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The efficiency of monopolistic provision of public goods through simultaneous bilateral bargaining*

Noriaki Matsushima† Ryusuke Shinohara‡

Abstract

We examine a monopolistic supplier's decision about a pure public good when he/she must negotiate with beneficiaries of the good. In our model, while the level of the public good is decided unilaterally by the supplier, the cost share of the public good is negotiated between the supplier and beneficiaries. Our bargaining model is built on simultaneous bilateral bargaining and the *bargaining power* of the supplier is a key factor for the analysis. We show that under some mild conditions, the supplier produces the public good at a Pareto-efficient level in equilibrium if and only if his/her bargaining power is sufficiently weak. In addition, under some reasonable parametric functions, we show that the equilibrium likelihood of the efficient provision of the public good diminishes as the number of beneficiaries increases. We show by a numerical example that the source of the inefficient provision of the public good when the supplier's bargaining power is sufficiently strong may be the excessive supply of the public good.

Keywords Public good; Simultaneous bilateral bargaining; Supplier bargaining power; Nash bargaining solution.
JEL Classification C78, D42, H41, H44.

1 Introduction

We consider a situation in which there is a single supplier of a pure public good and beneficiaries of the good. The supplier unilaterally decides the level of the public good

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and the cost of the public good is distributed to the beneficiaries through negotiation. We investigate under which conditions the supplier decides to provide the public good at a Pareto-efficient level given cost-sharing negotiation.

As an example of our situation, we consider a public project with interregional spillover. In a given country, there is a river that passes through region U in its upstream area and regions D1–D3 in its downstream area, as depicted in Figure 1. The central government constructs a dam in region U for river administration, which benefits region U as well as the downstream regions. Although the central government unilaterally decides the scale of the dam, the cost of the dam is shared by each regional government. The central government bilaterally negotiates how to share the cost with each regional government. When the central government cares about the budget surplus (the sum of transfers from the regional governments minus the cost of the public good), does the central government achieve the efficient provision of the public good? The analysis of our situation seems significant from the viewpoint of the provision of public goods in the real world. To the best of our knowledge, there are few models to capture such bargaining situations.

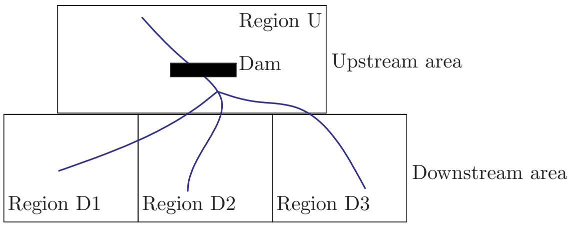

Figure 1: Sharing the construction cost of a dam

In the real world, we can observe intergovernmental negotiations on cost sharing when the central government undertakes public projects of river administration that benefit multiple districts. In Japan, according to the River Act, the central government can force the local government of the district where the project is undertaken (region U in the above example) to defray some fixed rate of the cost. The central government can also demand compensation from other local governments benefiting from spillover of the project (the local governments in the downstream area in the above example). However, the River Act does not clarify the specific rules of such compensation.[1] According to Kobayashi and Ishida (2012), for most central government projects that benefit several prefectures, the ways to share costs are negotiated between the regional development bureaus, which are delegated by the central government, and the relevant prefectures.[2]

[1]Articles 59, 60, and 63 of the River Act require cost sharing for river administration. Kobayashi and Ishida (2012) summarize the rule in Section III–4. In addition, the Road Act has similar clauses for road administration (see Section IV–4 in Kobayashi and Ishida, 2012).

[2]As another example, we can consider the construction of the new national stadium in Tokyo. The scale of the stadium is decided by the Japan Sport Council, which is an extra-governmental organization of the Ministry of Education, Culture, Sports, Science and Technology (MEXT), but the division of the construction cost is now being negotiated between the MEXT and the Tokyo metropolitan government. The total cost of the stadium's construction has been estimated at 169.2 billion yen. Because the stadium significantly benefits the Tokyo area, the central government has asked the Tokyo metropolitan government to pay 50 billion yen toward the construction cost. (See: "Govt to talk with Masuzoe on new venue," *The*

To address these questions, we construct a three-stage game with complete information. Players are the single supplier of a pure public good and consumers (beneficiaries) who benefit from the public good but cannot produce the good on their own. In the first stage, the supplier plans the level of the public good. The supplier does not incur costs at this stage. The final decision is made in the third stage. In the second stage, the supplier bilaterally and simultaneously negotiates with each consumer about his/her contribution to the level of the public good decided in the first stage. In this stage, if n consumers exist, the supplier has n simultaneous bilateral bargaining sessions. When negotiating, the supplier and each consumer in the session anticipate the outcome of the other bilateral bargaining sessions since all sessions are simultaneous. The anticipation by one session for the other sessions affects the surplus of this session. In equilibrium, each session correctly anticipates the outcome of the other sessions. We assume that through a *Nash bargaining solution*, each session shares the surplus correctly anticipated. The supplier and consumer in each session share the surplus in proportion to their *bargaining powers*, which are given exogenously. In the third stage, given the level of the public good in the first stage and the transfers from the consumers in the second stage, the supplier decides whether he/she will provide the public good at the level decided at the first stage. His/her payoff comprises the total transfers from consumers minus the cost of the public good if he/she provides the public good, and zero otherwise.

The relationship between the supplier's decision about the public good and his/her bargaining power is of interest to us. In intergovernmental negotiations in the real world, various factors would influence the relative powers of negotiators, for example, economic size, population size, and capabilities of governments, such as bargaining skills and information possessed by government negotiators (see, e.g., Schneider, 2005; Bailer, 2010). Some scholars state that another important bargaining power source is the saliency that each government attaches to a negotiation. Keohane and Nye (1977) contend that countries that are highly interested in a negotiation topic are forced to make larger concessions. Their view is consistent with some studies of a game-theoretic analysis of bargaining, such as Rubinstein (1982), which states that a less patient player gains less in bargaining if a bargaining participant that has a higher interest in the negotiation topic is eager to reach an agreement at an earlier round of the negotiation (thus, he/she is less patient). Some real-world intergovernmental negotiations have been studied in relation to bargaining powers (e.g., Moravcsik, 1993; Schneider et al., 2010).

Our results show that the supplier's bargaining power is a key factor for the efficient provision of the public good. We first show that under some mild conditions, there is a threshold value of the supplier's bargaining power below which an equilibrium exists at which the supplier provides the public good efficiently (see Theorem 1). That is, the supplier with "sufficiently weak" bargaining power provides the public good efficiently. This supplier receives transfers from all consumers and his/her payoff is "sufficiently close to zero." Interestingly, this result shows that whether the supplier provides the public good efficiently depends crucially on his/her attitude toward budget balance in the negotiation. We can interpret Theorem 1 as follows: a supplier who is more willing to accept a "sufficiently near balanced-budget" outcome is more likely to achieve the efficient provision of the public good (see the third last paragraph in Subsection 3.3).

Japan News. February 28, 2014.)

In addition, we show a numerical example in which the supplier with bargaining power that is beyond a threshold provides the public good over the efficient level. That is, the supplier with "sufficiently strong" bargaining power may provide the public good excessively (see Subsection 4.2). Moreover, based on Theorem 1, we show that as the number of consumers increases, the efficient provision of the public good is less likely to be observed in equilibrium; however, even if the number of consumers is very large, the efficient provision of the public good in equilibrium may be observed with a certainly high likelihood (see Proposition 4 and the discussion thereafter). This shows that there may be a sufficiently weak supplier that provides the public good efficiently even if he/she negotiates with many consumers.

Our results show an interesting tendency in terms of bargaining and internalization of beneficiaries' preferences in comparison with Ray and Vohra (1997, 2001) and Dixit and Olson (2000). Those studies examine public good provision through bargaining under complete information. Ray and Vohra (1997, 2001) introduce two models of coalition formation in which a grand coalition is the only one that can provide the public good efficiently. They show that in their models, the grand coalition does not necessarily form in equilibrium owing to free riding. Dixit and Olson (2000) investigate a voluntary participation property of efficient bargaining, like Coasian bargaining. They show that the voluntary participation of players (beneficiaries of the public good) is very difficult owing to free riding, which leads to inefficient provision of the public good. Although our model totally differs from those models, we end up with the same result: that bargaining does not always achieve efficiency. However, we must note that the reason is completely different. In the existing research, the inefficiency is due to the failure of the internalization of beneficiaries' preferences. In Ray and Vohra's models, if the grand coalition forms, the bargaining within the coalition internalizes the preferences of all beneficiaries and this provides the public good efficiently. However, the grand coalition does not form; hence, the internalization fails. A similar argument applies to Dixit and Olson (2000). On the other hand, in our model, in equilibrium, the supplier sets the public good in the first stage so that he/she receives transfers from all consumers; when this occurs, their preferences are considered in the supplier's objective (see (3) at $m = n$). In this sense, the internalization of the consumer's preferences succeeds through bargaining. Nevertheless, the supplier may not provide the public good efficiently. This is related to the supplier's incentive to provide a sufficiently high level of the public good.[3] Proposition 4 in this study contributes to the literature of the group size effects of public good provision after the seminal work by Olson (1965), as well as Chamberlin (1974), Bergstrom et al. (1986), Pecorino and Temimi (2008), and Pecorino (2015). In Subsection 4.1, we discuss the group size effect in our model in comparison with Dixit and Olson (2000).

In addition, studies of interregional negotiation over public good provision in political economics are relevant to us. Lülfesmann (2002), Gradstein (2004), and Luelfesmann et al. (2015) study interregional negotiation between a region providing a public good (the "supplier," in our terminology) and the region benefiting from it (the "consumer," in our terminology). However, their concern and ours are very different. Their objective is to analyze to what extent the bargaining outcome is distorted by political factors, such as strategic delegation and majority decisions that neglect minorities. Although we are not

[3]Refer to the discussion immediately after Theorem 1.

concerned with distortion by political factors, we obtain an implication for this line of research, which is discussed in Subsection 5.4.

Our results have similarities and dissimilarities with the results in some studies of vertical contracting with externalities. The common agency game is a noncooperative game model of vertical contracting. When the game is applied to public good provision, first, each beneficiary of a public good simultaneously offers a schedule of contributions to the public good to a profit-maximizing supplier of it. The schedule is contingent on a public good level that the supplier provides in the next stage. Second, given the offered schedules, the supplier chooses the level of the public good and provides it at that level.[4] The results of Bernheim and Whinston (1986) and Laussel and Le Breton (2001) are applicable to public good provision. By their results, the common agency game always has a Nash equilibrium at which the supplier provides the public good efficiently (Bernheim and Whinston, 1986). The supplier's payoff at the Nash equilibrium is zero if beneficiaries have *comonotonic* benefit functions (Laussel and Le Breton, 2001).[5] A common feature between the common agency game for public good provision and ours is that the public good is provided by a single supplier. However, the transfers to the supplier are determined in totally different ways. This difference leads to different results. What is interesting is that in our model, although comonotonicity holds, the supplier does not necessarily provide the public good efficiently or his/her payoff is not necessarily zero in equilibrium. Hence, the supplier's decision for the public good and his/her payoff in equilibrium changes according to the interaction between the supplier and beneficiaries. Another interesting point is that in our model, the supplier also provides the public good efficiently when his/her equilibrium payoff is "sufficiently close" to zero, although our game differs from the common agency game. In this sense, in our model too, the efficient provision of the public good seems to be related to the "zero-payoff property" of the supplier.[6]

Segal (1999) investigates vertical contracting through the ultimatum game under some general setting, which is applicable to public good provision. When this game is applied to public good provision, in the first stage, the supplier of a public good makes offers on the level of the public good as well as transfers from beneficiaries to the supplier, and then, each beneficiary independently decides to accept the offers or not. The take-it-or-leave-it offers mean that the supplier has complete bargaining power. By Proposition 2 of Segal (1999), in equilibrium of the offer game, the supplier of the public good with complete bargaining power never produces the public good over the efficient level. By contrast, in our model, the supplier with sufficiently strong bargaining power may oversupply the public good. We carefully discuss the difference in the results in Section 4.2.

Finally, we mention some other related studies. Based on the simultaneous bilateral bargaining model, Chipty and Snyder (1999), Raskovich (2003), and Matsushima and Shinohara (2014) study vertical contracting between the supplier of an intermediate good

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[4] Laussel and Le Breton (1998) and Martimort and Moreira (2010) examine public good provision by the common agency under incomplete information while Bernheim and Whinston (1986) and Laussel and Le Breton (2001) analyze such provision under complete information. Our model is based on the complete information game.

[5] In a one-dimensional public good space, comonotonicity requires that if the level of the public good increases, then the benefit from it for all consumers increases. In the model of this study, this condition also holds.

[6] We present a relevant discussion in the third last paragraph of Subsection 3.3.

and its buyers, while Marshall and Merlo (2004) investigate pattern bargaining of labor contracts. Brito and Oakland (1980) and Brennan and Walsh (1981) study the monopolistic provision of excludable public goods. None of them study the provision of pure public goods. Raskovich (2001), a working-paper version of Raskovich (2003), examines a voluntary contribution game to a discrete pure public good, which completely differs from our model built on simultaneous bilateral bargaining.

The remainder of this paper is organized as follows. Section 2 presents the basic model and Section 3 presents its results. Section 4 presents an analysis under parametric functions. Section 5 discusses the extension of the basic model. Section 6 concludes.

2 The model

Consider an economy with a *pure* public good and a private good (money), with a supplier of the public good and $n \geq 2$ consumers of the public good. Only the supplier can provide the public good. To consume the public good, each consumer needs to pay some amount of money and delegate the provision of it to the supplier. The supplier's objective is assumed to maximize the budget surplus (the sum of the payments from consumers minus the cost of the public good). The supplier provides the public good since he/she can receive transfers from consumers by providing it.[7]

The set of players is denoted by $\{s, 1, \dots, n\}$, where s represents the supplier and i ($i = 1, \dots, n$) represents a consumer. Let N be the set of consumers. Then, the set of players is denoted by $\{s\} \cup N$. The level of the public good is typically denoted by $g \geq 0$ and consumer i's transfer to the supplier is denoted by T_i. The cost function of the public good is denoted by $c : \mathbb{R}_+ \to \mathbb{R}_+$ such that $c(0) = 0$, c is an increasing, convex (sometimes, weakly convex), and twice continuously differentiable function. When the supplier provides g units of the public good and receives payment T_i from each $i \in N$, its payoff is $\sum_{i \in N} T_i - c(g)$. Each consumer i receives payoff $v(g) - T_i$, where $v : \mathbb{R}_+ \to \mathbb{R}_+$ is a benefit function from the public good such that $v(0) = 0$ and v is an increasing, concave (sometimes, weakly concave), twice continuously differentiable function. Note that in this model, each consumer has the same benefit function v. We further impose the following conditions on $v(g)$ and $c(g)$.

Assumption 1
(1.1) [$\lim_{g \to 0} v'(g) = \infty$, $\lim_{g \to \infty} v'(g) = 0$, $\lim_{g \to 0} c'(g) = 0$, and $\lim_{g \to \infty} c'(g) = \infty$], [$\lim_{g \to 0} v'(g) = \infty$, $\lim_{g \to \infty} v'(g) = 0$, and $c'(g)$ is finite] or [$v'(g)$ is finite, $\lim_{g \to 0} c'(g) = 0$, and $\lim_{g \to \infty} c'(g) = \infty$].
(1.2) For all $g > 0$, $\dfrac{c'(g)}{c(g)} > \dfrac{v'(g)}{v(g)}$.

Condition (1.1) guarantees interior solutions of the model. Condition (1.2) implies that $c(g)/v(g)$ is increasing in g, which is crucial for the subsequent analysis. We consider these conditions not to be restrictive because they are satisfied by many benefit and cost functions. For example, the assumptions are satisfied if $v(g) = \theta g^{\alpha}$ and $c(g) = \gamma g^{\delta}$ where

[7] A discussion on the supplier's objective is presented in Section 5.

α, θ, γ, and δ are positive constants such that $\alpha \in (0,1]$, $\delta \geq 1$, and $\alpha \neq \delta$ (see also Proposition 3 in Subsection 4.1). By the l'Hôpital's rule and (1.1), we obtain

$$\lim_{g \to 0} \frac{c(g)}{v(g)} = 0. \tag{1.3}$$

We model a three-stage game to analyze public good provision through simultaneous bilateral bargaining. In the first stage, the supplier chooses the level of the public good $g \geq 0$. In this stage, the supplier does not provide the public good at this level or incur the cost. He/she makes a final decision for public good provision in the third stage. In the second stage, the supplier and each consumer $i \in N$ *bilaterally* and *simultaneously* negotiate over the division of the joint surplus, so that $T_i \geq 0$ (a transfer from consumer i to the supplier) is determined. Note that in this stage, the supplier faces n independent bilateral negotiations. We assume that the outcome of the second stage is determined as follows. First, the outcome of each negotiation is given by the (asymmetric) Nash bargaining solution, in the belief that the bargaining outcomes with the other parties are determined in the same way. Second, the joint surplus of one bilateral bargaining is divided between the consumer and the supplier in the proportion of $1-\beta$ to β, in which $\beta \in [0,1]$ represents the supplier's bargaining power. In the third stage, the supplier decides whether to execute a *project* $(g, (T_i)_{i \in N})$, a tuple of the level of the public good, and the transfers. If he/she executes the project, then he/she provides g units of the public good and receives T_i from each consumer i. As a result, the supplier's payoff is $\sum_{i \in N} T_i - c(g)$ and each consumer i's payoff is $v(g) - T_i$. Otherwise, no public good is provided and no money is transferred. Then, the supplier's and each consumer's payoffs are zero.

We solve this game by backward induction. We simply solve the supplier's payoff maximization problem in the first and third stages. We examine the second-stage outcome by simultaneously applying the Nash bargaining solution to each bilateral negotiation. Under simultaneous bilateral negotiations, the negotiators in each bilateral negotiation need to anticipate the outcomes of the other negotiations. The anticipation of one negotiation affects the disagreement payoff and the surplus of the negotiation. If a consumer and the supplier in a negotiation anticipate that the other consumers' transfer covers the cost of the public good $c(g)$, then this consumer can free ride the public good. Thus, this consumer's disagreement payoff is the free-riding payoff $v(g)$. The supplier can receive $\sum_{j \neq i} T_j$ even if he/she does not reach an agreement with this consumer. Thus, the supplier's disagreement payoff is $\sum_{j \neq i} T_j - c(g)$. By contrast, if the negotiators anticipate that the other consumers' transfers fall short of the cost, then the failure of their negotiation means that no public good is provided and no transfer is made. Hence, the disagreement payoffs to the supplier and the consumer are both zero. The joint surplus of each bilateral negotiation also depends on the anticipation of the other negotiations, accordingly. When applying the Nash bargaining solution to all negotiations, we assume that the negotiators in each bilateral negotiation have consistent beliefs about the other negotiations: the outcome of each negotiation is predicted correctly by the others.[8]

[8]The method of simultaneous application of the Nash bargaining solution follows Chipty and Snyder (1999) and Raskovich (2003). However, note that in their models, free riding is impossible. See, for example, the condition "$v_i(0, q_{-i}) = 0$" in Raskovich (2003, p. 410, 12th line from the bottom).

A noncooperative foundation for the asymmetric Nash bargaining solution has been presented by several studies. As Binmore et al. (1986) show, the subgame perfect equilibrium outcome in Rubinstein's bargaining model with alternating offers and risk of breakdown approximates the Nash bargaining solution. In addition, in Binmore et al. (1986, pp. 186–187), the relationship between the Nash bargaining solution and asymmetric bargaining power is discussed. Hence, we can approximately interpret that in the second-stage game of our model, each pair of supplier and consumer plays the alternating offer bargaining game with risk of breakdown, anticipating the other sessions' outcomes. Therefore, our solution by backward induction is consistent with a subgame perfect Nash equilibrium.

3 Analysis

The analysis for $\beta = 0$ is trivial since the supplier's payoff is always zero for any of his/her choices of g in the first stage. We focus on the case of $\beta \in (0, 1]$.

We first show that *under some condition introduced as Condition 1 later, a threshold value of the supplier's bargaining power* $\bar{\beta}$ *exists such that if* $0 \leq \beta \leq \bar{\beta}$, *there is an equilibrium at which the public good is provided efficiently* (Proposition 1). We construct the equilibrium by backward induction.

3.1 The third stage: The supplier's execution

We start with the third stage. Clearly, given $(g, (T_i)_{i \in N})$, the supplier executes it if $\sum_{i \in N} T_i > c(g)$, he/she is indifferent between execution and nonexecution if $\sum_{i \in N} T_i = c(g)$, and he/she does not execute otherwise. Henceforth, we assume that when $\sum_{i \in N} T_i = c(g)$, the supplier does not execute.

3.2 The second stage: Simultaneous bilateral bargaining

3.2.1 Second-stage equilibria

In the second stage, who contributes to the project and how much money each contributor transfers to the supplier are determined. Who contributes to the project depends on who is pivotal to the project, defined below.

Definition. Let $g \geq 0$ be a level of the public good. Let T_j be a transfer from consumer $j \in N$. Consumer $i \in N$ is *pivotal* to the execution of the project $(g, (T_j)_{j \in N})$ if $\sum_{j \in N \setminus \{i\}} T_j \leq c(g) < \sum_{j \in N} T_j$.

The pivotal consumers are defined based on the third-stage equilibrium.[9] The transfer from the pivotal consumer is necessary for the supplier to provide the public good. If the bargaining with the pivotal consumer breaks down, the supplier does not execute the project in the third stage.

We derive the level of transfer from each consumer to the supplier. We first consider the case in which consumer i is not pivotal to the project $(g, (T_j)_{j \in N})$, that is, the case in which either $\sum_{j \in N} T_j \leq c(g)$ or $\sum_{j \in N \setminus \{i\}} T_j > c(g)$ is satisfied. In the former case, the

[9] The definition of the pivotal consumers is the same as that of Raskovich (2003).

supplier chooses not to execute at the third stage. Hence, the surplus of the bargaining is zero. The latter case means that the supplier executes the project at the third stage, irrespective of whether the bargaining with consumer i succeeds. Hence, the supplier's net surplus from this bargaining is $T_i + \sum_{j \neq i} T_j - c(g) - (\sum_{j \neq i} T_j - c(g)) = T_i$ and consumer i's net surplus is $v(g) - T_i - v(g) = -T_i$; the joint surplus of this bargaining is zero. In any case, the joint surplus of the bargaining with the nonpivotal consumer i is zero, which implies $T_i = 0$; consumer i free rides the public good.

We next consider the case in which consumer i is pivotal to $(g, (T_j)_{j \in N})$, that is, the case in which $\sum_{j \in N \setminus \{i\}} T_j \leq c(g) < \sum_{j \in N} T_j$. In this case, if this bargaining breaks down, then the supplier chooses nonexecution in the third stage and the supplier's and consumer i's disagreement payoffs are zero. If an agreement is reached in the bargaining, then the supplier's payoff is $\sum_{j \in N} T_j - c(g)$ and consumer i's payoff is $v(g) - T_i$. Then, the joint surplus of this bargaining is

$$v(g) - T_i + T_i + \sum_{j \in N \setminus \{i\}} T_j - c(g) = v(g) + \sum_{j \in N \setminus \{i\}} T_j - c(g).$$

This joint surplus is divided between the supplier and consumer i in the proportion of β and $1 - \beta$. Hence,

$$T_i = v(g) - (1-\beta) \left(v(g) + \sum_{j \in N \setminus \{i\}} T_j - c(g) \right) = \beta v(g) + (1-\beta) c(g) - (1-\beta) \sum_{j \in N \setminus \{i\}} T_j. \quad (1)$$

Finally, we examine how many consumers become pivotal and how much money the pivotal consumers transfer to the supplier. Suppose that m pivotal consumers $(1 \leq m \leq n)$ exist. Let $M \subseteq N$ be the set of pivotal consumers. Since condition (1) holds for all m consumers, solving the system of those equations yields $(T_j^m)_{j \in M}$ such that for each $j \in M$,

$$T_j^m = \frac{\beta v(g) + (1-\beta) c(g)}{\beta + (1-\beta) m} \ (> 0 \text{ if } g > 0). \quad (2)$$

The payoff to the supplier is

$$\pi_S^m(g) = \sum_{j \in M} T_j^m - c(g) = \frac{\beta(m v(g) - c(g))}{\beta + (1-\beta) m}, \quad (3)$$

the payoff to the pivotal consumer $i \in M$ is

$$v(g) - T_i^m = \frac{(1-\beta)(m v(g) - c(g))}{\beta + (1-\beta) m}, \quad (4)$$

and the payoff to the nonpivotal consumer $i \in N \setminus M$ is $v(g)$ (The superscript "m" of T_j^m and π_S^m refers to the number of pivotal consumers). Henceforth, we call pivotal consumers *contributors* and nonpivotal consumers *free riders*.

The sum of the joint surplus of the bargaining sessions with m pivotal consumers is $m v(g) - c(g)$ and the supplier's share of the surplus and the pivotal consumer's share are $\beta / (\beta + (1-\beta) m)$ and $(1-\beta)/(\beta + (1-\beta) m)$, respectively. Since $\beta + (1-\beta) m$ is the sum of the bargaining powers over the supplier and m pivotal consumers, the surplus is distributed in proportion to the bargaining power.[10]

Lemma 1 shows that the equilibrium number of contributors is determined according to the level of the public good in the first stage.

Lemma 1
(1.a) For any given $g \geq 0$ such that $nv(g) \leq c(g)$, the equilibrium number of contributors at g is zero.
(1.b) For any given $g > 0$ such that $nv(g) > c(g)$, m is the equilibrium number of contributors at g if and only if

$$\frac{c(g)}{v(g)} < m \leq \frac{c(g)}{\beta v(g)} + 1. \tag{5}$$

At least one integer m exists that satisfies (5).

Proof. (1.a) If g satisfies $nv(g) \leq c(g)$, there is no surplus in any bilateral bargaining. Hence, no consumer pays a positive fee; the number of contributors is zero.

(1.b) m is the number of contributors if and only if $\sum_{j \in M} T_j^m - T_i^m \leq c(g) < \sum_{j \in M} T_j^m$ for each $i \in M$. From (2), these inequalities hold if and only if $(m-1)\beta v(g) \leq c(g) < mv(g)$, implying (5). Clearly, an integer m exists that satisfies $c(g)/v(g) < m \leq (c(g)/v(g)) + 1$. Hence, at least one integer m exists that satisfies (5) since $\beta \leq 1$.

■

Note that if $nv(g) > c(g)$, then the pivotal condition restricts the number of contributors, which means that the supplier does not necessarily receive positive transfers from all consumers. We summarize the second-stage equilibria as follows:

The second-stage equilibria. After the supplier decides the level of the public good g in the first stage, the equilibrium outcome of the second-stage subgame is $(M, (T_j^m)_{j \in N})$ where M is the set of contributors, the equilibrium number of contributors $m = |M|$ is determined according to Lemma 1, and the equilibrium transfer is

$$T_j^m = \begin{cases} \dfrac{\beta v(g) + (1-\beta)c(g)}{\beta + (1-\beta)m} & \text{if } j \in M, \\ 0 & \text{if } j \in N \backslash M. \end{cases} \tag{6}$$

3.2.2 A second-stage equilibrium that maximizes the supplier's payoff

Note that for some level of the public good in the first stage, there may be multiple second-stage equilibria that support different equilibrium numbers of contributors because (5) may include multiple integers. For the backward induction analysis, we focus on the equilibrium that maximizes the supplier's payoff among the second-stage equilibria in every second-stage subgame.

To investigate which second-stage equilibrium maximizes the supplier's payoff, we start with the restatement of (5), which is based on the level of the public good. Define $\bar{g}^m$ for

[10] $(T_j^m)_{j \in M}$ is supportable by other bargaining models. See Subsection 5.1.

each $m \in \{1, \ldots, n\}$ and $\underline{g}^m$ for each $m \in \{2, \ldots, n\}$ such that

$$m = \frac{c(\bar{g}^m)}{v(\bar{g}^m)} \quad \text{and} \quad m = \frac{c(\underline{g}^m)}{\beta v(\underline{g}^m)} + 1. \tag{7}$$

Note that $c(g)/v(g)$ is increasing in g by (1.2) in Assumption 1. Thus, by (1.2) in Assumption 1 and (1.3), $\bar{g}^m$ can be defined uniquely for each $m \in \{1, \ldots, n\}$ and $\underline{g}^m$ can be defined uniquely for each $m \in \{2, \ldots, n\}$. We adopt the convention that $\underline{g}^1 \equiv 0$ by (1.3).

Lemma 2
(2.a) For any given $g > 0$ such that $nv(g) > c(g)$, m is the equilibrium number of contributors at g if and only if

$$\underline{g}^m \leq g < \bar{g}^m. \tag{5'}$$

(2.b) For each $m \in \{2, \ldots, n\}$, $\underline{g}^m < \bar{g}^{m-1}$ if $\beta < 1$ and $\underline{g}^m = \bar{g}^{m-1}$ if $\beta = 1$.

Proof. (2.a) is the restatement of (1.b) in Lemma 1. We now show (2.b). By the definition, $c(\bar{g}^{m-1})/v(\bar{g}^{m-1}) = m - 1$ and $c(\underline{g}^m)/(\beta v(\underline{g}^m)) = m - 1$. By those conditions, we obtain$c(\underline{g}^m)/v(\underline{g}^m) = \beta(m-1) \leq m - 1 = c(\bar{g}^{m-1})/v(\bar{g}^{m-1})$. Since $c(g)/v(g)$ is increasing in g, we obtain $\underline{g}^m \leq \bar{g}^{m-1}$. ■

Lemma 2 shows that $\underline{g}^m$ and $\bar{g}^m$ make the lower and upper bounds of the public good level where m is the equilibrium number of contributors in the second stage. In addition, Lemma 2 shows the condition under which the second-stage subgame has multiple equilibria that support different numbers of contributors. (2.b) shows that for each $m \in \{2, \ldots, n\}$, $\underline{g}^{m-1} \leq g < \bar{g}^{m-1}$ (the range of $m-1$ contributors) and $\underline{g}^m \leq g < \bar{g}^m$ (the range of m contributors) overlap if and only if $\beta < 1$. Thus, in the case of $\beta < 1$, if the supplier chooses g between $\underline{g}^m$ and $\bar{g}^{m-1}$ in the first stage, multiple second-stage equilibria exist that support the existence of $m - 1$ contributors and that of m contributors in the subsequent second stage.

When there are multiple numbers of contributors attained at equilibria of a second-stage subgame, we focus on *the equilibrium that maximizes the supplier's payoff from the set of the second-stage equilibria*. Lemma 3 is helpful to clarify which second-stage equilibrium maximizes the supplier's payoff.

Lemma 3 For each $g > 0$ and each $m \in \{1, \ldots, n-1\}$, $\pi_S^m(g) \leq \pi_S^{m+1}(g)$ with strict inequality if $\beta > 0$.

Proof. From (3), we obtain

$$\pi_S^{m+1}(g) - \pi_S^m(g) = \frac{\beta(\beta v(g) + (1-\beta)c(g))}{(\beta + (1-\beta)(m+1))(\beta + (1-\beta)m)} \geq 0 \text{ with strict inequality if } \beta > 0. \quad ■$$

Lemma 3 shows that for each $g > 0$, $\pi_S^m(g)$ is nondecreasing in m. By Lemma 3, in the second stage, immediately after the supplier chooses g such that $nv(g) > c(g)$, *the second-stage equilibrium that maximizes the supplier's payoff within the set of the second-stage equilibria is the equilibrium at which the number of contributors is the maximal integer* *among m that satisfies* (5).

The selected second-stage equilibrium. For each $g \geq 0$, denote the equilibrium number of contributors by $m(g) \in \{0, \dots, n\}$. After the supplier decides the level of the public good g in the first stage, the equilibrium outcome of the second-stage subgame is $(M, (T_j^{m(g)})_{j \in N})$ where M is the set of contributors, the equilibrium number of contributors $m(g) = |M|$ is

$$m(g) = \begin{cases} 0 & \text{if } g = \underline{g}^1 \ (= 0), \\ 1 & \text{if } g \in (\underline{g}^1, \underline{g}^2), \\ k & \text{if } g \in [\underline{g}^k, \underline{g}^{k+1}) \quad (k \in \{2, \dots, n-1\}), \\ n & \text{if } g \in [\underline{g}^n, \bar{g}^n), \\ 0 & \text{if } g \geq \bar{g}^n, \end{cases} \tag{8}$$

and equilibrium transfer $(T_j^{m(g)})_{j \in N}$ conforms to (6).

We can learn from (8) that the supplier does not always receive transfers from all consumers. The equilibrium number of contributors increases as the level of the public good increases; hence, the supplier sets the level of the public good "sufficiently high" if he/she receives transfers from all consumers.

Finally, we remark that we do not explicitly model how the supplier selects $m(g)$ contributors out of n identical consumers when $0 < m(g) < n$. However, note that since the consumers are identical, the way to select contributors does not affect the main results of this study. The random selection of $m(g)$ consumers out of n consumers could be one of the ways to select.

3.3 The first stage: The supplier's decision about the level of the public good

Given that the supplier receives transfers from $m(g)$ contributors in the second stage for each $g \geq 0$, we investigate the public good level that maximizes the supplier's payoff.

3.3.1 Some preliminaries

As a reference level for the optimal g for the supplier, we define

$$g(m) \equiv \arg\max_{g \geq 0} \ mv(g) - c(g).$$

From the maximization problem, there are several observations, as follows.

1. By (3), if the number of contributors is fixed at m, then $\pi_S^m(g)$ is maximized at $g = g(m)$.

   This implies that $\pi_S^m(g)$ is increasing in $g \in [0, g(m))$ and decreasing in $g \in [g(m), \infty)$.

2. $g(m) < g(m+1)$ for each $m \in \{1, \dots, n-1\}$.

3. $g(n)$ is the (Pareto) efficient level of the public good because $g(n)$ maximizes $nv(g) - c(g)$.

Corollary 1 shows that $\pi_S^m(g(m))$ is increasing in m.

Corollary 1 For each $m \in \{1, \ldots, n-1\}$, $\pi_S^m(g(m)) < \pi_S^{m+1}(g(m+1))$.

Proof. By Lemma 3, $\pi_S^m(g(m)) \leq \pi_S^{m+1}(g(m))$. From the abovementioned first and second observations, $\pi_S^{m+1}(g(m)) < \pi_S^{m+1}(g(m+1))$. ■

Assumption 1 ensures that $c(g)/v(g)$ has an inverse function and by (7), it is given as $G(\cdot)$ such that

$$\underline{g}^m = G(\beta(m-1)).$$

Obviously, $G(\beta(m-1))$ is continuous and increasing in the value of $\beta(m-1)$.

For each $g \geq 0$ and $m \in (1, n]$, define $\beta(g, m) \in \mathbb{R}_+$ such that

$$\beta(g, m) \equiv \frac{c(g)}{v(g)} \cdot \frac{1}{m-1}. \tag{9}$$

Note that if $\beta = \beta(g, m)$, $g = \underline{g}^m$ $(= G(\beta(g,m)(m-1)))$. Up to now, we have supposed that m is an integer. However, even if we extend m in the domain of $\beta(g, m)$ to *real numbers greater than one*, we can also define $\beta(g, m)$. For mathematical tractability, we suppose that m of $\beta(g, m)$ is a real number greater than one.

We now examine how $\beta(g(m), m)$ and $\beta(g(m), m+1)$ react to the change of m.[11] As we see later, $\beta(g(m), m)$ marks the threshold level of the supplier's bargaining power below which the supplier produces the public good efficiently. By (9),

$$\beta(g(m), m) = \frac{c(g(m))}{v(g(m))} \cdot \frac{1}{m-1} \quad \text{and} \quad \beta(g(m), m+1) = \frac{c(g(m))}{v(g(m))} \cdot \frac{1}{m}. \tag{10}$$

Thus,

$$\begin{aligned} g(m) &= \underline{g}^m \quad \text{when } \beta = \beta(g(m), m) \text{ and} && (11)\\ g(m) &= \underline{g}^{m+1} \quad \text{when } \beta = \beta(g(m), m+1). && (12) \end{aligned}$$

By differentiating (10) with respect to m, we obtain Lemma 4.

Lemma 4 It follows that

$$\frac{\partial \beta(g(m), m)}{\partial m} < 0 \quad \text{if and only if} \quad \frac{dg(m)}{dm} < \left.\frac{\partial G(\beta(m-1))}{\partial m}\right|_{\beta(m-1)=\beta(g(m),m)(m-1)} \tag{13}$$

$$\text{and} \quad \frac{\partial \beta(g(m), m+1)}{\partial m} < 0 \quad \text{if and only if} \quad \frac{dg(m)}{dm} < \left.\frac{\partial G(\beta m)}{\partial m}\right|_{\beta m=\beta(g(m),m+1)m}. \tag{14}$$

Proof of Lemma 4 is in the Appendix. The magnitude of the relationship between the differential coefficient of $g(m)$ and that of $G(\beta(m-1))$ is not clear. Hence, we need to consider various cases on the slope of $\beta(g(m), m)$ and $\beta(g(m), m+1)$ in m. Henceforth, our analysis is built on the following "reasonable" condition.

[11] Note that $g(m)$ can be defined for any real number $m > 1$.

Condition 1 $\beta(g(m), m)$ and $\beta(g(m), m+1)$ are nonincreasing in m.

(13) and (14) provide an economic interpretation of Condition 1. $dg(m)/dm$ measures a marginal increase in the efficient level of the public good through a marginal increase in m under m contributors. Note that $G(\beta(m-1))$ ($G(\beta m)$, resp.) is the minimum level of the public good at which m ($m+1$, resp.) consumers contribute. In other words, $G(\beta(m-1))$ ($G(\beta m)$, resp.) is the level that is needed for none of m ($m+1$, resp.) consumers to free ride. Hence, $\partial G(\beta(m-1))/\partial m$ ($\partial G(\beta m)/\partial m$, resp.) is a marginal increase in the level of the public good through a marginal increase in m such that none of m ($m+1$, resp.) consumers to free ride. (13) ((14), resp.) imposes that the marginal increase of the public good to prevent m contributors from free riding must be larger than that to supply the public good efficiently under m contributors. Furthermore, Condition 1 is satisfied under many benefit and cost functions (see Proposition 3 in Subsection 4.1). For analytical completeness, we discuss the case without Condition 1 in the final paragraph of Section 3.

Lemma 5 shows the relative relationship between $\beta(g(m), m)$ and $\beta(g(m), m+1)$.

Lemma 5
(5.a) $\lim_{m\to 1} \beta\left(g(m), m\right) = \infty$ and $\lim_{m\to 1} \beta\left(g(m), m+1\right) = \dfrac{c(g(1))}{v(g(1))}$.
(5.b) For each $m > 1$, $\beta\left(g(m), m+1\right) < \beta\left(g(m), m\right)$.

Proof. (5.a) By (10), as $m \to 1$,

$$\beta\left(g(m), m\right) = \frac{c(g(m))}{v(g(m))(m-1)} \to \infty \text{ and } \beta\left(g(m), m+1\right) = \frac{c(g(m))}{v(g(m))m} \to \frac{c(g(1))}{v(g(1))}.$$

(5.b) is immediate from (10). ■

Figure 2 is an example that reflects Lemma 5 and Condition 1. Let $\beta' \in (0, 1]$ be a fixed value of bargaining power. This figure illustrates a case in which $\beta(g(m), m)$ and $\beta(g(m), m+1)$ are decreasing in m and there are m' and m'' such that $\beta(g(m'), m') = \beta'$ and $\beta(g(m''), m''+1) = \beta'$.

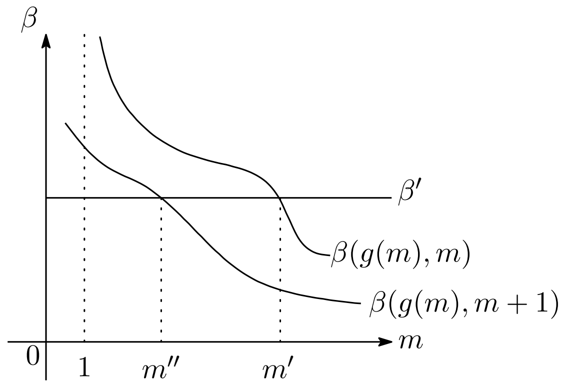

Figure 2: The case when m' and m'' exist for $\beta' > 0$

Note that while $g(m)$ is dependent on m but independent of β, $\underline{g}^m$ $(= G(\beta(m-1)))$ and $\underline{g}^{m+1}$ $(= G(\beta m))$ depend on m and β. Lemma 6 shows how the relationship between $g(m)$ and $\underline{g}^m$ (or $\underline{g}^{m+1}$) changes according to the values of m and β.

Lemma 6 Let $\beta \in (0,1]$ and $m \in (1,n]$. Then,

$$\underline{g}^{m+1} \leq g(m) \quad \text{if } \beta \leq \beta\left(g(m), m+1\right), \tag{15}$$

$$\underline{g}^{m} \leq g(m) < \underline{g}^{m+1} \quad \text{if } \beta\left(g(m), m+1\right) < \beta \leq \beta\left(g(m), m\right), \text{ and} \tag{16}$$

$$g(m) < \underline{g}^{m} \quad \text{if } \beta > \beta\left(g(m), m\right). \tag{17}$$

Proof. By (11) and (12),

$$g(m) = G\left(\beta\left(g(m), m\right)(m-1)\right) \text{ and } g(m) = G\left(\beta\left(g(m), m+1\right) m\right).$$

Since $G\left(\beta(m-1)\right)$ and $G\left(\beta m\right)$ are increasing in β, we obtain $g(m) < G(\beta(m-1)) = \underline{g}^m$ if and only if $\beta > \beta(g(m), m)$; $g(m) < G(\beta m) = \underline{g}^{m+1}$ if and only if $\beta > \beta(g(m), m+1)$. Thus, we obtain (15)–(17). ■

Lemma 7 shows what level of the public good g maximizes $\pi_S^m(g)$ under the constraint $g \in [\underline{g}^m, \underline{g}^{m+1})$ for each m and β.

Lemma 7 Let $m \in \{1, \ldots, n\}$ and $\beta \in (0,1]$. Then,

(7.a) If $\beta \leq \beta\left(g(m), m+1\right)$, then $\underline{g}^{m+1} \leq g(m)$ (see (15)); hence, $\pi_S^m(g)$ is increasing within the interval $[\underline{g}^m, \underline{g}^{m+1})$. Thus, there is no maximizer when restricting to this interval.

(7.b) If $\beta\left(g(m), m+1\right) < \beta \leq \beta\left(g(m), m\right)$, then $g(m) \in [\underline{g}^m, \underline{g}^{m+1})$ (see (16)); hence, within the interval $[\underline{g}^m, \underline{g}^{m+1})$, $\pi_S^m(g)$ is maximized at $g = g(m)$.

(7.c) If $\beta\left(g(m), m\right) < \beta$, then $g(m) < \underline{g}^m$ (see (17)); hence, within the interval $[\underline{g}^m, \underline{g}^{m+1})$, $\pi_S^m(g)$ is maximized at $g = \underline{g}^m$.

3.3.2 Analysis of the supplier's choice of the level of the public good

Proposition 1 *Under Assumption 1 and Condition 1, in the first stage induced by the second- and third-stage equilibrium, $g(n)$ maximizes the supplier's payoff if β satisfies*[12]

$$0 \leq \beta \leq \min_{m \in \{1, \ldots, n\}} \beta\left(g(m), m\right). \tag{18}$$

Therefore, if β satisfies (18)*, then an equilibrium exists at which the supplier produces the public good efficiently.*

Imposing an additional condition for monotonicity of π_S^m (Condition 2 below), we show (18) is also a necessary condition for the existence of equilibria at which the supplier provides the efficient level of the public good.

[12] We adopt the convention that $\beta(g(1), 1) = \infty$ for mathematical consistency.

Condition 2 $\pi_S^m(\underline{g}^m) < \pi_S^{m+1}(\underline{g}^{m+1})$ for each $m \in \{1, \ldots, n-1\}$.

Proposition 2 *Under Assumptions 1 and Conditions 1–2, an equilibrium exists at which the supplier produces the public good efficiently only if β satisfies* (18).

Proofs of Propositions 1 and 2 are in the Appendix. We immediately obtain Theorem 1 from Propositions 1 and 2.

Theorem 1 *Under Assumption 1 and Conditions 1–2, an equilibrium exists at which the supplier produces the efficient level of the public good ($g(n)$) if and only if β satisfies $0 \leq \beta \leq \min_{m\in\{1,\ldots,n\}} \beta\,(g(m), m) = \beta\,(g(n), n)$.*

As Theorem 1 proves, $\beta(g(n), n)$ marks the threshold value of the supplier's bargaining power below which the supplier produces the public good efficiently. The supplier provides the public good efficiently at an equilibrium if and only if his/her bargaining power is "sufficiently weak" under several assumptions and conditions. At this equilibrium, the supplier receives transfers from all consumers, as we see in the proof of Proposition 1, and his/her payoff is "sufficiently close" to zero because β is sufficiently small.

We present an intuition of Theorem 1. We can confirm from Lemma 3 that for any level of the public good, the more consumers from which the supplier receives transfers, the higher is the supplier's payoff; hence, the supplier desires to receive transfers from all consumers. Since the public good is pure, the supplier must set the level of the public good "sufficiently high" in order to receive transfers from all consumers; if the level of the public good is low so that the cost of the public good is compensated by the transfers from fewer than n consumers, then free riders exist. However, there is another problem: the supplier may choose an excessively higher level of the public good. If the supplier's bargaining power is strong, the transfers from contributors are large. Then, there may be a case in which the cost of $g(n)$ is compensated by fewer than n consumers. In this case, even if the supplier chooses $g(n)$, there are free riders. Then, the supplier sets the level of the public good over $g(n)$ to eliminate the free riders because he/she wants to receive transfers from all consumers. This leads to excessive provision of the public good.[13] Therefore, the supplier provides the public good efficiently if and only if his/her bargaining power is sufficiently weak such that the cost of $g(n)$ is compensated by all consumers, but not by fewer than n consumers.

We can interpret Theorem 1 in relation to the efficiency of the supplier's decision and his/her "willingness" to accept budget balance. Suppose that the supplier receives transfers from m consumers. By (2), if $\beta = 0$, each contributor pays $c(g)/m$ and the sum of the transfers is equal to $c(g)$; this supplier accepts budget balance through negotiation. If $\beta = 1$, the supplier extracts full benefits $v(g)$ from all m consumers; this supplier does not accept budget balance through negotiation. Since each consumers' transfer decreases in β, and hence, the budget surplus also decreases, we can interpret that the weaker the supplier's bargaining power is, the nearer to a balanced-budget outcome will the supplier accept in the negotiation. From this viewpoint, we conclude that if a supplier is willing

[13]In Subsection 4.2, we provide an example in which the public good is excessively provided in equilibrium.

to accept a "sufficiently near" balanced-budget outcome, he/she is more likely to achieve the efficient provision of the public good.

Note that Condition 2 is imposed only in Proposition 2. As we see in Proposition 3 in Subsection 4.1, Condition 2 is satisfied under various benefit and cost functions.

Finally, we mention how the results change without Condition 1.[14] Even if Condition 1 is violated, we can show that the supplier supplies the public good efficiently if his/her bargaining power is sufficiently weak. Hence, a similar result to Proposition 1 holds in the case. However, we point out the possibility that the supplier supplies the public good efficiently even if his/her bargaining power is sufficiently strong; the result may be different from Proposition 2 if Condition 1 is not satisfied. Again, we emphasize that Condition 1 is reasonable in our analysis because it satisfies the reasonable parametric benefit and cost functions, which are introduced in Section 4.

4 Analysis under parametric functions

4.1 Number of consumers and likelihood of allocative efficiency

We consider an example in which $v(g) = \theta g^\alpha$ and $c(g) = \gamma g^\delta$, where α, θ, δ, and γ are positive constants such that $\alpha \in (0, 1]$, $\delta \geq 1$, and $\alpha \neq \delta$. Note that $v(g)$ and $c(g)$ satisfy Assumption 1. From (7) and (10), we obtain

$$\begin{aligned}
&\underline{g}^m = \left(\frac{\beta\theta(m-1)}{\gamma}\right)^{\frac{1}{\delta-\alpha}}, \quad \bar{g}^m = \left(\frac{m\theta}{\gamma}\right)^{\frac{1}{\delta-\alpha}}, \quad g(m) = \left(\frac{m\alpha\theta}{\delta\gamma}\right)^{\frac{1}{\delta-\alpha}}, \\
&\beta(g, m) = \frac{\gamma}{\theta(m-1)} g^{\delta-\alpha}, \quad \beta(g(m), m) = \frac{\alpha}{\delta}\left(\frac{1}{m-1} + 1\right), \quad \text{and} \quad \beta(g(m), m+1) = \frac{\alpha}{\delta}.
\end{aligned} \tag{19}$$

We can see from (19) that $\beta(g(m), m)$ is decreasing in m and $\beta(g(m), m+1)$ is constant in m. Thus, Condition 1 holds. Moreover,

$$\begin{aligned}
\pi_S^m(\underline{g}^m) &= \frac{\beta}{1 + (1-\beta)(m-1)} \left(\frac{\beta\theta(m-1)}{\gamma}\right)^{\frac{\alpha}{\delta-\alpha}} \left(m\theta - \gamma \cdot \frac{\beta\theta(m-1)}{\gamma}\right) \\
&= \left(\frac{(\beta\theta)^\delta}{\gamma^\alpha}\right)^{\frac{1}{\delta-\alpha}} (m-1)^{\frac{\alpha}{\delta-\alpha}}.
\end{aligned}$$

Since $\alpha/(\delta - \alpha) > 0$, $\pi_S^m(\underline{g}^m)$ is increasing in m. Thus, Condition 2 holds.

Proposition 3 *Assumption 1, Conditions 1, and 2 hold if $v(g) = \theta g^\alpha$ and $c(g) = \gamma g^\delta$, where α, θ, δ, and γ are positive constants such that $\alpha \in (0, 1]$, $\delta \geq 1$, and $\alpha \neq \delta$.*

By Proposition 3, Theorem 1 applies to this example.

Corollary 2 Suppose that $v(g) = \theta g^\alpha$ and $c(g) = \gamma g^\delta$, where α, θ, δ, and γ are positive constants such that $\alpha \in (0, 1]$, $\delta \geq 1$, and $\alpha \neq \delta$. The supplier provides the public good efficiently at an equilibrium if and only if $\beta \in [0, \beta(g(n), n)]$, where $\beta(g(n), n) = \alpha n/(\delta(n-1))$.

[14]The formal analysis for this paragraph is available upon request

Proposition 4 is directly from Corollary 2.

Proposition 4 *The set of the supplier's bargaining powers in which the supplier provides the public good efficiently at an equilibrium,* $[0, \beta(g(n), n)]$*, shrinks and converges to* $[0, \alpha/\delta]$ *as* n *becomes large if* $v(g) = \theta g^{\alpha}$ *and* $c(g) = \gamma g^{\delta}$*, where* α*,* θ*,* δ*, and* γ *are positive constants such that* $\alpha \in (0, 1]$*,* $\delta \geq 1$*, and* $\alpha \neq \delta$*.*

By Proposition 4, we can examine the equilibrium likelihood of the efficient provision of the public good by the supplier. Although there may be various measures for this likelihood in the literature, the length of the interval $[0, \beta\,(g(n), n)]$ would be appropriate as the measure of this likelihood in our model. Since $\beta(g(n), n)$ decreases in n, the length becomes shorter as n becomes larger. Thus, we could conclude that the equilibrium likelihood of the efficient provision of the public good becomes lower as the number of consumers increases. However, note that the likelihood of the efficient provision of the public good does not necessarily vanish even if the number of consumers approaches infinity. This implies that there may be a sufficiently weak supplier who provides the public good efficiently even if he/she negotiates with many consumers.

A comparison of our results with those of Dixit and Olson (2000) might be important because whether beneficiaries are *pivotal* to public good provision plays a role in both sets of results. Dixit and Olson (2000) examine a voluntary participation game in bargaining for public good provision. In their game, each beneficiary simultaneously decides whether to participate in the bargaining. The public good is discrete: a threshold number exists such that one unit of the public good is provided if at least the threshold number of beneficiaries participates and no public good is provided otherwise. If the public good is provided, its cost is shared by participants and nonparticipants can free ride. They assume that the provision of the public good is efficient. They examine mixed-strategy Nash equilibria of that game and show that each beneficiary's equilibrium probability of participation diminishes to zero as the number of the players increases. Thus, the likelihood that the public good is provided efficiently in equilibrium is extremely low when the number of beneficiaries is large.[15] Intuitively, in their game, the probability of being *pivotal* to the public good provision affects the participation probability of each beneficiary.[16] A beneficiary participates if and only if he/she is pivotal because otherwise, the public good is not provided or he/she cannot benefit from it. As the number of beneficiaries increases, the probability of being pivotal becomes lower, which reduces each player's participation probability.

Note that in our analysis, the probability of being pivotal is irrelevant. As discussed in Section 3, in our model, the supplier has an incentive to set the level of the public good such that all consumers are pivotal to the provision of the public good. This is the same, even when the number of consumers is very large. Hence, Proposition 4 comes from a different reason to that of Dixit and Olson (2000).

[15] A similar implication can be obtained from the results of the voluntary participation game for a public good mechanism (Saijo and Yamato, 1999, 2010; Shinohara, 2009; Healy, 2010; Furusawa and Konishi, 2011; Konishi and Shinohara, 2014).

[16] The meaning of pivotal here is the same as that in our analysis: without a pivotal beneficiary, the public good is not provided.

Although our model and measurement of likelihood of efficiency are completely different from Dixit and Olson's (2000), we share a similar implication: as the number of consumers (beneficiaries) increases, the likelihood of achieving efficiency in equilibrium decreases. Our result differs from that of Dixit and Olson (2000) in that the likelihood of the efficient provision of the public good does not necessarily vanish even if the number of consumers becomes large. Note that α/δ is close to one if α and δ are close to one.

4.2 Equilibrium level of the public good when the supplier's bargaining power is sufficiently strong

Theorem 1 proves that if $\beta > \min_{m\in\{1,\ldots,n\}} \beta(g(m), m)$, there is no equilibrium that supports the efficient provision of the public good. We discuss which of underprovision and overprovision of the public good causes inefficiency in public good provision.

Consider an example in which $n = 11$, $v(g) = \sqrt{g}$ and $c(g) = g$. Then,

$$g(m) = \left(\frac{m}{2}\right)^2, \ \underline{g}^m = (\beta(m-1))^2, \ \beta(g(m), m) = \frac{m}{2(m-1)}, \text{ and } \beta(g(m), m+1) = \frac{1}{2}.$$

Note that $\min_{m\in\{1,\ldots,n\}} \beta(g(m), m) = \beta(g(11), 11) = 0.55$. In the Appendix, we show that there is an equilibrium at which the supplier chooses $\underline{g}^{11}$ in the first stage. Since $g(11) < \underline{g}^{11}$, the supplier produces the public good over the efficient level at this equilibrium. Recall that in the paragraphs after Theorem 1, we mentioned that the supplier has an incentive to increase the level of the public good to receive transfers from all consumers. This example presents a case in which this incentive leads to the overprovision of the public good in equilibrium when the supplier's bargaining power is strong.

This example produces a result that differs from that of Segal (1999), who proposes a vertical contracting model under some general setting, which is applicable to public good provision. When his model is applied to public good provision, the supplier of a public good has bilateral negotiations with each consumer; in the bilateral negotiation with consumer $i \in N$, the supplier first makes a take-it-or-leave-it offer (g_i, T_i) to consumer i, where g_i is the individual contribution level of the public good from i and T_i is i's transfer to the supplier. Each consumer knows the offer to him/her as well as the offers to the others. Then, each consumer simultaneously decides whether to accept the offer. All offers that are accepted by consumers are executed: if a set of consumers $\mathcal{A} \subseteq N$ accepts the offer, then the total level of the public good is $\sum_{j\in\mathcal{A}} g_j$, the supplier's payoff is $\sum_{j\in\mathcal{A}} T_j - c(\sum_{j\in\mathcal{A}} g_j)$, the payoff to acceptor $i \in \mathcal{A}$ is $v(\sum_{j\in A} g_j) - T_i$ and that to rejector $l \in N\backslash\mathcal{A}$ is $v(\sum_{j\in\mathcal{A}} g_j)$. Take-it-or-leave-it offers mean that the supplier has complete bargaining power. By Proposition 2 of Segal (1999), in equilibrium, the supplier with complete bargaining power never produces the public good over the efficient level. By contrast, as the abovementioned example shows, in our model, the supplier with complete bargaining power ($\beta = 1$) may overprovide the public good.

In Segal's (1999) model, the reason why the supplier never excessively produces a public good is that the supplier must care about consumers' free riding. Given that the supplier makes offers $(g_j, T_j)_{j\in N}$ that all consumers accept, if consumer i rejects an offer, i can enjoy $v(\sum_{j\in N\backslash\{i\}} g_j)$ because (g_j, T_j) for each $j \in N\backslash\{i\}$ are accepted. Thus, the supplier with complete bargaining power can extract full marginal benefit of consumer i

from accepting the offer, $v(\sum_{j\in N} g_j) - v(\sum_{j\in N\setminus\{i\}} g_j)$, but not full benefit, $v(\sum_{j\in N} g_j)$: $T_i = v(\sum_{j\in N} g_j) - v(\sum_{j\in N\setminus\{i\}} g_j)$ for each $i \in N$ in equilibrium.

In our model, when the supplier with complete bargaining power make offers that the consumers accept, he/she does not have to care about consumers' free riding. This is because the supplier can make all consumers pivotal to the execution of the project $(g, (T_j))_{j\in N}$ by setting the level of the public good sufficiently high; if one of the consumers rejects an offer, then the project is not executed. This means that consumers receive nothing if they decline the supplier's offer. Thus, the supplier with complete bargaining power can extract full benefit, $v(g)$, from each consumer: $T_i = v(g)$ for each $i \in N$. The transfers that the supplier receives in our model are likely to be greater than those in Segal's (1999) model. This difference in transfer values leads to the difference in the supplier's decision.

Finally, we note that the possibility cannot be denied that the supplier with strong bargaining power underprovides the public good under other benefit and cost functions.[17] Thus, the inefficiency in public good provision is due to the overprovision of the public good in some cases and the underprovision of the public good in other cases.

5 Discussion

5.1 Bargaining procedures

The outcome through simultaneous bilateral bargaining can be obtained through other multilateral bargaining models.[18] First, (2)–(4) are consistent with the outcome of multilateral Nash bargaining among the supplier and m pivotal consumers. We can easily confirm that $(T_j^m)_{j\in M}$ in (2) maximizes the Nash product $(\sum_{j\in M} T_j - c(g))^\beta \prod_{j\in M} (v(g) - T_j)^{1-\beta}$. Second, we can confirm that for each level of the public good in the first stage $g \geq 0$, the payoffs attained at the simultaneous bilateral bargaining belong to the core of the cooperative game $(\{s\} \cup N, w^g)$ where $w^g : 2^{\{s\}\cup N} \to \mathbb{R}_+$ is the characteristic function such that for any nonempty subset $\mathcal{C} \subseteq N$, $w^g(\{\emptyset\}) = w^g(\{s\}) = w^g(\mathcal{C}) = 0$; $w^g(\{s\} \cup \mathcal{C}) = \max\{|\mathcal{C}|v(g) - c(g), 0\}$ where $|\mathcal{C}|$ represents the cardinality of $\mathcal{C}$. Thus, we can say that simultaneous bilateral bargaining provides an outcome attained through some multilateral negotiation.

We consider other kinds of bargaining models between the supplier and consumers instead of simultaneous and bilateral bargaining, and we discuss how the main result changes under other bargaining models.

First, we consider sequential bargaining between the supplier and each consumer. We replace the second-stage bargaining game of the basic model with the following sequential bilateral bargaining game: the supplier first negotiates with consumer 1, second negotiates with consumer 2 after the bilateral bargaining with consumer 1, ..., and finally, bilaterally negotiates with consumer n after the bilateral bargaining sessions with the other consumers. Each bilateral bargaining session is assumed to be Nash bargaining. The supplier has only one bilateral negotiation with each of n consumers. The equilibrium transfer

[17]The formal analysis of the case in which the supplier has strong bargaining power is available upon request.

[18]The formal analysis of Subsection 5.1 is available upon request.

attained through this sequential bargaining is the same as that in (6). Hence, a result similar to Theorem 1 can be obtained under sequential bilateral bargaining.

Second, we apply solutions of the cooperative game to the second-stage bargaining. Since several famous solutions, such as the Shapley value, kernel, and nucleolus, satisfy the *equal treatment property* (ETP), we focus on the solution with the ETP.[19] Consider the cooperative game $(\{s\} \cup N, w^g)$ defined above. Since the consumers have identical benefit functions in the basic model, all distinct consumers i and j are *interchangeable* in the sense that for each coalition $D \subseteq \{s\} \cup N$ that contains neither i nor j, $w^g(D \cup \{i\}) - w^g(D) = w^g(D \cup \{j\}) - w^g(D)$. The solution of the cooperative game assigns how to distribute the total surplus $w^g(\{s\} \cup N)$. If the solution satisfies the ETP, then it assigns the same distribution to all interchangeable players (i.e., all consumers in our model). Formally, let $(u_i)_{i \in \{s\} \cup N} \in \mathbb{R}_+^{n+1}$ be payoffs that are assigned by the solution with the ETP and that satisfy $u_s + \sum_{i \in N} u_i = w^g(\{s\} \cup N)$. Then, $u_1 = \cdots = u_n$. Thus, a surplus-sharing ratio exists that is common to all consumers, $r_c \in [0, 1/n]$, such that $u_i = r_c w^g(\{s\} \cup N)$ for each $i \in N$. In addition, we have $u_s = (1 - nr_c) w^g(\{s\} \cup N)$. If the supplier chooses g in the first stage, then his/her payoff attained at the solution is

$$\pi_S^{\mathrm{ETP}}(g) \equiv \begin{cases} (1 - nr_c)(nv(g) - c(g)) & \text{if } \ nv(g) - c(g) > 0, \\ 0 & \text{if } \ nv(g) - c(g) \leq 0. \end{cases}$$

Given this supplier's payoff function, it is supported at an equilibrium that the supplier chooses $g(n)$ in the first stage because $g(n)$ maximizes the total surplus $nv(g) - c(g)$. Therefore, under the solution satisfying the ETP, the supplier always provides the public good efficiently, which is different from the main result of this study. This difference comes from the ETP. Under the solution with the ETP, all consumers obtain the same payoff, which means they all transfer the same amount of money to the supplier. Thus, the solution under the ETP is assumed implicitly to have a function that prevents consumers from free riding.

However, in contradiction, it is hard to state that the solution with the ETP has the function of preventing free riding. This is demonstrated by the following example. Let g be such that $(n-1)v(g) > c(g)$. Let $i \in N$ be a consumer who obtains the payoff $u_i^{\mathrm{ETP}} \equiv r_c(nv(g) - c(g))$ under the solution with the ETP. Since $(n-1)v(g) > c(g)$, then $w^g(\{s\} \cup N \backslash \{i\}) > 0$, which means that the coalition $\{s\} \cup N \backslash \{i\}$ produces g units of the public good even if consumer i opts out from $\{s\} \cup N$. Thus, since the public good is pure, consumer i can enjoy the free-riding payoff $v(g)$ even if he/she opts out of N. The free-riding payoff $v(g)$ clearly outperforms u_i^{ETP} since $r_c \leq 1/n$ and $u_i^{\mathrm{ETP}} \leq v(g) - (c(g)/n) < v(g)$. Thus, the solution with the ETP might be inappropriate for the analysis of bargaining over pure public good provision.

5.2 Heterogeneous consumers

We prove that it is partially correct that the supplier produces the public good inefficiently if his/her bargaining power is sufficiently strong, even when consumers are heterogeneous.[20]

[19] See Peleg and Sudhölter (2007) for the definitions of those solutions.

[20] See the Appendix for detailed derivation.

We consider the case in which there are two consumers ($N = \{1, 2\}$) and they have different benefit functions and bargaining power to the supplier. The payoff to consumer $i \in N$ is $v_i(g) - T_i$, where $v_i : \mathbb{R}_+ \to \mathbb{R}_+$ is consumer i's benefit function from the public good that satisfies $v_i(0) = 0$, $v_i' > 0$, $v_i'' \leq 0$, and twice continuous differentiability. Let $\beta_i \in [0, 1]$ be the supplier's bargaining power with consumer $i \in N$. The cost function $c(g)$ satisfies $c(0) = 0$, $c' > 0$, $c'' > 0$, and twice continuous differentiability. To make the analysis simpler, our analysis is based on the assumption that $v_i(g) = \lambda_i g$ for each $i \in N$, where $0 < \lambda_1 < \lambda_2$, $c(g) = g^2/2$, and $\beta_1 \leq \beta_2$. The timing of the game is the same as that for the basic model.

We can derive equilibria in each stage, similarly to the case of identical consumers. From the analysis, we obtain that consumers 1 and 2 are both pivotal to (g, T_1, T_2) if and only if g satisfies

$$2\left(\lambda_2 - \frac{(1-\beta_2)\beta_1}{\beta_2}\lambda_1\right) \leq g < 2(\lambda_1 + \lambda_2). \tag{20}$$

It is true that if the efficient level of the public good, $\lambda_1 + \lambda_2$, is produced at an equilibrium of the game, then both consumers 1 and 2 are pivotal to the efficient provision (see Claim 1 in the Appendix). That is, by (20), if $\lambda_1 + \lambda_2$ is produced in equilibrium, then

$$\lambda_1 + \lambda_2 \geq 2\left(\lambda_2 - \frac{(1-\beta_2)\beta_1}{\beta_2}\lambda_1\right) \quad \text{or} \quad \beta_2 \leq \frac{2\lambda_2\beta_1}{\lambda_2 - \lambda_1 + 2\lambda_2\beta_1}. \tag{21}$$

Therefore, if the supplier's bargaining power with consumer 2 is sufficiently strong in the sense that $\beta_2 \in ((2\lambda_2\beta_1)/(\lambda_2 - \lambda_1 + 2\lambda_2\beta_1), 1]$, then the supplier does not produce the public good efficiently in equilibrium.

Since $2\lambda_2\beta_1/(\lambda_2 - \lambda_1 + 2\lambda_2\beta_1)$ is increasing in β_1, the interval $(2\lambda_2\beta_1/(\lambda_2 - \lambda_1 + 2\lambda_2\beta_1), 1]$ shrinks as β_1 increases. However, since this interval is nonempty at any value of β_1, the implication is true for any β_1.

In conclusion, in this extended model, the Pareto-efficient allocation is not achieved in equilibrium if the supplier's bargaining power with *some* (not*every*) consumer (consumer 2 in the above analysis) is sufficiently strong. In this sense, the main result in the case of identical consumers remains partially true.

5.3 Commitment to the level of public goods before negotiation

In the basic model, the supplier is the decision maker of the level of the public good and commits to a level before negotiating.[21] In contrast to this, we now consider a new simultaneous bilateral bargaining model in which the supplier and each consumer negotiate the level of the public good as well as transfer. This model consists of two stages. In the first stage, the supplier and each consumer $i \in N$ bilaterally negotiate the joint production level of the public good $g_i \geq 0$ and the transfer to the supplier from consumer i $T_i (\geq 0)$. We assume that every bilateral negotiation is simultaneous and Nash bargaining, as in the basic model. Hence, g_i is negotiated so as to maximize the joint surplus of the bilateral negotiation between the supplier and consumer i; T_i is determined so as to share the maximized joint surplus in proportion to the bargaining power (the supplier's bargaining power and consumer i's bargaining power is β and $1 - \beta$, as in the basic model). Let

[21] The formal analysis in Subsection 5.3 is available upon request.

$(g_j, T_j)_{j\in N}$ be the outcome of the simultaneous bilateral bargaining. In the second stage, the supplier decides whether he/she executes $(g_j, T_j)_{j\in N}$. If so, then his/her payoff is $\sum_{j\in N} T_j - c(\sum_{j\in N} g_j)$. Otherwise, the payoff is zero.

As in the basic model, the joint surplus of each bilateral negotiation takes different forms, depending on whether each consumer is pivotal. Consumer $i \in N$ is said to be *pivotal* to $(g_j, T_j)_{j\in N}$ if $\sum_{j\in N} T_j \geq c(g_i + \sum_{j\neq i} g_j)$ and $\sum_{j\neq i} T_j < c(\sum_{j\neq i} g_j)$. If consumer i is pivotal to the supplier's second-stage decision, then the joint surplus of the bilateral bargaining session is $v(g_i + \sum_{j\neq i} g_j) - c(g_i + \sum_{j\neq i} g_j) + \sum_{j\neq i} T_j$. If consumer i is not pivotal, then the joint surplus of the bilateral bargaining session is $v(g_i + \sum_{j\neq i} g_j) - v(\sum_{j\neq i} g_j) - [c(g_i + \sum_{j\neq i} g_j) - c(\sum_{j\neq i} g_j)]$.

We can confirm that in this model, irrespective of whether consumer i is pivotal, the joint surplus maximization of the supplier and consumer i is equivalent with the maximization of $v(g_i + \sum_{j\neq i} g_j) - c(g_i + \sum_{j\neq i} g_j)$ given $(g_j)_{j\neq i}$ through the choice of g_i. That is, in each bilateral negotiation, the total surplus $nv(g) - c(g)$ is never maximized, which induces the inefficient provision of the public good. Formally we can show that in every equilibrium of this game, $g(1)$ units of the public good are provided.

The implications of the analysis are as follows: (i) The efficient level of the public good $g(n)$ is never supported at any equilibrium in the new model (recall $g(1) < g(n)$). (ii) The commitment to the level of the public good before negotiation, like in the first stage in the basic model, is necessary to achieve the efficient allocation.

5.4 Implications of results for political bargaining

We can fit our basic model to the story of intergovernmental negotiation like the one in Section 1. Suppose that in several regions there are citizens which have identical preferences over consumption of private and public goods. In each region, one of the citizens is elected as a governor, who participates in a negotiation and acts according to his/her interests.[22][23] The central government determines the level of the public good. It has no source of outside funding, and hence, the cost of the public good must be compensated by beneficiary regions: whether there is a budget surplus is essential for this central government.[24] If we interpret "consumers" in the basic model as these regional governors and "supplier" as this central government, we can apply our model to the intergovernmental negotiation.

We have not examined the case of $n = 1$. In this case, obviously, the supplier always produces the public good efficiently for any value of the supplier's bargaining power in equilibrium. That is, the bilateral bargaining procedure itself works successfully when the supplier negotiates with one consumer. This is very different from the result in the case of $n \geq 2$. The difference between the cases of $n \geq 2$ and $n = 1$ is the existence of externalities between bilateral negotiations. Thus, we conclude that the externalities between bilateral bargaining sessions cause the Pareto inefficiency.

[22] Since every resident in the regions is identical, we can provide another interpretation that local governments of the regions act in the best interest of their representative constituencies. Luelfesmann et al. (2015) provide a similar interpretation.

[23] We can observe an approach that the representatives of regions act in their best interests in may studies (see, for example, Gradstein (2004)).

[24] See more discussions for the supplier's objective function in Section 6.

Lülfesmann (2002), Gradstein (2004), and Luelfesmann et al. (2015) study political bargaining over the provision of interregional public goods. In their models, there are two regions: one supplies a public good and another benefits from the public good. There is only one bilateral session in which the representatives of those two regions negotiate. The bilateral bargaining is assumed to be resolved through the Nash bargaining solution. However, the authors show that the allocation through the bargaining may be inefficient because of political factors, such as strategic delegation and majority decisions that neglect the minority, which distort the bargaining outcome. In their models, the bargaining procedure itself is not a source of the Pareto inefficiency.

On the other hand, from our result, we could say that the bargaining procedure itself becomes a source of the Pareto inefficiency as well as the political factors if the models of political bargaining in the earlier studies are extended to the case in which there are "many" regions benefiting from the public good. When the representative of the supplier region and that of each beneficiary region negotiate in a manner similar to our simultaneous bilateral negotiation, the negotiation itself does not work efficiently, as seen in the analysis of the basic model. Not only the political factors but also the bargaining procedure matter in the extended model.

6 Concluding remarks

We examine a supplier's decision about a public good in the face of bargaining with beneficiaries of the good. Our bargaining model is built on simultaneous bilateral bargaining. Our results show that the supplier's bargaining power is a key factor for his/her decision. We show that under some conditions, the supplier produces the public good efficiently at an equilibrium if and only if his/her bargaining power is sufficiently weak (Theorem 1). This can be interpreted in the relationship between the efficiency of the supplier's decision and his/her willingness to accept a budget surplus; if a supplier is willing to accept a sufficiently near balanced-budget outcome, he/she is more likely to achieve the efficient provision of the public good. A supplier with strong bargaining power who obtains a large surplus provides the public good inefficiently. In some case, this inefficiency stems from the excessive provision of the public good (Subsection 4.2). In addition, we investigate the likelihood of the efficient public good provision in equilibrium. Under the reasonable parametric benefit and cost functions, the equilibrium likelihood of the efficient provision of the public good diminishes as the number of consumers increases. However, in some case, the equilibrium likelihood is sufficiently high even if the number of consumers is very large (Subsection 4.1). This implies that there may be a sufficiently weak supplier who efficiently produces the public good even if he/she negotiates with "many" consumers.

We assume that the supplier is a budget-surplus (or profit) maximizer. However, the supplier's objective may differ depending on the situation. If we consider public good provision by the central government, then it might be reasonable to consider that the level of the public good and the cost distribution are determined by the national legislature. Then, the central government's objective is the maximization of the welfare of the majority of the legislature. The maximization of social welfare could be another objective of the central government.[25] However, we consider that maximization of budget

[25] In our bargaining model, the supplier takes the contributors' joint surplus into account and the supplier

surplus can be an objective of the government if the decision for the public good is based on a bureaucratic system and bureaucrats maximize their profits, like the approach of public choice theory. Moreover, the model of profit-maximizing governments has been studied by several researchers, such as Mansoorian and Myers (1997). In some studies on mechanism design for public good provision, the central authority is assumed to be a profit maximizer (see, e.g., Güth and Hellwig, 1986; Lu and Quah, 2009). Finally, whether there is a budget surplus seems essential to public projects when there is no source of outside funding. In this case, it may not be problematic that the government takes account of the budget surplus.

Appendix

Proof of Lemma 4

We prove only (13) ((14) can be proven similarly). Differentiating $\beta(g(m), m)$ with respect to m yields

$$\frac{\partial\beta(g(m), m)}{\partial m} = \frac{1}{m-1} \cdot \frac{c'(g(m))v(g(m)) - c(g(m))v'(g(m))}{(v(g(m)))^2} \cdot \left(\frac{d\,g(m)}{d\,m} - \frac{(v(g(m)))^2}{c'(g(m))v(g(m)) - c(g(m))v'(g(m))} \cdot \underbrace{\frac{c(g(m))}{v(g(m))(m-1)}}_{(22.1)}\right). \quad (22)$$

We first observe that $c'(g(m))v(g(m)) - c(g(m))v'(g(m)) > 0$ by (1.2) of Assumption 1. Second, we observe that $G(\beta(m-1))$ is the inverse function of $c(g)/v(g)$ and

$$\frac{\partial G(\beta(m-1))}{\partial m} = \frac{dG(\beta(m-1))}{d\beta(m-1)} \cdot \beta = \frac{(v(g))^2}{c'(g)v(g) - c(g)v'(g)} \cdot \beta.$$

Since $G\left(\beta(g(m), m)(m-1)\right) = g(m)$ and (22.1) is equal to $\beta(g(m), m)$, then

$$\left.\frac{\partial G(\beta(m-1))}{\partial m}\right|_{\beta(m-1)=\beta(g(m),m)(m-1)} = \frac{(v(g(m)))^2}{c'(g(m))v(g(m)) - c(g(m))v'(g(m))} \cdot \frac{c(g(m))}{v(g(m))(m-1)}.$$

We obtain (13) from (22). ■

Proof of Proposition 1

Suppose that $0 \leq \beta \leq \min_{m\in\{1,\ldots,n\}} \beta\left(g(m), m\right)$. If $\beta = 0$, the supplier's payoff is zero, irrespective of his/her choice of g. Thus, $g(n)$ is one of the optimal levels for the supplier. Hereafter, we restrict our focus to the case in which $0 < \beta \leq \min_{m\in\{1,\ldots,n\}} \beta\left(g(m), m\right)$. Note that this condition and (16) imply

$$\underline{g}^m \leq g(m) \text{ for each } m \in \{1, \ldots, n\}.$$

We obtain the following three subcases in order to examine the relationship between $\underline{g}^{m+1}$ and $g(m)$ for each m:

receives contributions from all consumers in equilibrium. Hence, the supplier considers the total surplus to some extent (see the numerator on the right-hand side of (3)).

- **Subcase 1.1:** $\min_{m\in\{1,\dots,n\}} \beta\left(g(m), m+1\right) \le \beta \le \max_{m\in\{1,\dots,n\}} \beta\left(g(m), m+1\right)$.
- **Subcase 1.2:** $\beta < \min_{m\in\{1,\dots,n\}} \beta\left(g(m), m+1\right)$.
- **Subcase 1.3:** $\max_{m\in\{1,\dots,n\}} \beta\left(g(m), m+1\right) < \min_{m\in\{1,\dots,n\}} \beta\left(g(m), m\right)$ and $\max_{m\in\{1,\dots,n\}} \beta\left(g(m), m+1\right) < \beta \le \min_{m\in\{1,\dots,n\}} \beta\left(g(m), m\right)$.

Typical situations in the three cases are summarized in Figure 3. Subcases 1.1, 1.2, and 1.3 correspond to the case of $\beta = \beta_{11}$, that of $\beta = \beta_{12}$, and that of $\beta = \beta_{13}$, respectively. Since we do not impose any restriction except nonincreasing $\beta(g(m), m+1)$, we must consider various possibilities. In some cases, as in the case of sufficiently small β, like $\beta = \beta_{12}$ (Subcase 1.2) or the case of sufficiently high β, like $\beta = \beta_{13}$ (Subcase 1.3), β may not intersect with $\beta(g(m), m+1)$ at any $m \in [1, n]$. In the other case, β may intersect with $\beta(g(m), m+1)$, not only at the unique $m \in [1, n]$, but also at multiple points, as in the case of $\beta = \beta_{11}$ (Subcase 1.1).

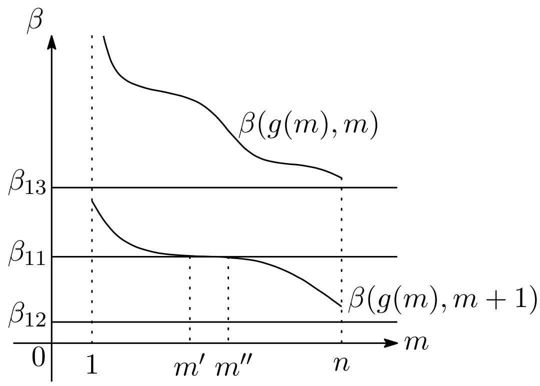

Figure 3: Sufficiency

(Subcase 1.1) In this subcase, at least one $m \in [1, n]$ exists such that $\beta = \beta(g(m), m+1)$. Denote the largest one among such ms by $m'' \in [1, n]$ (like in Figure 3). In addition, let $\overline{m+1}(\beta)$ be the greatest integer that is less than or equal to m''. Then, by (15), for each $m \in \{1, \dots, \overline{m+1}(\beta)\}$, $\underline{g}^{m+1} \le g(m)$; by (16), for each $m \in \{\overline{m+1}(\beta)+1, \dots, n\}$, $\underline{g}^{m} \le g(m) < \underline{g}^{m+1}$.[26]

First, consider the category $\{1, \dots, \overline{m+1}(\beta)\}$. The supplier receives transfers from the consumers of the number in this category if and only if the supplier chooses $g \in (0, \underline{g}^{\overline{m+1}(\beta)+1})$. Let $m \in \{1, \dots, \overline{m+1}(\beta)\}$. The supplier receives transfers from m consumers if and only if $g \in [\underline{g}^{m}, \underline{g}^{m+1})$. By (15), since $m \in \{1, \dots, \overline{m+1}(\beta)\}$, then $\underline{g}^{m+1} \le g(m)$. Thus, $\pi_S^m(g)$ is increasing in $[\underline{g}^{m}, \underline{g}^{m+1})$. Since $\pi_S^m(g)$ is continuous at every g, we can define $\pi_S^m(\underline{g}^{m+1})$ and

$$\lim_{g\uparrow \underline{g}^{m+1}} \pi_S^m(g) = \pi_S^m(\underline{g}^{m+1}) = \frac{\beta\left(mv(\underline{g}^{m+1}) - c(\underline{g}^{m+1})\right)}{\beta + (1-\beta)m}.$$

[26] If $n < \overline{m+1}(\beta)+1$, then $\{\overline{m+1}(\beta)+1, \dots, n\}$ is empty. Then, $\underline{g}^{m+1} \le g(m)$ for each $m \in \{1, \dots, n\}$.

Thus, $\sup_{g\in[\underline{g}^m,\,\underline{g}^{m+1})} \pi_S^m(g) = \pi_S^m(\underline{g}^{m+1})$.

For each $\mu \in \{1,\dots,\overline{m+1}(\beta)-1\}$, we obtain $\pi_S^{\mu}(\underline{g}^{\mu+1}) < \pi_S^{\mu+1}(\underline{g}^{\mu+1})$ by Lemma 3. By (15), $\underline{g}^{\mu+1} < \underline{g}^{\mu+2} \leq g(\mu+1)$, which implies $\pi_S^{\mu+1}(\underline{g}^{\mu+1}) < \pi_S^{\mu+1}(\underline{g}^{\mu+2})$. Hence, for each $\mu \in \{1,\dots,\overline{m+1}(\beta)-1\}$,

$$\sup_{g\in[\underline{g}^{\mu},\,\underline{g}^{\mu+1})} \pi_S^{\mu}(g) = \pi_S^{\mu}(\underline{g}^{\mu+1}) < \pi_S^{\mu+1}(\underline{g}^{\mu+2}) = \sup_{g\in[\underline{g}^{\mu+1},\,\underline{g}^{\mu+2})} \pi_S^{\mu+1}(g). \tag{23}$$

From (23), we obtain

$$\sup_{m\in\{1,\dots,\overline{m+1}(\beta)\}} \left(\sup_{g\in[\underline{g}^m,\,\underline{g}^{m+1})} \pi_S^m(g) \right) = \pi_S^{\overline{m+1}(\beta)}(\underline{g}^{\overline{m+1}(\beta)+1}).$$

This means that the supremum of the supplier's payoff in $[0, \underline{g}^{\overline{m+1}(\beta)+1})$ is $\pi_S^{\overline{m+1}(\beta)}(\underline{g}^{\overline{m+1}(\beta)+1})$.

Second, consider the category $\{\overline{m+1}(\beta)+1,\dots,n\}$. The supplier receives transfers from the consumers with the number in this category if and only if the supplier chooses $g \in [\underline{g}^{\overline{m+1}(\beta)+1}, \bar{g}^n)$. Let $m \in \{\overline{m+1}(\beta)+1,\dots,n\}$. By (16), $\underline{g}^m \leq g(m) < \underline{g}^{m+1}$. Then, $\pi_S^m(g)$ is maximized at $g = g(m)$. By Corollary 1, for each $\mu \in \{\overline{m+1}(\beta)+1,\dots,n-1\}$,

$$\pi_S^{\mu}(g(\mu)) < \pi_S^{\mu+1}(g(\mu+1)).$$

Thus, the maximum of the supplier's payoff in $[\underline{g}^{\overline{m+1}(\beta)+1}, \bar{g}^n)$ is $\pi_S^n(g(n))$.

Finally, we show that in Subcase 1.1, the supplier's payoff is maximized globally at $g = g(n)$. If $n = \overline{m+1}(\beta)$, then $\underline{g}^n < \underline{g}^{n+1} = g(n) < \bar{g}^n$ by (15).[27] Hence, the supplier can choose $g(n)$, receiving transfers from n consumers. By (23),

$$0 < \pi_S^1(\underline{g}^2) < \cdots < \pi_S^{n-1}(\underline{g}^n) < \pi_S^n(\underline{g}^{n+1}) = \pi_S^n(g(n)).$$

Hence, $g(n)$ maximizes the supplier's profit. If $\overline{m+1}(\beta) < n$, then

$$\begin{aligned} 0 &< \pi_S^1(\underline{g}^2) < \cdots < \pi_S^{\overline{m+1}(\beta)}(\underline{g}^{\overline{m+1}(\beta)+1}) \\ &< \pi_S^{\overline{m+1}(\beta)+1}(\underline{g}^{\overline{m+1}(\beta)+1}) \leq \pi_S^{\overline{m+1}(\beta)+1}(g(\overline{m+1}(\beta)+1)) \qquad (24) \\ &\leq \cdots \leq \pi_S^n(g(n)). \qquad (25) \end{aligned}$$

The first inequality in (24) comes from Lemma 3 and the second from $\underline{g}^{\overline{m+1}(\beta)+1} \leq g(\overline{m+1}(\beta)+1)$.[28] Hence, $g(n)$ maximizes the supplier's payoff.

(Subcase 1.2) In this subcase, we obtain $\underline{g}^m < \underline{g}^{m+1} < g(m)$ for each $m \in \{1,\dots,n\}$ by (15). The analysis of this subcase is almost the same as that of Subcase 1.1 with $n = \overline{m+1}(\beta)$. Similarly to the above analysis, we obtain that for each $m \in \{1,\dots,n\}$,

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[27] Note that $n = \overline{m+1}(\beta)$ if and only if $m'' = n$. If $m'' = n$, then $\beta(g(n), n+1) = \beta$. Hence, $\underline{g}^{n+1} = g(n)$.

[28] If $\overline{m+1}(\beta)+1 = n$, the last inequality in (24) becomes $\pi_S^n(\underline{g}^n) \leq \pi_S^n(g(n))$. If $\overline{m+1}(\beta)+1 < n$, then all inequalities in (25) hold strictly by Corollary 1.

$\pi_S^m(g)$ increases as g moves from $\underline{g}^m$ to $\underline{g}^{m+1}$. As in (23), we obtain that for each $m \in \{1, \dots, n-1\}$,

$$\lim_{g \uparrow \underline{g}^{m+1}} \pi_S^m(g) < \lim_{g \uparrow \underline{g}^{m+2}} \pi_S^{m+1}(g).$$

Since $\underline{g}^{n+1} < g(n)$, we finally obtain

$$0 < \pi_S^1(\underline{g}^2) < \cdots < \pi_S^{n-1}(\underline{g}^n) < \pi_S^n(\underline{g}^{n+1}) < \pi_S^n(g(n)).$$

Thus, the supplier chooses $g(n)$ at the first stage.

(Subcase 1.3) In this subcase, we obtain that $\underline{g}^m \leq g(m) < \underline{g}^{m+1}$ for each $m \in \{1, \dots, n\}$ by (16). For each $m \in \{1, \dots, n\}$, $\pi_S^m(g)$ is maximized at $g = g(m)$. By Corollary 1, $\pi_S^m(g(m)) < \pi_S^n(g(n))$ for each $m \in \{1, \dots, n-1\}$. Thus, the supplier chooses $g(n)$ at the first stage.

In conclusion, in any subcase, the supplier chooses $g(n)$ in equilibrium. ■

Proof of Proposition 2

To the contrary, suppose that $\beta > \min_{m \in \{1,\dots,n\}} \beta\left(g(m), m\right)$ $(= \beta\left(g(n), n\right))$ and an equilibrium exists at which the supplier produces the public good efficiently. Suppose that on the path of this equilibrium, the supplier chooses $g(n)$ in the first stage, he/she receives transfers from $\mu \in \{1, \dots, n\}$ consumers in the second stage, and he/she executes the project in the third stage. The supplier obtains $\pi_S^\mu(g(n))$ in this equilibrium.

Note that the supplier chooses a level of the public good in the set $[\underline{g}^\mu, \overline{g}^\mu]$ when the supplier receives positive transfers from μ consumers. Since the supplier chooses $g(n)$ given that he/she receives positive transfers from μ consumers, $g(n)$ must belong to $[\underline{g}^\mu, \overline{g}^\mu]$. By (17), we obtain $g(n) < \underline{g}^n$ from $\beta > \beta(g(n), n)$. Thus, in this equilibrium, the supplier never receives positive transfers from n consumers: $\mu < n$.

We need to consider two possibilities: $\underline{g}^\mu < g(n)$ and $\underline{g}^\mu = g(n)$. Suppose first that $\underline{g}^\mu < g(n)$. We obtain $g(\mu) < g(n)$ since $\mu < n$. Since $\pi_S^\mu(g)$ is decreasing in g if $g > g(\mu)$, then, if the supplier sets a level of the public good a bit lower than $g(n)$ in the interval $[\underline{g}^\mu, \overline{g}^\mu]$, the supplier's payoff increases (the supplier can choose such a level because of $\underline{g}^\mu < g(n)$). Second, suppose that $\underline{g}^\mu = g(n)$. By Condition 2 and $\mu < n$, $\pi_S^\mu(\underline{g}^\mu) = \pi_S^\mu(g(n)) < \pi_S^n(\underline{g}^n)$. In any case, the supplier does not choose $g(n)$ in the equilibrium, which is a contradiction. ■

Remark 1 Proof of Proposition 2 does not depend on the equilibrium selection in the second and third stages presented in Sections 3.1 and 3.2.2. In addition to the equilibrium presented in Section 3.1, in the third stage, there is an equilibrium at which the supplier chooses execution if and only if $\sum_{j \in N} T_j \geq c(g)$. Depending on which third-stage equilibrium we consider, the condition under which μ consumers transfer to the supplier is $g(n) \in [\underline{g}^\mu, \overline{g}^\mu)$ or $g(n) \in (\underline{g}^\mu, \overline{g}^\mu]$; that is, in any equilibrium, if μ consumers transfer to the supplier, then $g(n) \in [\underline{g}^\mu, \overline{g}^\mu]$. In the proof, we consider this interval.

Analysis in Subsection 4.2

Suppose that the second and third-stage equilibria are the same as those in Sections 3.1 and 3.2.2. We now prove that the supplier's payoff is maximized when he/she chooses $\underline{g}^n$. Since $n = 11$ and $\beta > \min_{m\in\{1,\dots,n\}} \beta(g(m), m)$, we obtain $\beta > 0.55$. Since $\beta(g(m), m+1) = 1/2$ and $\beta > 0.55$, it follows that $g(m) < \underline{g}^{m+1}$ for each m (see Lemma 6). We obtain that $\beta(g(m), m) = \beta$ at $m \equiv 2\beta/(2\beta - 1)$. Let $\tilde{m} \equiv 2\beta/(2\beta - 1)$. Then, $\underline{g}^m \leq g(m)$ if $m \leq \tilde{m}$ and $g(m) \leq \underline{g}^m$ if $m \geq \tilde{m}$ (see Lemma 6). In summary, $\underline{g}^m \leq g(m) < \underline{g}^{m+1}$ if $m \leq \tilde{m}$ and $g(m) \leq \underline{g}^m < \underline{g}^{m+1}$ if $m \geq \tilde{m}$.

We also obtain

$$\pi_S^m(g(m)) = \frac{\beta m^2}{4(1 + (1-\beta)(m-1))} \quad \text{and} \quad \pi_S^m(\underline{g}^m) = \beta^2(m-1).$$

Note that $\pi_S^m(g(m))$ and $\pi_S^m(\underline{g}^m)$ are increasing in m.

Since $\pi_S^m(g(m))$ is increasing in m, we obtain $\pi_S^m(g(m)) < \pi_S^{\tilde{m}}(g(\tilde{m}))$ for each $m < \tilde{m}$. By (7.b) of Lemma 7, in $[0, \underline{g}^{\tilde{m}})$, the supplier's payoff is maximized at $g = g(\tilde{m})$ and the maximized payoff is

$$\pi_S^{\tilde{m}}(g(\tilde{m})) = \frac{\beta^2}{2\beta - 1}.$$

Since $\pi_S^m(\underline{g}^m)$ is increasing in m, we obtain $\pi_S^m(\underline{g}^m) < \pi_S^{11}(\underline{g}^{11})$ for each $m \in [\tilde{m}, 11)$. By (7.c) of Lemma 7, in $[\underline{g}^{\tilde{m}}, \infty)$, it is maximized at $g = \underline{g}^{11}$ and the maximized payoff is $\pi_S^{11}(\underline{g}^{11}) = 10\beta^2$. The difference between those payoffs is

$$\pi_S^{11}(\underline{g}^{11}) - \pi^{\tilde{m}}(g(\tilde{m})) = \frac{\beta^2(20\beta - 11)}{2\beta - 1} > 0 \text{ since } \beta > 0.55.$$

Since $\pi_S^m(g(m))$ is increasing in m, we also obtain $\pi_S^{11}(\underline{g}^{11}) - \pi_S^m(g(m)) > 0$ for all integers m such that $m \leq \tilde{m}$. Therefore, in this numerical example, the supplier oversupplies the public good when his/her bargaining power is sufficiently high.

Analysis in Subsection 5.2

The analysis is based on the assumption that $v_1(g) < v_2(g)$ for all $g > 0$ and $\beta_1 \leq \beta_2$. Under this assumption, for any level of the public good, consumer 2's benefit from the public good is greater than consumer 1's; the supplier's bargaining power with consumer 2 is not lower than that with consumer 1.

The analysis of the third stage is the same as that of the case of identical consumers. Based on the third-stage equilibrium, consumer $i \in N$ is said to be *pivotal* to a project (g, T_i, T_j) $(i \neq j)$ if $T_i + T_j > c(g) \geq T_j$.

In the second stage, if consumer i is nonpivotal to (g, T_i, T_j), then $T_i = 0$. If consumer i is pivotal, then

$$T_i = v_i(g) - (1-\beta_i)(v_i(g) + T_j - c(g)) = \beta_i v_i(g) - (1-\beta_i)(T_j - c(g)). \tag{26}$$

If consumer i is pivotal and consumer j is not, then $T_i = \beta_i v_i(g) + (1-\beta_i)c(g)$ and $T_j = 0$. The supplier's payoff is $\pi_S^{\{i\}}(g) \equiv \beta_i(v_i(g) - c(g))$ (The upper-script letter $\{i\}$ means that the supplier receives transfers from consumer i). Consumer i's payoff is $(1-\beta_i)(v_i(g)-c(g))$ and consumer j's payoff is $v_j(g)$.

If consumers 1 and 2 are pivotal, then (26) is satisfied for each $i \in N$; hence,

$$T_i = \frac{\beta_i v_i(g) - (1-\beta_i)\beta_j(v_j(g) - c(g))}{\beta_i + \beta_j - \beta_i\beta_j}, \tag{27}$$

where $j \in N \backslash \{i\}$.[29] The supplier's payoff is

$$\pi_S^N(g) \equiv T_1 + T_2 - c(g) = \frac{\beta_1\beta_2}{\beta_1 + \beta_2 - \beta_1\beta_2}(v_1(g) + v_2(g) - c(g))$$

(The upper-script letter N means that the supplier receives transfers from consumers 1 and 2).

Consumer i is pivotal to (g, T_1, T_2) if and only if $T_i + T_j > c(g) \geq T_j$ ($j \in N\backslash\{i\}$). By (27), $T_i + T_j > c(g)$ if and only if $v_i(g) + v_j(g) > c(g)$. By (27), $c(g) \geq T_j$ if and only if

$$0 \geq \frac{1}{\beta_i + \beta_j - \beta_i\beta_j}(\beta_j(v_j(g) - c(g)) - (1-\beta_j)\beta_i v_i(g))$$

if and only if

$$v_j(g) - \frac{(1-\beta_j)\beta_i}{\beta_j} v_i(g) \leq c(g).$$

Therefore, consumer i is pivotal to (g, T_1, T_2) if and only if g satisfies

$$v_j(g) - \frac{(1-\beta_j)\beta_i}{\beta_j} v_i(g) \leq c(g) < v_i(g) + v_j(g) \quad (j \in N\backslash\{i\}). \tag{28}$$

Since $v_1(g) < v_2(g)$ for each $g > 0$ and $\beta_1 \leq \beta_2$,

$$v_1(g) - \frac{(1-\beta_1)\beta_2}{\beta_1} v_2(g) < v_2(g) - \frac{(1-\beta_2)\beta_1}{\beta_2} v_1(g).$$

Thus, by (28), consumers 1 and 2 are pivotal to (g, T_1, T_2) if and only if g satisfies

$$v_2(g) - \frac{(1-\beta_2)\beta_1}{\beta_2} v_1(g) \leq c(g) < v_1(g) + v_2(g). \tag{29}$$

Clearly, (29) does not hold at $g = 0$.

To simplify the discussion on (29), we further assume that for each $i \in N$, $v_i(g) = \lambda_i g$, where $0 < \lambda_1 < \lambda_2$, and $c(g) = g^2/2$. Under those functions, consumers 1 and 2 are pivotal if and only if g satisfies

$$2\left(\lambda_2 - \frac{(1-\beta_2)\beta_1}{\beta_2}\lambda_1\right) \leq g < 2(\lambda_1 + \lambda_2). \tag{30}$$

Let $g(N) \equiv \arg\max_{g \geq 0} \sum_{j \in N} v_j(g) - c(g)$ and $g(\{i\}) \equiv \arg\max_{g \geq 0} v_i(g) - c(g)$ for each $i \in N$. Then, $g(N) = \lambda_1 + \lambda_2$ and $g(\{i\}) = \lambda_i$ for each $i \in N$. Note that whether $g(N) = \lambda_1 + \lambda_2$ satisfies (30) depends on the values of λ_i and β_i ($i \in N$).

In Claim 1, we show that the efficient provision of the public good is achieved at an equilibrium of the game only if consumers 1 and 2 are both pivotal to the efficient provision.

[29] We assume in the subsequent analysis that $T_i > 0$ for each $i \in N$ when both consumers are pivotal.

Claim 1 There is an equilibrium of the game that supports the provision of $g(N)$ units of the public good only if

$$g(N) \geq 2\left(\lambda_2 - \frac{(1-\beta_2)\beta_1}{\beta_2}\lambda_1\right). \tag{31}$$

Proof. Suppose, to the contrary, that (31) does not hold, but an equilibrium exists at which $g(N)$ units of the public good are provided.

Since (31) does not hold, it is impossible that both consumers 1 and 2 are pivotal to the provision of $g(N)$ units of the public good. Hence, one of the consumers is pivotal to the provision. For each $g > 0$, if consumer 1 is solely pivotal to the provision of g units of the public good, then $0 < g \leq 2\lambda_1$.[30] This, together with $2\lambda_1 < g(N)$, implies that consumer 1 cannot be solely pivotal to the provision of $g(N)$ units of the public good. Therefore, if the supplier chooses $g(N)$ in the first stage, then consumer 2 is solely pivotal to the provision of it.

Take $\tilde{g}$ such that $\max\{2\lambda_1, \lambda_2\} < \tilde{g} < g(N)$. Note that at $\tilde{g}$, consumer 1 cannot be solely pivotal since $2\lambda_1 < \tilde{g}$ and consumer 2 can be pivotal since $\tilde{g} < 2\lambda_2$. The supplier's payoff when he/she chooses $\tilde{g}$ is $\pi_S^{\{2\}}(\tilde{g}) = \beta_2(v_2(\tilde{g}) - c(\tilde{g}))$ and his/her payoff when he/she chooses $g(N)$ is $\pi_S^{\{2\}}(g(N)) = \beta_2(v_2(\lambda_1 + \lambda_2) - c(\lambda_1 + \lambda_2))$. Since $\lambda_2 < \tilde{g} < g(N)$ and $v_2(g) - c(g)$ is maximized at $g = g(\{2\}) = \lambda_2$, we obtain $\pi_S^{\{2\}}(g(N)) < \pi_S^{\{2\}}(\tilde{g})$, which contradicts that $g(N)$ units of the public good are provided in equilibrium. ■

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Online Appendix for "The efficiency of monopolistic provision of public goods through simultaneous bilateral bargaining"

Noriaki Matsushima and Ryusuke Shinohara

A.1 Equilibrium level of the public good when β is sufficiently high

We focus on the analysis of the first stage, given the second- and third-stage equilibrium in Subsections 3.1 and 3.2.2.

Assume Conditions 1 and 2. Suppose that $\beta > \min_{m\in\{1,\ldots,n\}} \beta\left(g(m),m\right)$. By Lemma 5, $\beta(g(m),m)$ approaches infinity as m decreases to one. Hence, for each $\beta \in [0,1]$, if $\beta > \min_{m\in\{1,\ldots,n\}} \beta\left(g(m),m\right)$, there is $\mu \in (1,n]$ such that $\beta = \beta\left(g(\mu),\mu\right)$. However, we cannot generally say whether β intersects with $\beta(g(m),m+1)$. Hence, we need to consider the following two subcases.

- **Subcase 2.1:** $\min_{m\in\{1,\ldots,n\}} \beta\left(g(m),m\right) < \beta \le \max_{m\in\{1,\ldots,n\}} \beta\left(g(m),m+1\right)$ and $\min_{m\in\{1,\ldots,n\}} \beta\left(g(m),m\right) < \max_{m\in\{1,\ldots,n\}} \beta\left(g(m),m+1\right)$.
- **Subcase 2.2:** $\beta > \min_{m\in\{1,\ldots,n\}} \beta\left(g(m),m\right)$ and $\beta > \max_{m\in\{1,\ldots,n\}} \beta\left(g(m),m+1\right)$.

Typical situations in these two subcases are summarized in Figure 4. Subcases 2.1 and 2.2 correspond to the cases of $\beta = \beta_{21}$ and $\beta = \beta_{22}$, respectively. Since we do not impose any restriction except nonincreasing $\beta(g(m),m)$ and $\beta(g(m),m+1)$, we must consider various possibilities. In some cases, as in the case of $\beta = \beta_{22}$ in this figure, β intersects with $\beta(g(m),m)$ at some $m \in (1,n]$, but not with $\beta(g(m),m+1)$ at any $m \in [1,n]$ (Subcase 2.2). In other cases, as in the case of $\beta = \beta_{21}$ in this figure, β intersects with $\beta(g(m),m)$ and $\beta(g(m),m+1)$ at some points (Subcase 2.1).

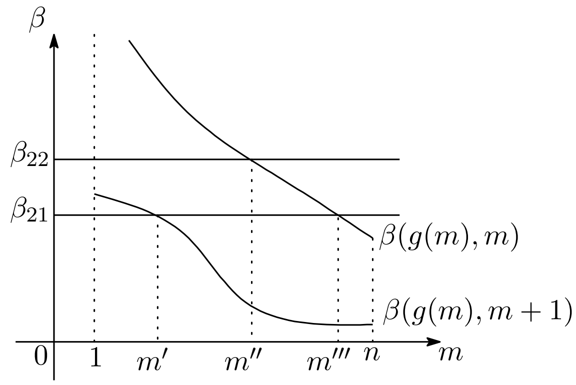

Figure 4: The case in which β is sufficiently high

(Subcase 2.1) In this subcase, at least one $m''' \in (1, n]$ exists such that $\beta = \beta(g(m'''), m''')$ and at least one $m' \in [1, n]$ exists such that $\beta = \beta(g(m'), m'+1)$ (see m' and m''' in Figure 4). Such m' and m''' are not necessarily unique. Let $\bar{m}'$ be the maximal among such m's and let $\bar{m}'''$ be the maximal among such m'''s. Let $\overline{m}(\beta) \in \{1, \ldots, n\}$ be the maximal integer that is less than or equal to $\bar{m}'''$ and let $\overline{m+1}(\beta) \in \{1, \ldots, n\}$ be the maximal integer that is less than or equal to $\bar{m}'$. Clearly, $\overline{m+1}(\beta) \leq \overline{m}(\beta)$.[31] We set three categories for the number of contributors:

Category I $\equiv \left\{m \in \{1, \ldots, n\} : m \leq \overline{m+1}(\beta)\right\}$.

Category II $\equiv \left\{m \in \{1, \ldots, n\} : \overline{m+1}(\beta) < m \leq \overline{m}(\beta)\right\}$.

Category III $\equiv \{m \in \{1, \ldots, n\} : \overline{m}(\beta) < m\}$.

As Claim A1 shows, while Categories I and III are nonempty, Category II may be empty.

Claim A1 In Subcase 2.1, Categories I and III are nonempty. Category II is empty if and only if $\overline{m+1}(\beta) = \overline{m}(\beta)$.

Proof. Category I is nonempty since $1 \leq \bar{m}'$ implies $1 \leq \overline{m+1}(\beta)$. The statement for Category II is trivial. Category III is nonempty since $\bar{m}''' < n$ implies $\overline{m}(\beta) < n$. ■

By (15)–(17), for each $m \in \{1, \ldots, n\}$,

$$\begin{aligned} &\underline{g}^{m+1} \leq g(m) \quad \text{if } m \in \text{Category I} \\ &\underline{g}^{m} \leq g(m) < \underline{g}^{m+1} \quad \text{if } m \in \text{Category II} \\ &g(m) < \underline{g}^{m} \quad \text{if } m \in \text{Category III.} \end{aligned} \tag{32}$$

We first examine what is the best choice for the supplier within each category.

Category I. The supplier receives transfers from the number of consumers in this category if and only if he/she chooses the level of the public good in $(0, \underline{g}^{\overline{m+1}(\beta)+1})$. The analysis for Category I is similar to that for Subcase 1.1 in the proof of Proposition 1. By (32), for each m in Category I, $\underline{g}^{m+1} \leq g(m)$; hence, for each m in this category, $\pi_S^m(g)$ is increasing in the interval $[\underline{g}^m, \underline{g}^{m+1})$. The supplier receives transfers from m contributors if and only if he/she chooses the level in the interval $\left[\underline{g}^m, \underline{g}^{m+1}\right)$. By the continuity of $\pi_S^m(g)$ at every g, we can define $\pi_S^m(\underline{g}^{m+1})$ and

$$\lim_{g \uparrow \underline{g}^{m+1}} \pi_S^m(g) = \pi_S^m(\underline{g}^{m+1}) = \frac{\beta\left(mv(\underline{g}^{m+1}) - c(\underline{g}^{m+1})\right)}{1 + (1-\beta)(m-1)}.$$

Thus, we obtain $\sup_{g \in [\underline{g}^m, \underline{g}^{m+1})} \pi_S^m(g) = \lim_{g \uparrow \underline{g}^{m+1}} \pi_S^m(g)$. Similarly to (23), we obtain that for each pair m, $m+1$ in Category I,

$$\lim_{g \uparrow \underline{g}^{m+1}} \pi_S^m(g) < \lim_{g \uparrow \underline{g}^{m+2}} \pi_S^{m+1}(g). \tag{33}$$

By (33), the supremum of the supplier's payoff in $[0, \underline{g}^{\overline{m+1}(\beta)+1})$ is $\pi_S^{\overline{m+1}(\beta)}(\underline{g}^{\overline{m+1}(\beta)+1})$.

[31] We obtain $\bar{m}' < \bar{m}'''$. However, there may be a case in which $\overline{m+1}(\beta) = \overline{m}(\beta)$.

Category II. The supplier receives transfers from the number of consumers in this category if and only if he/she chooses the level in $[\underline{g}^{\overline{m+1}(\beta)+1}, \underline{g}^{\overline{m}(\beta)+1})$. The analysis for this category is similar to that for Subcase 1.2 in the proof of Proposition 1. By (32), for each m in Category II, $g(m) \in [\underline{g}^m, \underline{g}^{m+1})$; hence, for each m in this category, $g(m)$ maximizes $\pi_S^m(g)$ subject to $g \in [\underline{g}^m, \underline{g}^{m+1})$. By Corollary 1, for each pair m, $m+1$ in Category II,

$$\pi_S^m(g(m)) < \pi_S^{m+1}(g(m+1))$$

In conclusion, the maximum payoff to the supplier in $[\underline{g}^{\overline{m+1}(\beta)+1}, \underline{g}^{\overline{m}(\beta)+1})$ is $\pi_S^{\overline{m}(\beta)}(g(\overline{m}(\beta)))$ within this category.

Category III. The supplier receives transfers from the number of consumers in this category if and only if he/she chooses the level of the public good in $[\underline{g}^{\overline{m}(\beta)+1}, \overline{g}^n)$. By (32), for each m in Category III, $g(m) < \underline{g}^m$; hence, for each m in this category, $\underline{g}^m$ maximizes $\pi_S^m(g)$ subject to $g \in [\underline{g}^m, \underline{g}^{m+1})$ and the supplier receives the payoff $\pi_S^m(\underline{g}^m)$. By Condition 2, the maximal payoff to the supplier in $[\underline{g}^{\overline{m}(\beta)+1}, \overline{g}^n)$ is $\pi_S^n(\underline{g}^n)$.

Second, we examine what the best choice is for the supplier across the categories.

Claim A2 In equilibrium, the supplier never chooses $g \in [0, \underline{g}^{\overline{m+1}(\beta)+1})$.

Proof. We obtain

$$\begin{aligned}
\lim_{g \uparrow \underline{g}^{\overline{m+1}(\beta)+1}} \pi_S^{\overline{m+1}(\beta)}(g) &= \pi_S^{\overline{m+1}(\beta)}\left(\underline{g}^{\overline{m+1}(\beta)+1}\right) \\
&< \pi_S^{\overline{m+1}(\beta)+1}\left(\underline{g}^{\overline{m+1}(\beta)+1}\right) \quad \text{(by Lemma 3).} \qquad (34)
\end{aligned}$$

Since Category II may be empty and Category III is nonempty by Claim A1, $\overline{m+1}(\beta)+1$ belongs to Category II if Category II is nonempty and belongs to Category III if Category II is empty. In any case, if the supplier chooses $\underline{g}^{\overline{m+1}(\beta)+1}$ and receives transfers from $\overline{m+1}(\beta)+1$ contributors, then he/she can obtain the payoff $\pi_S^{\overline{m+1}(\beta)+1}(\underline{g}^{\overline{m+1}(\beta)+1})$. Note that $\pi_S^{\overline{m+1}(\beta)}(\underline{g}^{\overline{m+1}(\beta)+1})$ is the supremum of the supplier's payoff in Category I. Hence, in equilibrium, the supplier never chooses g such that $g < \underline{g}^{\overline{m+1}(\beta)+1}$. ∎

Claim A3 In equilibrium, the supplier chooses $g(\overline{m}(\beta))$ if $\pi_S^{\overline{m}(\beta)}(g(\overline{m}(\beta))) > \pi_S^n(\underline{g}^n)$, which implies that the supplier underprovides the public good since $g(\overline{m}(\beta)) < g(n)$. The supplier chooses $\underline{g}^n$ if $\pi_S^{\overline{m}(\beta)}(g(\overline{m}(\beta))) < \pi_S^n(\underline{g}^n)$, which implies that the supplier overprovides the public good since $g(n) < \underline{g}^n$.

Proof. The supplier's choice of the level of the public good is immediate from the previous analysis. We obtain $g(\overline{m}(\beta)) < g(n)$ since $\min_{m \in \{1,\dots,n\}} \beta(g(m), m) < \beta$ implies $\overline{m}(\beta) < n$. We have $g(n) < \underline{g}^n$ since n belongs to Category III. ∎

By Claims A2 and A3, in this subcase, the supplier never chooses $g(n)$ in equilibrium.

(Subcase 2.2) The analysis of this subcase is almost the same as that of Subcase 2.1 in which Category I is empty. In this subcase, at least one $m'' \in (1, n]$ exists such that $\beta = \beta(g(m''), m'')$ (see m'' of Figure 4). Let $\bar{m}''$ be the maximal among such m'' s. Let $\overline{m}(\beta) \in \{1, \ldots, n\}$ be the maximal integer that is less than or equal to $\bar{m}''$. We set two categories for the number of contributors as follows

Category A $:= \{m \in \{1, \ldots, n\} : m \leq \overline{m}(\beta)\}$.

Category B $:= \{m \in \{1, \ldots, n\} : \overline{m}(\beta) < m\}$.

Note that both categories are nonempty because $1 \leq \overline{m}(\beta) < n$.[32] For each $m \in \{1, \ldots, n\}$, $\underline{g}^m \leq g(m)$ if $m \in$ Category A and $g(m) < \underline{g}^m$ if $m \in$ Category B. In addition, note that in this subcase, for each $m \in \{1, \ldots, n\}$, $g(m) < \underline{g}^{m+1}$. In summary,

$$\begin{aligned} &\underline{g}^m \leq g(m) < \underline{g}^{m+1} \text{ if } m \in \text{Category A} \\ &g(m) < \underline{g}^m \ (< \underline{g}^{m+1}) \text{ if } m \in \text{Category B}. \end{aligned}$$

For each $m \in$ Category A, $g(m)$ maximizes $\pi_S^m(g)$ subject to $g \in [\underline{g}^m, \underline{g}^{m+1})$. For each $m \in$ Category B, $\underline{g}^m$ maximizes $\pi_S^m(g)$ subject to $g \in [\underline{g}^m, \underline{g}^{m+1})$. As in Subcase 2.1, we obtain Claim A4, which is similar to Claim A3, and the proof of Claim A4 is the same as that of Claim A3.

Claim A4 In equilibrium, the supplier chooses $g\left(\overline{m}\left(\beta\right)\right)$ if $\pi_S^{\overline{m}(\beta)}\left(g\left(\overline{m}\left(\beta\right)\right)\right) > \pi_S^n\left(\underline{g}^n\right)$, which implies that the supplier underprovides the public good since $g\left(\overline{m}(\beta)\right) < g(n)$. The supplier chooses $g = \underline{g}^n$ if $\pi_S^{\overline{m}(\beta)}\left(g(\overline{m}(\beta))\right) < \pi_S^n\left(\underline{g}^n\right)$, which implies that the supplier overprovides the public good since $g(n) < \underline{g}^n$.

By Claim A4, in this subcase, the supplier never chooses $g(n)$ in equilibrium.

In conclusion, in any subcase, the supplier chooses an inefficient level of the public goods, that is, a level higher or lower than the efficient level.

A.2 Analysis without condition 1

We discuss whether the supplier provides the public good efficiently if and only if his/her bargaining power is sufficiently weak without Condition 1. We show that in a case in which $\beta(g(m), m)$ and $\beta(g(m), m+1)$ are U-shaped, the supplier provides the public good efficiently if his/her bargaining power is sufficiently low; however, he/she may provide the public good efficiently even if his/her bargaining power is sufficiently high.

We focus on the analysis of the first stage, given the second- and third-stage equilibrium in Subsections 3.1 and 3.2.2. We consider a case in which $\beta(g(m), m)$ and $\beta(g(m)m+1)$ are depicted in Figures 5 and 6. In this case, Condition 1 is not satisfied. Let $\underline{\beta} \equiv \min_{m\in\{1,\ldots,n\}} \beta(g(m), m)$. Let $\beta' \in [0, 1]$ be the supplier's bargaining power. In Figure 5, $\beta' \leq \underline{\beta}$ (in this sense, the supplier's bargaining power is sufficiently weak), while in Figure 6, $\underline{\beta} < \beta'$ (in this sense, it is sufficiently strong).

[32]Since $\min_{m\in\{1,\ldots,n\}} \beta(g(m), m) < \beta$, $\overline{m}(\beta) < n$. Since $\beta(g(m), m)$ approaches infinity as m approaches 1, $1 \leq \overline{m}(\beta)$.

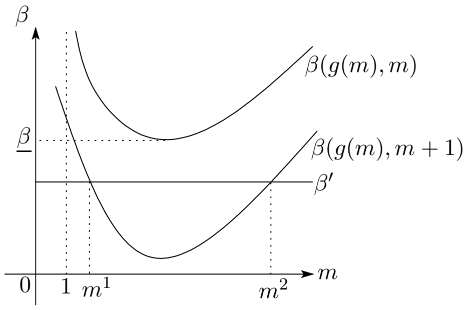

Figure 5: The case in which $\beta' \leq \underline{\beta}$

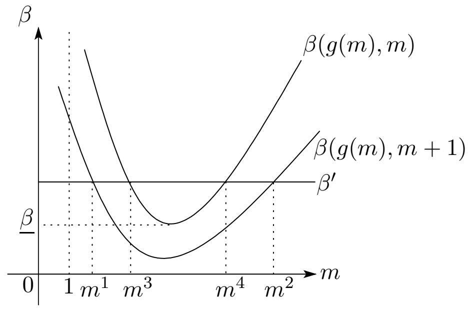

Figure 6: The case in which $\beta' > \underline{\beta}$

We first consider the case in Figure 5, in which $\beta' \leq \underline{\beta}$. We categorize the number of consumers as $M_{51} \equiv \{m \in \{1, \ldots, n\} : m \in (1, m^1) \text{ or } m > m^2\}$ and $M_{52} \equiv \{m \in \{1, \ldots, n\} : m \in [m^1, m^2)\ \}$. Then, by $\beta' \leq \underline{\beta}$,

$$\begin{aligned} \underline{g}^m < \underline{g}^{m+1} \leq g(m) \qquad & \text{if } m \in M_{51} \text{ and} \\ \underline{g}^m \leq g(m) < \underline{g}^{m+1} \qquad & \text{if } m \in M_{52}, \end{aligned}$$

similarly to (15) and (16).[33] By similar reason to (23),

$$\pi_S^m(\underline{g}^{m+1}) < \pi_S^\mu(\underline{g}^{\mu+1}) \text{ for each pair } m,\ \mu \in M_{51} \text{ such that } m < \mu. \tag{35}$$

By Corollary 1,

$$\pi^m(g(m)) < \pi^\mu(g(\mu))\text{for each pair } m,\ \mu \in M_{52} \text{ such that } m < \mu.$$

We prove that in the case in Figure 5, the supplier's payoff is maximized globally at $g = g(n)$. Suppose first that $n \in M_{51}$. By (35), $\pi_S^m(\underline{g}^{m+1}) < \pi_S^n(\underline{g}^{n+1})$ for each $m \in M_{51}$ such that $m \neq n$. Since $n \in M_{51}$ implies $\underline{g}^{n+1} \leq g(n)$, $\pi_S^n(\underline{g}^{n+1}) \leq \pi_S^n(g(n))$. Thus, under the constraint that g is the level of the public good at which the number of contributors belongs to M_{51}, the supplier's payoff $\pi_S^m(g)$ is maximized at $m = n$ and $g = g(n)$. For each $m \in M_{52}$, by Corollary 1 and $m < n$, $\pi_S^m(g(m)) < \pi_S^n(g(n))$. Thus, there is no $m \in M_{52}$ such that $\pi_S^m(g(m)) > \pi_S^n(g(n))$. In conclusion, if $n \in M_{51}$, then the supplier's payoff is maximized at $g = g(n)$.

Suppose second that $n \in M_{52}$. This case is similar to Subcase 1.1 in the proof of Proposition 1 because $\beta(g(m), m+1)$ intersects β' at once. Then, by analysis that is similar to Subcase 1.1 in the proof of Proposition 1, we confirm that the supplier's payoff is maximized at $g = g(n)$. In conclusion, in the case in Figure 5, the supplier's payoff is maximized at $g = g(n)$.

Second, we consider the case in Figure 6, in which $\beta' > \underline{\beta}$. In this case, the supplier may or may not provide the public good good efficiently. By (15), (16), and (17),

$$\begin{aligned} &\underline{g}^m < \underline{g}^{m+1} < g(m) \text{ if } m \in (1, m^1) \text{ or } m > m^2, \\ &\underline{g}^m \leq g(m) \leq \underline{g}^{m+1} \text{ if } m \in [m^1, m^3] \cup [m^4, m^2], \text{ and} \\ &g(m) < \underline{g}^m < \underline{g}^{m+1} \text{ if } m \in (m^3, m^4). \end{aligned}$$

[33] Note that (15), (16), and (17) hold without Condition 1.

Since $\beta' > \underline{\beta}$, $n > m^3$.[34] Thus, we need to consider the following three cases: the case with $n \in (m^3, m^4)$, the case with $n \in [m^4, m^2]$, and the case with $n > m^2$. Suppose first that $n \in (m^3, m^4)$. Then, the analysis is the same as Section A.1 (note that we do not have to consider the interval $[m^4, \infty)$). In this case, the supplier never supplies the public good efficiently at any equilibrium.

Suppose second that $n \in [m^4, m^2]$. Similarly to Claim A2, the equilibrium number of consumers from which the supplier receives positive transfers does not belong to $(1, m^1)$. By Corollary 1, $\pi_S^n(g(n)) > \pi_S^m(g(m))$ for any other integer $m \in [m^1, m^3] \cup [m^4, m^2]$. Let $\hat{\mu}$ be the largest integer in (m^3, m^4). Then, by Condition 2, $\pi_S^{\hat{\mu}}(\underline{g}^{\hat{\mu}}) > \pi_S^m(\underline{g}^m)$ for any other integer $m \in (m^3, m^4)$. In addition, we obtain

$$\pi_S^{\hat{\mu}}(\underline{g}^{\hat{\mu}}) < \pi_S^{\hat{\mu}+1}(\underline{g}^{\hat{\mu}}) \leq \pi_S^{\hat{\mu}+1}(g(\hat{\mu}+1))$$

because the first inequality follows from Lemma 3 and the second from $\hat{\mu}+1 \in [m^4, m^2]$. Moreover, we obtain

$$\pi_S^{\hat{\mu}+1}(g(\hat{\mu}+1)) \leq \pi_S^n(g(n))$$

because $\hat{\mu}+1 \leq n$. In conclusion, $\pi_S^{\hat{\mu}}(\underline{g}^{\hat{\mu}}) < \pi_S^n(g(n))$. Therefore, the supplier chooses $g(n)$ in the first stage.

Suppose finally that $n > m^2$. Since $n > m^2$, $\underline{g}^n < \underline{g}^{n+1} < g(n)$ and $g(n) < \overline{g}^n$. The supplier prefers to receive transfers from n consumers to less than n and greater than m^2 consumers. In addition to the analysis for the second case, if we note that $\pi_S^m(g(m)) < \pi_S^n(g(n))$ for each integer $m \in [m^4, m^2]$, then we confirm that the supplier chooses $g(n)$ in the first stage.

In conclusion, in the case in Figure 6, the supplier with sufficiently high bargaining power provides the public good inefficiently if and only if $n \in (m^3, m^4)$; the supplier may provide the public good efficiently if his/her bargaining power is sufficiently strong.

A.3 Analysis in Subsection 5.1

Simultaneous bilateral bargaining and the core

We show that the payoffs attained with simultaneous bilateral bargaining belong to the core of some appropriate cooperative game.

Let $\mathcal{N} = \{s\} \cup N$, where s is the supplier and $N = \{1, \ldots, n\}$ is the set of consumers with $n \geq 2$. For each $g > 0$ such that $nv(g) > c(g)$, define the characteristic function $w^g : 2^{\mathcal{N}} \to \mathbb{R}_+$ as follows: for any nonempty subset $\mathcal{C} \subseteq N$, $w^g(\{\emptyset\}) = w^g(\{s\}) = w^g(\mathcal{C}) = 0$; $w^g(\{s\} \cup \mathcal{C}) = \max\{|\mathcal{C}|v(g) - c(g), 0\}$.[35] By this definition, the cooperation of the supplier and consumers is necessary for a positive surplus. If $\mathcal{C}$ is sufficiently large, $|\mathcal{C}|v(g) > c(g)$ holds; hence, the supplier provides g (decided in the first stage) and the consumers in $\mathcal{C}$ pay the fee to the supplier. *The supplier is assumed to commit the level of the public good decided in the first stage. In the negotiation in coalitions, only the surplus division is considered.* A payoff profile $u = (u_i)_{i \in N}$ belongs to the *core* of $(\mathcal{N}, w^g)$ if $\sum_{i \in C} u_i \geq w^g(C)$ for each $C \subseteq \mathcal{N}$ and $\sum_{i \in \mathcal{N}} u_i = w^g(\mathcal{N})$.

[34] Suppose that $\underline{\beta} < \beta'$ and $n \leq m^3$. Then, by Figure 6, $\underline{\beta} = \beta(g(n), n) \geq \beta'$, which contradicts $\beta' > \underline{\beta}$.

[35] $|\mathcal{C}|$ represents the cardinality of $\mathcal{C}$.

Let $\beta \in [0,1]$ be the supplier's bargaining power. Let $g > 0$ be a level of the public good such that $nv(g) - c(g) > 0$. Let m be a number of pivotal consumers. Note then that $mv(g) > c(g)$ holds by the pivotal condition. Let $M \subseteq N$ be a set of contributors. Let u^s, u^c, and u^f denote the payoffs to the supplier, the contributor, and the free rider through the simultaneous bilateral bargaining in the second stage, respectively. Then,

$$\begin{aligned} u^s &= \pi_S^m(g) = \frac{\beta(mv(g) - c(g))}{\beta + (1-\beta)m} \geq 0, \\ u^c &= v(g) - T_i^m = \frac{(1-\beta)(mv(g) - c(g))}{\beta + (1-\beta)m} \geq 0, \text{ and} \\ u^f &= v(g) > 0. \end{aligned}$$

Proposition A1 A payoff profile $(u_i)_{i \in \mathcal{N}}$ such that $u_s = u^s$, $u_i = u^c$ for any $i \in M$, and $u_i = u^f$ for any $i \in N \backslash M$ belongs to the core of $(\mathcal{N}, w^g)$.

Proof. Trivially, $\sum_{i \in C} u_i \geq w^g(C)$ for each $C \subseteq \mathcal{N}$ such that $w^g(C) = 0$. We need to consider the coalition $\{s\} \cup \mathcal{C}$ such that $\mathcal{C} \subseteq N$ and $w^g(\{s\} \cup \mathcal{C}) > 0$; that is, $|\mathcal{C}|v(g) > c(g)$. Let $\mathcal{C}^c \subseteq \mathcal{C}$ be the set of contributors in $\mathcal{C}$ and let $\mathcal{C}^f \subseteq \mathcal{C}$ be the set of free riders in $\mathcal{C}$. We obtain

$$\sum_{i \in \{s\} \cup \mathcal{C}} u_i = \frac{\beta + (1-\beta)|\mathcal{C}^c|}{\beta + (1-\beta)m}(mv(g) - c(g)) + |\mathcal{C}^f| v(g).$$

Then, we obtain

$$\frac{\beta + (1-\beta)|\mathcal{C}^c|}{\beta + (1-\beta)m}(mv(g) - c(g)) - (|\mathcal{C}^c|v(g) - c(g)) = \frac{(m - |\mathcal{C}^c|)}{\beta + (1-\beta)m}(\beta v(g) + (1-\beta)c(g)) \geq 0.$$

Thus, $\sum_{i \in \{s\} \cup \mathcal{C}} u_i \geq |\mathcal{C}^c|v(g) - c(g) + |\mathcal{C}^f|v(g) = w^g(\{s\} \cup \mathcal{C})$. In conclusion, $(u_i)_{i \in \mathcal{N}}$ belongs to the core of $(\mathcal{N}, w^g)$. ■

Remark A1 We can prove that the core of $(\mathcal{N}, w^g)$ coincides with the set of payoffs $\{(u_i)_{i \in \mathcal{N}} | \ u_i \geq 0$ for each $i \in \mathcal{N}$ and $\sum_{i \in \mathcal{N}} u_i = nv(g) - c(g)\}$.

Simultaneous bargaining and the multilateral Nash bargaining solution

We clarify the relationship between the outcome through simultaneous bilateral bargaining and the multilateral Nash bargaining solution. We suppose that the supplier and m pivotal consumers come to the same negotiation table and multilaterally negotiate the level of transfer. We analyze the outcome of this multilateral negotiation through the multilateral Nash bargaining solution. That is, $(T_i)_{i \in M}$ is determined so as to maximize the Nash product $(\sum_{i \in M} T_i - c(g))^\beta \prod_{i \in M}(v(g) - T_i)^{1-\beta}$. For each pivotal consumer $i \in M$, partially differentiating the Nash product with respect to T_i yields

$$\beta(v(g) - T_i) = (1-\beta)\left(\sum_{j \in M} T_j - c(g)\right) \text{ or } T_i = \beta v(g) + (1-\beta)c(g) - (1-\beta)\sum_{j \in M \backslash \{i\}} T_j,$$

which is equivalent to (1). Thus, the outcome of the multilateral Nash bargaining solution is the same as that of the simultaneous bilateral bargaining through the Nash bargaining solution.

Sequential bilateral bargaining

We replace the second-stage bargaining of the basic model with the following *sequential* bilateral bargaining model. The supplier first negotiates with consumer 1, second negotiates with consumer 2 after the bilateral bargaining with consumer 1, ..., and finally, bilaterally negotiates with consumer n. The supplier has only one bilateral negotiation with each consumer.

We focus on the third-stage equilibrium at which for each project $(g, (T_j)_{j\in N})$, the supplier executes the project if and only if $\sum_{j\in N} T_j > c(g)$.

Given this third-stage equilibrium, we solve the second-stage sequential bargaining by the backward application of the Nash bargaining solution. Given the outcome $(g, (T_j)_{j\neq n})$, the supplier negotiates with consumer n.

- If $\sum_{j\in N\setminus\{n\}} T_j > c(g)$ or if $\sum_{j\in N\setminus\{n\}} T_j \leq c(g)$ and $v(g) \leq c(g) - \sum_{j\in N\setminus\{n\}} T_j$, then the surplus of the bargaining session of the supplier and consumer n is zero; hence, $T_n = 0$.
- If $\sum_{j\in N\setminus\{n\}} T_j \leq c(g)$ and $v(g) > c(g) - \sum_{j\in N\setminus\{n\}} T_j$, the surplus of this bargaining is positive and consumer n pays to the supplier $T_n = v(g) - (1-\beta)(v(g) + \sum_{j\in N\setminus\{n\}} T_j - c(g)) = \beta v(g) - (1-\beta)(\sum_{j\in N\setminus\{n\}} T_j - c(g))$.

Given this outcome of the Nash bargaining solution, we next investigate bilateral bargaining with consumer $n-1$.

We investigate the bilateral bargaining with consumer $k \in \{1, \ldots, n-1\}$, assuming that we have investigated the bilateral bargaining with consumer n to consumer $k+1$ by the backward application of the Nash bargaining solution. Let $(T_j)_{j=k+1}^{n}$ be the Nash bargaining outcome of the bargaining with consumers $k+1$ to n. Before reaching the bargaining with consumer k, the supplier negotiates with consumer 1 to $k-1$. Let $(T_j)_{j=1}^{k-1}$ be the outcome of the bargaining before the bargaining with consumer k. Given $(T_j)_{j=1}^{k-1}$, the supplier and consumer k negotiate, anticipating that $(T_j)_{j=k+1}^{n}$ is obtained in the subsequent negotiations.

- If $\sum_{j=1}^{k-1} T_j + \sum_{j=k+1}^{n} T_j > c(g)$ or if $\sum_{j=1}^{k-1} T_j + \sum_{j=k+1}^{n} T_j \leq c(g)$ and $v(g) \leq c(g) - (\sum_{j=1}^{k-1} T_j + \sum_{j=k+1}^{n} T_j)$, then the surplus of the bargaining session of the supplier and consumer k is zero; hence, $T_k = 0$.
- If $\sum_{j=1}^{k-1} T_j + \sum_{j=k+1}^{n} T_j \leq c(g)$ and $v(g) > c(g) - \sum_{j=1}^{k-1} T_j - \sum_{j=k+1}^{n} T_j$, the surplus of this bargaining is positive and consumer k pays to the supplier $T_k = v(g) - (1-\beta)(v(g) + \sum_{j\neq k} T_j - c(g)) = \beta v(g) - (1-\beta)(\sum_{j\neq k} T_j - c(g))$.

Given this Nash bargaining outcome, we return to the bilateral bargaining with consumer $k-1$. Repeating a similar procedure until reaching the bargaining with consumer 1, we finally obtain the following results. For each $k \in N$, $T_k = \beta v(g) - (1-\beta)(\sum_{j\neq k} T_j - c(g))$ if

$$\sum_{j\in N\setminus\{k\}} T_j \leq c(g) \text{ and } v(g) > c(g) - \sum_{j\in N\setminus\{k\}} T_j \tag{36}$$

and $T_k = 0$ if

$$\sum_{j \in N \setminus \{k\}} T_j > c(g) \quad \text{or} \quad \left[\sum_{j \in N \setminus \{k\}} T_j \le c(g) \text{ and } v(g) \le c(g) - \sum_{j \in N \setminus \{k\}} T_j \right]. \tag{37}$$

Claim A5 shows that (36) and (37) are equivalent to the pivotal condition and the nonpivotal condition, respectively.[36]

Claim A5 (36) if and only if $\sum_{j \in N \setminus \{k\}} T_j \le c(g) < \sum_{j \in N} T_j$.

Proof. ($\Rightarrow$) Obviously, $\sum_{j \in N \setminus \{k\}} T_j \le c(g)$. Since $T_k = \beta v(g) + (1-\beta)c(g) - (1-\beta) \sum_{j \in N \setminus \{k\}} T_j$, we obtain

$$\sum_{j \in N \setminus \{k\}} T_j + T_j - c(g) = \beta \left(v(g) + \sum_{j \in N \setminus \{k\}} T_j - c(g) \right) > 0.$$

($\Leftarrow$) Obviously, $\sum_{j \in N \setminus \{k\}} T_j \le c(g)$. Suppose that $\sum_{j \in N} T_j > c(g)$. Since $T_k \le v(g)$, we obtain $c(g) < \sum_{j \in N} T_j \le v(g) + \sum_{j \in N \setminus \{k\}} T_j$. ■

By Claim A5, for each $k \in N$, if (36), then k is pivotal to a project $(g, (T_j)_{j \in N})$ and $T_k = \beta v(g) - (1-\beta)(\sum_{j \ne k} T_j - c(g))$; if (37), then k is nonpivotal and $T_k = 0$. This is the same as the outcome of the simultaneous bilateral bargaining. Therefore, even under the sequential bilateral negotiations, Theorem 1 still holds.

A.4 Analysis in Subsection 5.3

In the basic model, the supplier is the decision maker about the level of the public good and commits to this level before negotiating. By contrast, we now consider a new simultaneous bilateral bargaining model in which the supplier and each consumer negotiate the level of the public good as well as the transfer. This model consists of two stages: first, the supplier and each consumer $i \in N$ bilaterally negotiate the joint production level of the public good $g_i \ge 0$ and the transfer to the supplier from consumer i $T_i \ge 0$. We assume, as in the basic model, that each bilateral is Nash bargaining and simultaneous. Hence, g_i is negotiated so as to maximize the joint surplus of the bilateral negotiation between the supplier and consumer i; T_i is determined so as to share the maximized joint surplus in proportion to the bargaining power (the supplier's bargaining power and consumer i's bargaining power are β and $1-\beta$, as in the basic model). Let $(g_j, T_j)_{j \in N}$ be the outcome of the simultaneous bilateral bargaining. In the second stage, the supplier decides whether he/she executes $(g_j, T_j)_{j \in N}$. He/she provides $\sum_{j \in N} g_j$ and receives $\sum_{j \in N} T_j$ if and only if he/she executes.

Consumer $i \in N$ is said to be *pivotal* to $(g_j, T_j)_{j \in N}$ if $\sum_{j \in N} T_j \ge c(g_i + \sum_{j \ne i} g_j)$ and $\sum_{j \ne i} T_j < c(\sum_{j \ne i} g_j)$. As in the basic model, the joint surplus of each bilateral negotiation

[36] Note that the negation of (36) is equivalent to (37).

takes different forms, depending on whether each consumer is pivotal. If consumer i is pivotal, then the joint surplus of the bilateral negotiation is

$$v\Big(g_i + \sum_{j\neq i} g_j\Big) - c\Big(g_i + \sum_{j\neq i} g_j\Big) + \sum_{j\neq i} T_j.$$

If consumer i is not pivotal, then the joint surplus of the bilateral negotiation is

$$\begin{aligned} & v\Big(g_i + \sum_{j\neq i} g_j\Big) - T_i - v_i\Big(\sum_{j\neq i} g_j\Big) + \sum_{j\in N} T_j - c\Big(g_i + \sum_{j\neq i} g_j\Big) - \Bigg[\sum_{j\neq i} T_j - c\Big(\sum_{j\neq i} g_j\Big)\Bigg] \\ = \; & v\Big(g_i + \sum_{j\neq i} g_j\Big) - v_i\Big(\sum_{j\neq i} g_j\Big) - \Bigg[c\Big(g_i + \sum_{j\neq i} g_j\Big) - c\Big(\sum_{j\neq i} g_j\Big)\Bigg]. \end{aligned}$$

The supplier and consumer i maximize their surplus, anticipating the other bargaining outcomes $(g_j, T_j)_{j\neq i}$. Note that from the forms of the surplus, irrespective of whether consumer i is pivotal, the bilateral negotiation maximizes $v(g_i + \sum_{j\neq i} q_j) - c(g_i + \sum_{j\neq i} q_j)$ through the choice of g_i. Hence, if we denote the levels of the public good supported at an equilibrium by $(g_j^*)_{j\in N}$, then

$$\{g_i^*\} = \arg\max_{g_i\geq 0}\; v\Big(g_i + \sum_{j\neq i} g_j^*\Big) - c\Big(g_i + \sum_{j\neq i} g_j^*\Big) \text{ for each } i \in N.$$

We prove that $g(1)$ units of the public good are provided at any equilibrium. Recall that $g(1)$ maximizes $v(g) - c(g)$ and $g(1)$ is an interior solution by Assumption 1. Then, $v'(g(1)) = c'(g(1))$. To the contrary, suppose that for some $\hat{g} > g(1)$, equilibrium levels of the public good $(\hat{g}_j)_{j\in N}$ exist such that $\sum_{j\in N} \hat{g}_j = \hat{g}$. Since $v'(g(1)) = c'(g(1))$ and $g(1) < \hat{g}$, $v'(\hat{g}) < c'(\hat{g})$. Since $\hat{g}_i + \sum_{j\neq i} \hat{g}_j = \hat{g}$ for each $i \in N$, $v'(\hat{g}_i + \sum_{j\neq i} \hat{g}_j) < c'(\hat{g}_i + \sum_{j\neq i} \hat{g}_j)$ for each $i \in N$. Since $\hat{g} > 0$, at least one $k \in N$ exists such that $\hat{g}_k > 0$. Since $v'(\hat{g}_k + \sum_{j\neq k} \hat{g}_j) < c'(\hat{g}_k + \sum_{j\neq k} \hat{g}_j)$, the bargaining surplus between the supplier and consumer k increases by decreasing the level of the public good a little from $\hat{g}_k$, which contradicts $(\hat{g}_j)_{j\in N}$ being supported at an equilibrium.