\partial\pi(p_2, K)/\partial p_2 = y_2 + \frac{\partial\psi}{\partial p_2} y_3$.[13] Next noting $\partial\psi/\partial p_2 = -\eta$, $S_{ij} = \frac{\partial h_i}{\partial p_j} + \frac{\partial h_i}{\partial I} x_j$, and $M_2 = p_2 (\frac{\partial h_2}{\partial I} - \eta \frac{\partial h_3}{\partial I})$, the above is rewritten as Eq. (20).

Appendix II.

The technique is the same as that of Appendix I. By partially differentiating

$$\widetilde{Z}_2(p_2, K, \tau) = h_2[1, Tp_2, \psi(1, Tp_2), I(p_2, K, \tau)] - \{\eta(Tp_2) h_3[1, Tp_2, \psi(1, Tp_2), I(p_2, K, \tau)] + \lambda_2\}$$

where $I(p_2, K, \tau) = \pi(Tp_2, K) + (T-1) p_2 \widetilde{Z}_2(p_2, K, \tau) + R^*(p_2, p_3^*, K) K$ with respect to p_2 and τ respectively, and observing

$$M_2 = Tp_2 (\frac{\partial h_2}{\partial I} - \eta \frac{\partial h_3}{\partial I}), \; S_{ij} = \frac{\partial h_i}{\partial p_j} + \frac{\partial h_i}{\partial I} x_j, \; \frac{\partial\psi(1, Tp_2)}{\partial p_2} = -T\eta,$$

$$E = \frac{\partial h}{\partial p_2} x_3 + \frac{\partial \lambda_2}{\partial p_2},$$ we obtain the desired results.

Appendix III.

Since $\partial U/\partial x_1 = \mu$, $\partial U/\partial x_2 = \mu Tp_2$, $\partial U/\partial x_3 = \mu p_3$, where μ is the marginal utility of income (we assume $\mu = 1$), and $\partial\pi/\partial p_2 = y_2 - \eta x_3$, by partially differentiating $V(\tau, t)$ with respect to τ, and rearranging we obtain

$$\frac{\partial V}{\partial \tau} = \frac{\partial h_1}{\partial p_2}\frac{\partial q}{\partial \tau} - \eta \frac{\partial q}{\partial \tau}\frac{\partial h_1}{\partial p_3} + \frac{\partial h_1}{\partial I}(y_2 - \eta x_3)\frac{\partial q}{\partial \tau} + Tp_2 \{ \frac{\partial h_2}{\partial p_2}\frac{\partial q}{\partial \tau} - \eta \frac{\partial q}{\partial \tau}\frac{\partial h_2}{\partial p_3} + \frac{\partial h_2}{\partial I}(y_2 - \eta x_3)\frac{\partial q}{\partial \tau} \} + P_3 \{ \frac{\partial h_3}{\partial p_2}\frac{\partial q}{\partial \tau} - \eta \frac{\partial q}{\partial \tau}\frac{\partial h_3}{\partial p_2} + \frac{\partial h_3}{\partial I}(y_2 - \eta x_3)\frac{\partial q}{\partial \tau} \}$$
$$+ (\frac{\partial h_1}{\partial I} + Tp_2 \frac{\partial h_2}{\partial I} + p_3 \frac{\partial h_3}{\partial I})(-R\frac{\partial K}{\partial \tau} + R^* \frac{\partial K}{\partial \tau} + K \frac{dR^*}{dp_2}\frac{\partial p_2}{\partial \tau} + p_2 z_2 + \tau z_2 \frac{\partial p_2}{\partial \tau} + \tau p_2 \frac{\partial \widetilde{Z}_2}{\partial \tau}).$$

Next, observing $y_2 = x_2 - z_2$, $\frac{\partial h_1}{\partial I} + Tp_2 \frac{\partial h_2}{\partial I} + p_3 \frac{\partial h_3}{\partial I} = m_1 + m_2 + m_3 = 1$, and $S_{ij} = \frac{\partial h_i}{\partial p_j} + \frac{\partial h_i}{\partial I} x_j$, the above is rewritten as

$$\frac{\partial V}{\partial \tau} = \frac{\partial q}{\partial \tau}(S_{12} + S_{22}\, Tp_2 + S_{32}\, p_3) - \eta \frac{\partial q}{\partial \tau}(S_{13} + S_{23}\, Tp_2 + S_{33}\, p_3)$$
$$- z_2 \frac{\partial q}{\partial \tau} + (R^* - R)\frac{\partial K}{\partial \tau} + K \frac{dR^*}{dp_2}\frac{\partial p_2}{\partial \tau} + p_2 z_2 + \tau z_2 \frac{\partial p_2}{\partial \tau} + \tau p_2 \frac{\partial \widetilde{Z}_2}{\partial \tau}.$$

Since, $\begin{bmatrix} S_{12} & S_{22} & S_{32} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} \begin{bmatrix} 1 \\ Tp_2 \\ p_3 \end{bmatrix} = 0$, the above is rewritten as Eq. (43).

Eq. (44) can be proved similarly.

Notes

1) For the foreign country, Eq.s (2) and (3) are expressed as

$$I^* = \pi^* - r^* K$$
$$\pi^* = w^* L^* + r^* (C^* + K), \text{ so that}$$
$$I^* = w^* L^* + r^* C^*$$

2) Formally W_1 is defined as follows; let $Y_i \subset R^3$ be the production set of good i so that y_i amount of good i is *producible* by employing l_{iL}, l_{ik} amounts of labor and capital respectively *iff* $(y_i, -l_{iL}, -l_{iK}) \in Y_i$. Then the world production set V_1 is expressed as

$$V_1 = \{ z \mid z = y+y^*,\ y = (y_1, y_2), y^* = (y_1^*, y_2^*) \in R_+^2 \text{ such that } \exists (l_{iL}, l_{iK}) \text{ and } (l_{iL}^*, l_{iK}^*) \text{ with } (y_i, -l_{iL}, -l_{iK}) \in Y_i,\ (y^*, -l_{iL}^*, -l_{iK}^*) \in Y_i^*,\ i = 1,2\ \sum_{i=1}^{2} l_{iL} = L,\ \sum_{i=1}^{2} l_{iL}^* = L^*,\ \sum_{i=1}^{2} (l_{iK} + l_{iK}^*) = C + C^* \}.$$

Then W_1 is expressed as

$$W_1 = \{ z \mid \text{there exists no } z' \geq z \text{ such that } z' \in V_1 \}$$

where $z' \geq z$ means $z_i' \geqq z_i$, $i = 1,2$ but $z_i' > z_i$ for some i.

3) For both cases, L is assumed to be fixed.

4) $\pi = \pi (p_2, p_3, K) = Y_1 + p_2 Y_2 + p_3 Y_3$ is regarded as a function of p_2, p_3 and K and is called a support function, and $\partial\pi/\partial p_i = Y_i$, $l = 2,3$, $\partial\pi/\partial K = -r$. For the properties of the support function, see, e.g., Chipman (1972).

5) From Eq. (14), it follows $\partial\eta/\partial p_2 = \begin{vmatrix} b_{11} & b_{21} & b_{31} \\ b_{12} & b_{22} & b_{32} \\ g_1'' & g_2'' & g_3'' \end{vmatrix} g_1^2 / |B|^3 r$ where $g_i = g_i(\omega) = G_i(\omega, 1)$, $\omega = w/r$, $g_i' = b_{i1}$, $g_i - \omega g_i' = b_{i2}$, $g_i'' < 0$. Furthermore observe $\partial\lambda_2/\partial p_2 = - \begin{vmatrix} b_{11} & b_{21} & L \\ b_{12} & b_{22} & C-K \\ g_1'' & g_2'' & 0 \end{vmatrix}$ $g_1^2 / |B|^3 r$. Then $E = y_3\ \partial\eta/\partial p_2 + \partial\lambda_2/\partial p_2$

$$= - \begin{vmatrix} b_{11} & b_{21} & 0 \\ b_{12} & b_{22} & 0 \\ g_1'' & g_2'' & \sum_{i=1}^{2} g_i'' y_i \end{vmatrix} g_1^2 / |B|^3 r \text{ from Eq.s (8) and (9)}$$

$$= - (\sum_{i=1}^{3} g_i'' y_i) g_1^2 / |B|^2 r > 0.$$

6) Since $M_2 = p_2 (\frac{\partial h_2}{\partial I} - \eta \frac{\partial h_3}{\partial I})$, $M_2 \geqq 0$ if $m_i \geqq 0$, $i = 2,3$ *and* $\eta < 0$. $\eta < 0$ *iff* $b_{11}/b_{12} >$ Max $(b_{21}/b_{22}, b_{31}/b_{32})$ or $b_{11}/b_{12} <$ Min $(b_{21}/b_{22}, b_{31}/b_{32})$.

7) Das and Lee (1979) proposed a more general system of adjustment $\dot{p} = f\ [J_1\ (p_2, K)]$, $\dot{K} = g\ [J_2\ (p_2, K)]$ where $f', g' > 0$ and $f(0) = g(0) - r$ and r^* also depend on K because they employ the Batra-Casas model. They then claim that the system has the property of l.d.s. *iff* $J_{11} + J_{22} < 0, J_{11}J_{22} - J_{12}J_{21} > 0$. However unless the speed of adjustment of market i, k_i ($k_1 = f'(0)$, $k_2 = g'(0)$) is unity, their claim is false. In general the system has the property of l.d.s. *iff* $k_1 J_{11} + k_2 J_{22} < 0$, $J_{11}J_{22} - J_{12}J_{21} > 0$. However, it is known that this system has the property of l.d.s. independent of the speed of adjustment *iff* the matrix $J = \begin{vmatrix} J_{11} J_{12} \\ J_{12} J_{22} \end{vmatrix}$ is Hicksian, i.e., J_{11}, $J_{22} < 0$, $J_{11}J_{22} - J_{12}J_{21} > 0$, which was shown by Metzler (1945). But since $J_{22} = 0$, the matrix J is *not* Hicksian, which implies that their system does not have l.d.s. independent of the speed of adjustment.

8) For Cases 2 and 2*, R and R^* are defined implicitly by $p_i = G_i(w, r) = G_i^*(w^*, r), i = 1,2,$ $p_1 = 1$ where $\partial G_i/\partial w = b_{i1}, \partial G_i/\partial r = b_{i2}$ so that $\partial R/\partial p_2 = b_{11}/|B|$, etc.
9) The existence of the state of world diversification (i.e., the state where each country produces all three goods), with the introduction of a tariff and a tax, can be shown in this manner; observing that W_1 exists *iff* W_2 exists, such a state exists for some factor endowments *iff* $1 = G_1(w, r) = G_1^*(w^*, r), p_2 = G_2(w, r) = G_2^*(w^*, r)$ where $\tilde{G}_2(w, r) = G_2(w, r) / T$, $\tilde{G}_i^*(w^*, r) = G_i^*(w^*, r/(1-t))$ has a solution $p_2 > 0, w, w^*, r \geq 0$. Under certain sufficient conditions given by Uekawa (1972) such a solution exists.
10) A sufficient condition for $M_2 \geq M_2^*$ is given in Corollary 4.
11) Brecher and Feenstra (1983) obtained similar expressions (Eq.s (9) through (13)) and discussed their signs.
12) For topics such as the optimal tariff and tax formula – full optimum when both the tariff and the tax can vary, and constrained optimum when only one of them can vary – see Brecher and Feenstra (1983).
13) $\partial Y_1/\partial p_2 + p_2 \partial Y_2/\partial p_2 + p_3 \partial Y_3/\partial p_2 = 0$ is made use of.

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